Fuzzy strongly g*- Super closed Sets

M.K. Mishra[1],

M.Shukla[2]

Professor EGS PEC Nagapattinam,

Asst. Prof Arignar Anna College Karaikal

Abstract:

In this paper, we introduce and investigate the concept of fuzzy strongly generalized g* - Super closed sets (briefly fuzzy strongly g*-Super closed set) and investigate the relation between the associated fuzzy topology.

Keywords: fuzzy super closure, fuzzy super interior, fuzzy super open set, fuzzy super closed set g*-Super closed set, Fuzzy Strongly g*-super closed set

1. Preliminaries

Let X be a non empty set and I= [0,1]. A fuzzy set on X is a mapping from X in to I. The null fuzzy set 0 is the mapping from X in to I which assumes only the value is 0 and whole fuzzy sets 1 is a mapping from X on to I which takes the values 1 only. The union (resp. intersection) of a family $\{A_\alpha: \alpha\in\Lambda\}$of fuzzy sets of X is defined by to be the mapping sup A_α (resp. inf A_α) . A fuzzy set A of X is contained in a fuzzy set B of X if $A(x) \leq B(x)$ for each $x\in X$. A fuzzy point x_β in X is a fuzzy set defined by x_β (y) = β for y=x and x(y) =0 for $y \neq x$, $\beta\in[0,1]$ and $y \in X$ .A fuzzy point x_β is said to be quasi-coincident with the fuzzy set A denoted by $x_{\beta q}A$ if and only if $\beta + A(x) > 1$. A fuzzy set A is quasi – coincident with a fuzzy set B denoted by A_qB if and only if there exists a point $x\in X$ such that $A(x) + B(x) > 1$.$A \leq B$ if and only if $\rceil(A_qB^c)$.

A family τ of fuzzy sets of X is called a fuzzy topology [2] on X if 0,1 belongs to τ and τ is super closed with respect to arbitrary union and finite intersection .The members of τ are called fuzzy super open sets and their complement are fuzzy super closed sets. For any fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy super closed super sets of A and the interior of A (denoted by int(A))is the union of all fuzzy super open subsets of A.

Defination1.1:A subset A of a fuzzy topological space (X, τ) is called

1. Fuzzy Super closure $scl(A)=\{x\in X:cl(U)\cap A\neq\phi\}$
2. Fuzzy Super interior $sint(A) =\{x\in X:cl(U)\leq A\neq\phi\}$
3. Fuzzy super closed if $scl(A) \leq A$.
4. Fuzzy super open if 1-A is fuzzy super closed sint(A)=A

5. Fuzzy pre super open set if $A \leq int(cl(A))$ and fuzzy pre-Super closed set if $cl(int(A)) \leq A$.
6. Fuzzy semi super open set if $A \leq cl(int(A))$ and fuzzy semi super closed set if $int(cl(A)) \leq A$.
7. Fuzzy α-Super open set if $A \leq int(cl(int(A)))$ and an fuzzy α-Super closed set if $cl(int(cl(A))) \leq A$.
8. Fuzzy semi pre super open set (fuzzy β- A subset A of a fuzzy topological space (X,τ) is called super open set) if $A \leq cl(int(cl(A)))$ and fuzzy semi-pre super closed set if $int(cl(int(A))) \leq A$.
9. fuzzy g- super closed if $cl(A) \leq G$ whenever $A \leq G$ and G is super open.
10. fuzzy g- super open if its complement 1-A is fuzzy g- super closed.
11. Fuzzy δ- super closed set if $A = cl_\delta(A)$ where $cl_\delta(A) = \{x \in X : int(cl(U)) \cap A \neq \phi, U \in \tau \text{ and } x \in U\}$.
12. Fuzzy θ- super closed set if $A = cl_\theta(A)$ where $cl_\theta(A) = \{x \in X : (cl(U)) \cap A \neq \phi, U \in \tau \text{ and } x \in U\}$.
13. Fuzzy g-super closed if $cl(A) \leq U$, whenever $A \leq U$ and U is fuzzy super open in (X,τ).
14. Fuzzy semi generalized super closed set (briefly fuzzy sg-super closed) if $scl(A) \leq U$, whenever $A \leq U$, U is fuzzy semi super open in (X, τ).
15. Fuzzy generalized semi-super closed set (briefly gs-super closed) if $scl(A) \leq U$ whenever $A \leq U$, U is fuzzy super open in (X, τ).
16. Fuzzy generalized α -super closed (briefly fuzzy gα -super closed) if $\alpha cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy α-Super open in (X,τ).
17. Fuzzy α generalized super closed set (briefly fuzzy αg-super closed) if $\alpha cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy super open in (X, τ).
18. Fuzzy generalized semi pre-super closed set (briefly fuzzy gsp-super closed) if $spcl(A) \leq U$ whenever $A \leq U$ and fuzzy U is super open in (X, τ).
19. Fuzzy regular generalized super closed set (briefly fuzzy r-g-super closed) if $cl(A) \leq U$ whenever $A \leq U$ and U is fuzzy regular super open in(X, τ).
20. Fuzzy generalized pre super closed set (briefly fuzzy gp-super closed) if $pcl(A) \leq U$, whenever $A \leq U$ and U is fuzzy super open in (X, τ).
21. Fuzzy generalized pre regular super closed set (briefly fuzzy gpr-super closed) if $pcl(A) \leq U$, whenever $A \leq U$ and U is fuzzy regular Super open in (X, τ).
22. Fuzzy θ generalized super closed set (briefly fuzzy θg-super closