Heat Transfer to Power Law Non-Newtonian Fluids Flowing across Tube Bank: An Analytical Approach

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ABSTRACT: Analytical analysis of short contact heat transfer in tube banks has been presented. This analysis is based on the fact that the fluid while flowing across the cylindrical tubes in a bundle is not in continuous contact with the tube surface but comes in contact for a short period only. Therefore, the thermal boundary layer, formed on the tube surface is never fully developed. Consequently, its thickness is so small as compared to the hydrodynamic boundary layer that resistance to heat transfer is confined to a very thin region near the tube surface. This analysis resulted into the following relationship:

Nu = $1.85(\Delta)^{1/3}(\text{Re})^{1/3}(\text{Pr})^{1/3}(\text{EF})^{1/3}$

where, $\Delta=\left[\frac{2n+1}{3n}\right]$ is a non-Newtonian factor

EF = $\frac{D_H}{X_s}=\left[\frac{4}{\pi}\,\frac{\varepsilon}{1-\varepsilon}\right]$ is an entrance factor which accounts for the flow length X_s over the surface.

The following correlation was obtained from experimental data for laminar and turbulent flow heat transfer across present tube bank.

$$\text{Nu.Pr}^{-1/3}(\Delta)^{-1/3}\left\{\frac{4}{\pi}\left(\frac{\varepsilon}{1-\varepsilon}\right)\right\}^{-1/3} = 0.02 + 0.26\,\text{Re}^{1/3} + 0.097\,\text{Re}^{2/3}$$

Key Words: cross flow, entrance factor, heat transfer, Nusselt number, void fraction,

1. INTRODUCTION

The heat exchangers used mostly in industries, consist of a number of parallel tubes (forming tube bank) in a single shell. In cross flow heat exchangers, the flow directions are generally perpendicular to each other. With the growth of process industries such as petroleum, textile, pulp paper and pharmaceutical etc., these equipments find an important use in processing rheological complex fluids, better known as non-Newtonian fluids. In both in-line and staggered square tube arrangements in tube banks, a boundary layer that forms on the forward portion of the tube ultimately separates from the rear portion of the tube surface producing a turbulent wake. The fluid stream after contacting a part of the tube surface finds a gap and then moves in case of in-line tube arrangement or a sinusoidal path in case of staggered tube arrangement. Due to the short surface contact for a short duration the temperature distribution does not attain a fully developed profile and the gradient of temperature with respect to the direction perpendicular to the flow remains confined to a very thin region near to the surface. This reality of physical flow phenomenon and heat transfer has been taken into consideration in this empirical approach.

2. PRESENT APPROACH:

2.1 ANALYTICAL:

In the present work, concept of the converging diverging model for predicting pressure drop has been extended to predict the heat transfer from tubes in the tube bank. The flow pattern in tube banks is affected due to the presence of neighbouring tubes. As there is steady state flow of incompressible fluids over a horizontal plate channel in x-direction and effect due to viscous friction on temperature is negligible, hence pertinent heat transfer differential equations is as follows:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] \qquad (1)$$

The visualization of the physical picture of cross flow and heat transfer patterns from tube surfaces immersed in the fluid in a tube bank reveals that:

- A fluid stream after contacting a part of the tube surface finds a gap and then moves to the next tube following either a slightly curved path in case of in-line tube arrangement or a sinusoidal path in case of staggered tube arrangement, as substantiated by the visualization of flows given by Lohrish, W.L,(1929).

- The flowing fluid is not in continuous contact with the heat transfer surfaces but experiences a short contact for short duration. The temperature distribution does not attain a fully developed profile and the gradient of temperature with respect to the direction perpendicular to the flow remains confined to a very thin region near to the surface.

Thus, $v << u$ and $\frac{\partial^2 T}{\partial x^2} << \frac{\partial^2 T}{\partial y^2}$

Hence, the steady state differential heat transfer equation (1) reduces to

$$u\frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (2)$$

Assuming a high velocity across a relatively short contact surface of cylinders and the velocity distribution in the boundary layer confined to a thin region over the tube surface for laminar flow of a constant property fluid, the velocity profile becomes:

$$u = \left[\frac{\partial u}{\partial y}\right]_w y \quad (3)$$

And consequently equation (2) takes the form:

$$\left[\frac{\partial u}{\partial y}\right]_w y \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (4)$$

With the following boundary conditions,

at $x = 0$; $y > 0$; $T = T_b$

and at $x > 0$; $y = 0$; $T = T_w$,

transforming the partial differential equation (4) to an ordinary differential equation with the introduction of a new variable 'X', we get:

$$\frac{d^2T}{dX^2} + 3X^2 \frac{dT}{dX} = 0 \quad (5)$$

where,

$$X = y\left[\frac{\beta_v}{9\,\alpha\, x}\right]^{1/3} \text{ and } \beta_v = \left[\frac{du}{dy}\right]_w \quad (6)$$

$$\frac{\partial X}{\partial y} = \left[\frac{\beta_v}{9\,\alpha\, x}\right]^{1/3} \quad (7)$$

and the boundary conditions change to

$$\begin{Bmatrix} \text{at } X = \infty, & T = T_b \\ \text{at } X = 0, & T = T_w \end{Bmatrix}$$

Integration of equation (5) and evaluation of the integration constant, give following equation:

$$\frac{T - T_w}{T_b - T_w} = \frac{\int_0^X e^{-X^3} dX}{\int_0^\infty e^{-X^3} dX} = \frac{\int_0^X e^{-X^3} dX}{0.893} \quad (8)$$

because, evaluation of $\int_0^\infty e^{-x^3}\,dx$, using numerical technique, has been found to be 0.893.

Differentiating terms on both sides of the equation (8), temperature gradient at the wall becomes:

$$\frac{\partial T}{\partial X} = \frac{(T_b - T_w)}{0.893} e^{-x^3} \quad (9)$$

At constant x: $\frac{\partial T}{\partial y} = \frac{\partial T}{\partial X}\frac{\partial X}{\partial y}$ (10)

Substituting values from equation (9) and (7) in equation (10) we get,

$$\frac{\partial T}{\partial y} = \left[\frac{\beta_v}{9\,\alpha\, x}\right]^{1/3} \frac{(T_b - T_w)}{0.893} e^{-X^3} \quad (11)$$

Hence the temperature gradient at the wall(where, at y = 0, x = 0) becomes,

$$\left[\frac{\partial T}{\partial y}\right]_{y=0} = \left[\frac{\beta_v}{9\,\alpha\, x}\right]^{1/3} \left(\frac{T_b - T_w}{0.893}\right) \quad (12)$$

Taking h_x as heat transfer coefficient, i.e. heat transfer per unit time per unit area,

Then,

$$h_x(T_w - T_b) = -k\left[\frac{\partial T}{\partial y}\right]_{y=0} = -k\left[\frac{\beta_v}{9\,\alpha\, x}\right]^{1/3}\left(\frac{T_b - T_w}{0.893}\right) \quad (13)$$

Therefore, local heat transfer coefficient h_x is worked out as:

$$h_x = \frac{k}{0.893}\left[\frac{\beta_v}{9\,\alpha\, x}\right]^{1/3} \quad (14)$$

The equation (14) represents the local heat transfer coefficient at any position x. The velocity at the wall is constant for steady state fully developed laminar flow through closed channels of uniform cross section. In the present case, where the fluid flowing across the space between the two parallel tubes is of converging and then diverging cross section, the velocity gradient at the wall changes with the span of the cross section and therefore becomes a function of the curved path x_1 at position x along x- coordinate. Refer Fig.1

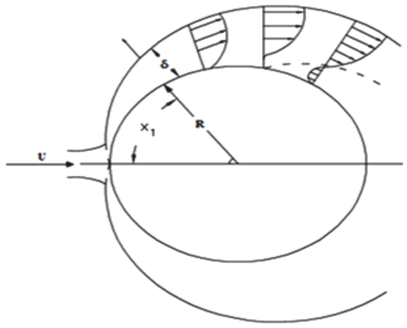

FIG. 1 DEVELOPMENT OF BOUNDARY LAYER ON A CIRCULAR CYLINDER

Assuming the width of the flow section at x_1 to be D_x and the velocity gradient β_v is approximated to be equal to gap D_x between the two plates. In a parallel plate channel, the fully developed velocity profile for laminar flow of a non-Newtonian power law fluid can be given by the following equation:

$$u = \frac{2n+1}{n+1} U\left[1-\left\{\frac{(D_x/2)-y}{D_x/2}\right\}^{\frac{n+1}{n}}\right] \text{ and } \left[\frac{du}{dy}\right]_{y=0} = \frac{2n+1}{3n}\left[\frac{12\,U}{2\,D_x}\right] \quad (15)$$

Inserting equation (15) for β_V at x_1 in equation (14) we get

$$h_{x_1} = \frac{k}{0.893}\left[\frac{2n+1}{3n}\right]^{1/3}\left[\frac{6\,U_x}{D_x}\,\frac{1}{9\,\alpha\,x_1}\right]^{1/3} \quad (16)$$

The velocity $U_x = (Q / D_x)$, where Q is volumetric flow rate per unit depth of the channel.

Inserting the value of U_x in equation (16) we get

$$h_{x_1} = \frac{k}{0.893}\left[\frac{2\,Q}{3\,\alpha}\right]^{1/3}\left[\frac{1}{x_1\;D_X^2}\right]^{1/3}\left[\frac{2n+1}{3n}\right]^{1/3} \quad (17)$$

In order to obtain the average heat transfer coefficient a heat balance is made assuming that the bulk temperature does not change appreciably during the short contact period. Let h_{av} be the average heat transfer coefficient and the length of contact area be equal to x_s such that

$$h_{av}\; x_s(T_w - T_b) = \int_0^{x_s} h_{x_1}\;(T_w - T_b)\, dx_1$$

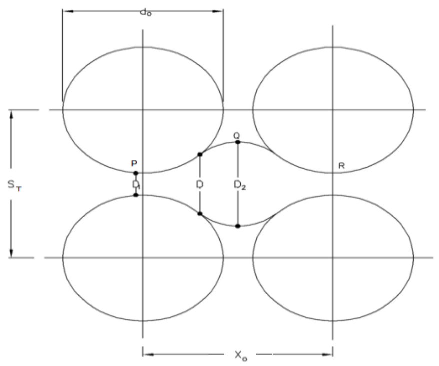

FIG. 2 CONVERGING DIVERGING PARALLEL PLATE CHANNEL MODEL

$$h_{av} = \frac{1}{x_s}\int_0^{x_0} h_{x_1}\; dx_1 = \frac{1}{x_s}\int_0^{x_0} h_{x_1}\;\frac{dx_1}{dx}\,dx \quad (18)$$

where, $x_s = \int_0^{x_0} \frac{dx_1}{dx}\,dx$

In converging-diverging channel model the surface area of the channel equals a part of the surface area of the tube involved in the heat transfer process. For example referring Fig. 2 the length PQ =1/2 (PQR) = x_s and the minimum cross sectional area is $D_1 = (S_T - d_0)$.

The gap D_x between the walls of the hypothetical continuous channel may be represented by the following expression:

$$D_x = \frac{D_1}{1-b}\left[1 - b\cos 2\pi\frac{x}{x_0}\right] \quad (19)$$

where, $b = \frac{D_2 - D_1}{D_2 + D_1}$

$$\frac{dD_X}{dx} = \frac{2\pi\, b\, D_1}{x_0(1-b)}\,\sin 2\pi\left(\frac{x}{x_0}\right) \quad (20)$$

$$\frac{dy}{dx} = \frac{1}{2}\frac{dD_X}{dx} \quad (21)$$

$$ds = \left[1 + \left\{\frac{dy}{dx}\right\}^2\right]^{1/2} dx$$

$$\text{or } ds = \left[1 + \left\{\frac{1}{2}\frac{dD_x}{dx}\right\}^2\right]^{1/2} dx \quad (22)$$

Equations (17) and (22) are substituted for h_{x_1} and (ds/dx) respectively in equation (18) and after simplification we get:

$$h_{av} = \left[\frac{2n+1}{3n}\right]^{1/3}\frac{1}{x_s}\int_0^{x_0}\frac{k}{0.893}\left\{\frac{2\,Q}{3\,\alpha}\right\}^{1/3}\left\{\frac{1}{x_1 D_X^2}\right\}^{1/3}\left[1 + \frac{1}{4}\left\{\frac{dD_X}{dx}\right\}^2\right]^{1/2} dx$$

$$h_{av} = \left[\frac{2n+1}{3n}\right]^{\frac{1}{3}}\frac{1}{x_s}\frac{k}{0.893}\left\{\frac{2\,Q}{3\,\alpha}\right\}^{\frac{1}{3}}\frac{x_0}{2\pi}\int_0^{\theta} x_1^{-\frac{1}{3}}\,D_x^{-\frac{2}{3}}[1 + B^2\sin^2\theta]^{\frac{1}{2}}d\theta \quad (23)$$

where, $B = \frac{\pi\, b\, D_1}{x_0(1-b)}$

The length of the curved surface x_1 for any x can be readily obtained by integrating equation (22)

$$x_1 = \frac{x_0}{2\pi}\int_0^{\theta}(1 + B^2\sin^2\theta)^{1/2}\,d\theta \quad (24)$$

Binomial expansion and subsequent integration of the term in brackets in the above equation gives

$$x_1 = \frac{x_0}{2\pi}\left[\theta + \frac{B^2}{4}\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right) - \frac{B^4}{8}\left(\frac{3\theta}{8} - \frac{3\sin 2\theta}{16} - \frac{\sin^2\theta\cos\theta}{4}\right) + \cdots\right]$$

For b <<1, B <<1, higher order terms in the above equation are neglected

$$x_1 = \frac{x_0}{2\pi}\left[\theta + \frac{B^2}{4}\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)\right] \quad (25)$$

so that for $\theta = \pi$, the equation reduces to $x_s = \frac{x_0}{2}\left[1 + \frac{B^2}{4}\right]$

Inserting the value of x_1 from the equation (24), D_x from equation (19) in equation (23) we get:

$$h_{av} = \left[\frac{2n+1}{3n}\right]^{\frac{1}{3}}\frac{1}{x_s}\frac{k}{0.893}\left\{\frac{2\,Q}{3\,\alpha}\right\}^{\frac{1}{3}}\left(\frac{x_0}{2\pi}\right)^{\frac{2}{3}}\left(\frac{D_1}{1-b}\right)^{-\frac{2}{3}} x$$

$$\int_0^{\pi}\left[\theta + \frac{B^2}{2}\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right)^{-\frac{1}{3}}(1 - b\cos\theta)^{-\frac{2}{3}}(1 + B^2\sin^2\theta)^{1/2}\right]d\theta \quad (26)$$

The integral term in the equation (26) is quite complex and it is difficult to get a practical solution.

It is seen that the equation for the average heat transfer coefficient

$$h_{av} = \frac{1}{x_s}\frac{k}{0.893}\int_0^{x_s}\left[\frac{\beta_v}{9\,\alpha\,x_1}\right]^{1/3} dx_1 \quad (27)$$

contains variable x_1 and velocity gradient β_v which itself is a function of D_x. However, if we consider β_v being equal to its average value β_{va} over the surface under consideration the variable remaining inside the integral is x_1 only.

Thus,

$$h_{av} = \frac{1}{x_s}\frac{k}{0.893}\left(\frac{\beta_{va}}{9\,\alpha}\right)^{1/3}\int_0^{x_s} x_1^{-1/3}\,dx_1 \quad (28)$$

$$\text{Or, } h_{av} = \frac{1}{x_s}\frac{k}{0.893}\left(\frac{\beta_{va}}{9\,\alpha}\right)^{1/3}\frac{3}{2}(x_s)^{2/3} \quad (29)$$

$$\text{Or, } h_{av} = \frac{3}{2}\frac{k}{0.893}\left(\frac{\beta_{va}}{9\,\alpha x_s}\right)^{1/3} \quad (30)$$

From equations (7) and (15)

$$\beta_v = \left(\frac{2n+1}{3n}\right)\left(\frac{12U}{2D_x}\right)$$

and hence,

$$\beta_{va} = \left(\frac{2n+1}{3n}\right)\left(\frac{12U}{D_H}\right) \quad (31)$$

Multiplying the equation (30) by mean hydraulic diameter D_H and then inserting the value of β_{va} from equation (31) we get

$$\frac{h_{av}D_H}{k} = \frac{3}{2}\left[\frac{D_H}{0.893}\left\{\left(\frac{2n+1}{3n}\right)\left(\frac{12U}{D_H}\right)\left(\frac{1}{9\alpha}\right)\left(\frac{1}{x_s}\right)\right\}^{1/3}\right] \quad (32)$$

Adjusting the term of the right hand side of the equation (32) into various non-dimensional groups, we get

$$\frac{h_{av}D_H}{k} = \frac{3}{2}\frac{1}{0.893}\left(\frac{12}{9}\right)^{1/3}\left(\frac{D_H}{\mu_{eff}}\frac{U\,\rho\,\mu_{eff}}{\rho\,\alpha}\frac{D_H}{x_s}\right)^{1/3}\left(\frac{2n+1}{3n}\right)^{1/3} \quad (33)$$

where, effective viscosity is

$$\mu_{eff} = K\left(\frac{2n+1}{3n}\right)^n\left(\frac{12U}{D_H}\right)^{n-1} \quad (34)$$

$$Nu = \frac{h_{av}D_H}{k} = 1.85\left(\frac{2n+1}{3n}\right)^{1/3} Re^{1/3}.Pr^{1/3}\left(\frac{D_H}{x_s}\right)^{1/3} \quad (35)$$

$$Nu = 1.85(\Delta)^{1/3}(Re)^{1/3}(Pr)^{1/3}(EF)^{1/3} \quad (36)$$

where, $\Delta = \left(\frac{2n+1}{3n}\right)$ is an non-Newtonian factor and EF $= \left(\frac{D_H}{x_s}\right)$ is an entrance factor which takes into account the flow length of the surface.

2.2 EXPERIMENTAL:

In the present work an attempt has been made to incorporate the effects of fluid rheology, tube diameter and tube configuration on the heat transfer phenomenon. The heat transfer measurements were carried out in a closed loop circulation rig with removable tubes (active and dummy both) such that different heat exchanger configurations could be tested.

2.2.1 OUTLINE OF THE EXPERIMENTAL PROGRAMME:

Heat transfer data have been obtained by measuring local surface temperatures of heat flux-probe and the bulk inlet and outlet temperature of the test fluid. The effect of neighbouring tubes on heat transfer from the heated tube was studied by placing dummy tubes around the heater tube one by one as shown in Fig. 4. Experiments were conducted using five different tube arrangements (discussed in details at the end of this chapter). In order to investigate the effect of tube diameter (or p/d) on heat transfer, different tubes of diameter 3.18 cm, 2.54 cm and 1.91 cm have been used.

2.2.2 TEST FLUID:

Water was selected as the test fluid because of its well known physical and thermal properties, as it is one of the most common Newtonian fluids. PVA and CMC solutions were found to obey the Ostwald power law model. These, being widely used in industries, are cheap and readily available and also for the fact that they do not suffer from thixotropy or ageing effects, were found to be the most suitable non-Newtonian test fluids. Two different concentrations of CMC solution viz. 1.0% and 1.5% and 0.5% PVA solutions were prepared.

2.2.3 EXPERIMENTAL SET UP:

The entire test rig may be divided into three major units:

(i) Test Section
(ii) Heating Arrangement
(iii) Heat Transfer & Flow measurement unit.

2.2.3.1 TEST SECTION:

The test section of dimension 23.5 cm (length) x 18.0 cm (width) x 14.7 cm (height) was fabricated with perspex sheets. Three tubes of diameter 2.54 cm were half exposed on either side walls. The test section was so fabricated that it could accommodate desired number of tubes of different diameters. The top portion of the test section was provided with through holes of diameter 3.18 cm while in the bottom portion, partial grooves were provided to support the base of

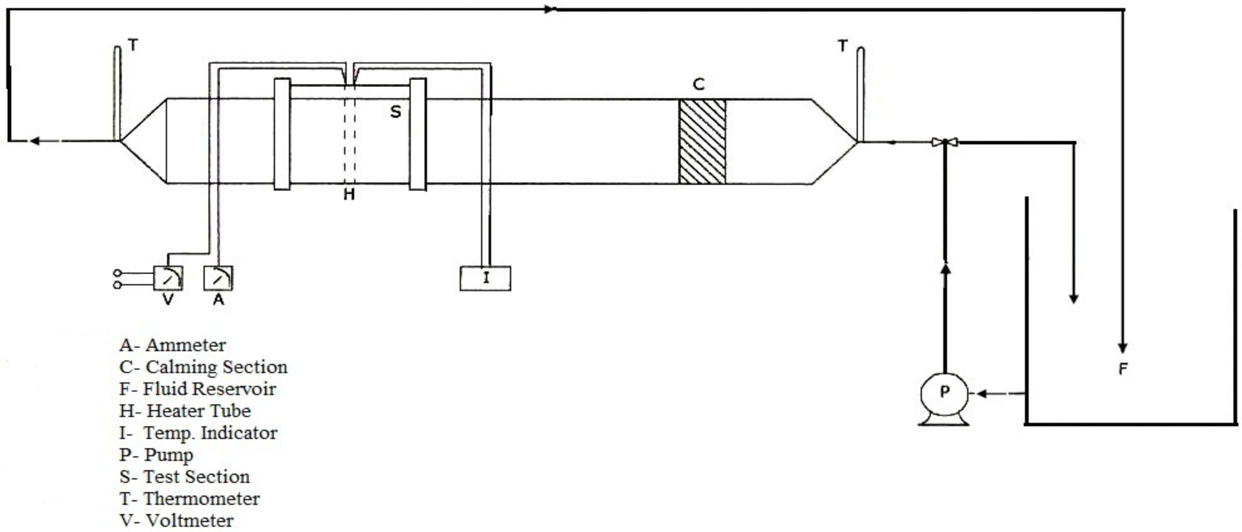

FIG. 3 SCHEMATIC DIAGRAM OF EXPERIMENTAL SET-UP

The schematic diagram of the experimental rig is shown in Fig.3. It consisted of a 2.0 H.P. pump (P), connected through 2 inch dia pipe to a steel reservoir (R) of size 122 cm length, 92 cm width and 108 cm depth. A $1\frac{1}{2}$ inch dia. bypass (B) was provided at the delivery side of the pump to regulate the flow. The entrance portion of the test rig had divergent portion in shape which finally matches the dimension of test section. A calming section was provided wire net at the entrance to eliminate the entrance effect and also to reduce eddies and reversal of flow.

the tubes against the pressure of flowing fluids. To cover the top portion of the test section a top cover plate made of perspex sheet was provided. The top cover plate was fastened with nuts and bolts thus making it removable, so that, whenever required the top cover plate could be removed and required number of dummy tubes could be placed in the test section.

The heat flux probe or active tube was tightly fitted in the removable top cover plate and its location was so adjusted that it acquired the central position in the test section. Arrangement was made to accommodate any of different sized tubes of three different diameters in the present work. To accommodate the corresponding dummy tube, plastic adopters (plugs) were used to accommodate the tubes in the top and bottom plates.. The heat flux probe was fabricated from a copper tube and a pencil heater was inserted inside the copper tube to fit in the tube tightly. Four slots of different length were cut at the outer surface of the heater rod which were at the location 0^0, 90^0, 180^0 and 270^0 i.e., diametrically opposite to each other. Copper-constantan thermocouple wire, embedded in each slot, measured the local surface temperature of the heated tube, which, when averaged gave the average surface temperature of the heating probe. To study the effect of neighbouring tubes an arrangement with longitudinal pitch S_L = 7.0 cm and transverse pitch S_T = 3.5 cm was used.

2.2.3.2. HEATING ARRANGEMENT:

A 300 watt pencil heater was placed inside the copper heating probe. The heater tube was provided an electrical heat flux through a rheostat. An ammeter (A) of range 0-2 Amp. and a voltmeter (V) of range 0-230 V were used to measure the current and voltage.

2.2.3.3. HEAT TRANSFER AND FLOW MEARSUREMENT UNIT:

The flow rate of the fluid was measured by collecting the fluid in a container for a known interval of time and weighing it on a balance having an accuracy of 2 gm.Two thermometers with accuracy of 0.1^0 C were used to measure the inlet and outlet temperatures of the fluid flowing through the test section. The average value of inlet and outlet temperature gave the bulk temperature of the fluid flowing across the tubes. A calibrated digital temperature indicator having a range of $0\text{-}300^0$C and a accuracy of 0.1^0C

was used to record the surface temperature of the heater tube through four thermocouples fitted at different locations on the tube surface.

2.2.4 EXPERIMENTAL PROCEDURE:

For a particular tube diameter and tube configuration the flow rate of a particular fluid was initially set to a predetermined value. A constant electrical heat flux was supplied to the heater tube. After about an interval of 15 minutes, when steady state was achieved, the temperature of the four thermocouples placed at four different locations on the heater tube were recorded with the help of a multi-channel digital temperature indicator. Simultaneously the inlet and outlet temperatures of the fluid were also noted.

The flow rate was varied and similar sets of data were recorded. This procedure was repeated for each tube diameter and tube arrangement. Similar sets of observations were made using different fluids for each configuration.

2.2.5 TUBE ARRANGEMENTS AND CORRESPONDING NOMENCLATURE:

Following tube arrangements and corresponding nomenclature have been used for this experimental study:

Single heater tube – S

Test cylinder with a dummy tube in front –F

Test cylinder with a dummy tube in front and one on the right hand side – FR

Test cylinder with a dummy tube in front and one each on left and right hand side – FRL

Test cylinder with dummy tubes one each in front, back, on left and right hand side of heater tube – FRLB

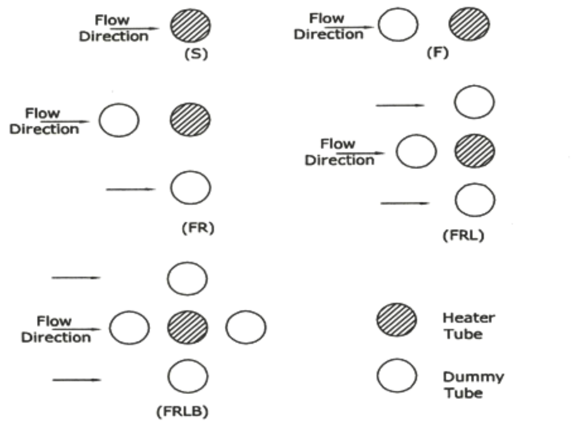

FIG. 4 ARRANGEMENT OF DUMMY TUBES AROUND HEAR TUBE

3.0 CORRELATION BASED ON EXPERIMENTAL DATA AND CONCLUSION:

Short contact heat transfer in tube banks has been proposed in the analysis here. This analysis is based on the fact that the fluid while flowing across the cylindrical tubes in a bundle is not in continuous contact with the tube surface but comes in contact for a short period only. Therefore, the thermal boundary layer, formed on the tube surface is never fully developed. Further, its thickness is so small as compared to the hydrodynamic boundary layer that the resistance to heat transfer is confined to a very thin region near the tube surface. The analysis resulted into the following relationship:

$$Nu = 1.85\,(\Delta)^{1/3} Re^{1/3} Pr^{1/3} (EF)^{1/3} \quad (37)$$

where, $\Delta = \left[\frac{2n+1}{3n}\right]$ is a non-Newtonian factor and,

$EF = \frac{D_H}{X_S}$ is an entrance factor which accounts for the flow length over the surface.

Hydraulic diameter is given by:

$$D_H = d_o \left[\frac{\varepsilon}{1-\varepsilon}\right] \quad (38)$$

The curved length x_s for the surface $= \frac{\pi}{4} d_o$ (39)

Therefore, the shape factor (EF), becomes:

$$EF = \frac{D_H}{X_s} = \left[\frac{4}{\pi}\,\frac{\varepsilon}{1-\varepsilon}\right] \quad (40)$$

Thus the final form of the equation is obtained as:

$$Nu = 1.85\,(\Delta)^{1/3} Re^{1/3} Pr^{1/3} \left[\frac{4}{\pi}\,\frac{\varepsilon}{1-\varepsilon}\right]^{1/3} \quad (41)$$

The value of the function $Nu.Pr^{-1/3} (\Delta)^{-1/3} [(4/\pi)\{\varepsilon/(1-\varepsilon)\}]^{-1/3}$ have been plotted against Re in (Fig. 5). A close observation of the plot reveals that there is a gradual increase in the slope with increasing Reynolds number. The value of the slope ranges from 1/3 to 2/3. At lower Reynolds numbers, there is considerable effect of convective heat transfer. To account for this, an analysis of the lower Reynolds number data ($Re \leq 60$) has been made which has resulted in the correlation of the following form:

$$Nu.Pr^{-1/3} (\Delta)^{-1/3} \left\{\frac{4}{\pi}\left(\frac{\varepsilon}{1-\varepsilon}\right)\right\}^{-1/3} = A_o + B_o\, Re^{1/3} \quad (42)$$

where value of A_o is found to be 0.02.

Although the short contact heat transfer analysis is applicable for low Reynolds number range, however, it can be extended to incorporate the data for turbulent regions also. For this purpose, a graph between the function $[Nu.Pr^{-1/3} (\Delta)^{-1/3} \{(4/\pi)\,(\varepsilon/(1-\varepsilon))\}^{-1/3} - A_o]\, Re^{-1/3}$ has been plotted against $Re^{1/3}$ (Fig. 6).

This resulted into following form of correlation:

$$\left[Nu.Pr^{-1/3} (\Delta)^{-1/3} \left\{\frac{4}{\pi}\left(\frac{\varepsilon}{1-\varepsilon}\right)\right\}^{-1/3} - A_o\right] Re^{-1/3} = B_o + C_o\, Re^{1/3} \quad (43)$$

Value of B_o and C_o is found to be 0.26 and 0.097, respectively and the equation takes the form:

$$Nu.Pr^{-1/3} (\Delta)^{-1/3} \left\{\frac{4}{\pi}\left(\frac{\varepsilon}{1-\varepsilon}\right)\right\}^{-1/3} = A_o + B_o\, Re^{1/3} + C_o\, Re^{2/3} \quad (44)$$

Therefore, the correlation obtained, which is applicable for both laminar and turbulent regimes, is

$$Nu.Pr^{-1/3} (\Delta)^{-1/3} \left\{\frac{4}{\pi}\left(\frac{\varepsilon}{1-\varepsilon}\right)\right\}^{-1/3} = 0.02 + 0.26\, Re^{1/3} + 0.097\, Re^{2/3}$$

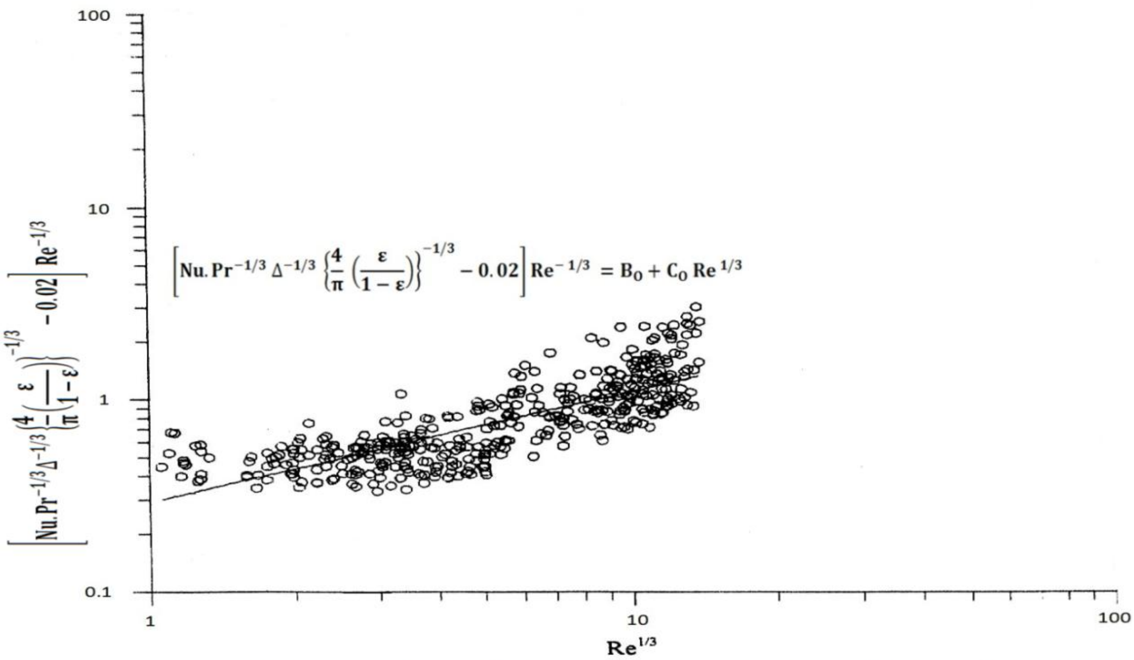

FIG. 6 HEAT TRANSFER FROM VARIOUS TUBE ASSEMBLAGES: PLOT OF $\left[Nu.Pr^{-1/3}\Delta^{-1/3}\left\{\frac{4}{\pi}\left(\frac{\varepsilon}{1-\varepsilon}\right)\right\}^{-1/3} - 0.02\right] Re^{-1/3}$ VERSUS Re1/3

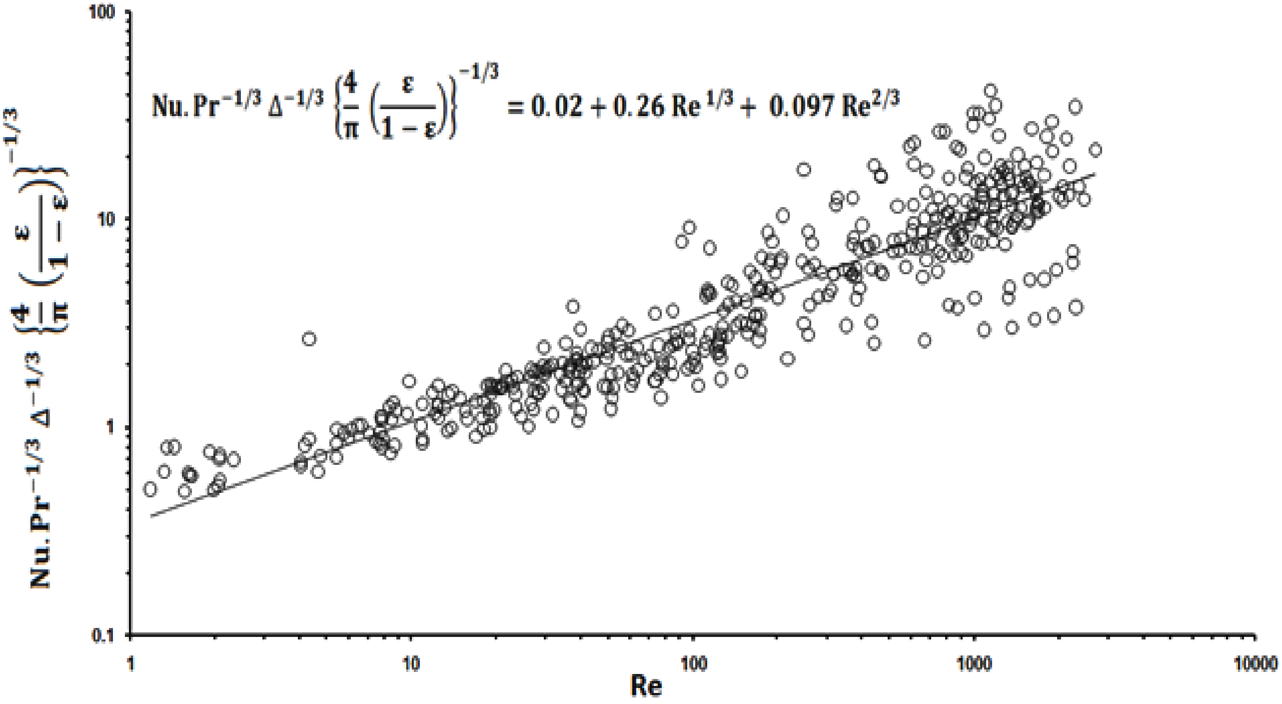

FIG. 5 GENERAL CORRELATION FOR HEAT TRANSFER FROM TUBE ASSEMBLAGES: PLOT OF $Nu.Pr^{-1/3}\Delta^{-1/3}\left\{\frac{4}{\pi}\left(\frac{\varepsilon}{1-\varepsilon}\right)\right\}^{-1/3}$ VERSUS REYNOLDS NUMBER

4. NOMENCLATURE

C_p specific heat at constant pressure, kJ/kg °C
d_o outside diameter of the tubes in the bank, m

D_X gap between two plates at any axial position in the converging-diverging parallel plate channel model, m
D_H hydraulic diameter of tube bank $\frac{d_o \varepsilon}{1-\varepsilon}$, m.
D_1 minimum gap at x = 0, m
D_2 maximum gap at x = $\frac{X_o}{2}$ m
g acceleration due to gravity, m^2/sec
g_c conversion factor, kg-m/kg_f sec^2
h heat transfer coefficient, W/m^2 °C
h_{av} average convective heat transfer coefficient, W/m^2 °C
h_{x_1} convective heat transfer coefficient at curved distance x_l from the reference point, W/m^2°C
k thermal conductivity, W/m^2 °C
K power law consistency constant, kg/m $sec^{(2-n)}$
L length of the tube bank, m
n flow behavior index
Q volumetric flow rate, m^3 /sec
S_L longitudinal pitch or spacing of longitudinal rows, m
S_T transverse pitch, m
t time, sec
T temperature, °C
ΔT temperature difference, °C
T_b bulk temperature of the fluid, °C
T_w wall temperature of the fluid, °C
u velocity component in x-direction, m/sec
U velocity, m/sec
U_s superficial, m/sec
v velocity component in y- direction, m/sec
x axial position in x direction, m
x_1 length of curved surface (Fig.1)
x_s 1/2 of length along curve PQR (Fig.2)
y axial position in y- direction, m

DIMENSIONLESS GROUPS:
Nu Nusselt number
Pr Prandtl number
Re Reynolds number

GREEK LETTERS:
α thermal diffusivity
β_V velocity gradient = $\left[\frac{du}{dy}\right]_w$
δ boundary layer thickness, m
Δ $\frac{2n+1}{3n}$ for parallel plate channel model
ε void fraction calculated from tube spacing and geometry
μ coefficient of viscosity, kg/m sec
μ_{eff} effective viscosity based on parallel plate tube model, :
μ_w viscosity at wall temperature, Ns $/m^2$
ρ density of the fluid, kg/m^3
θ angular position with respect to front stagnation point, degrees

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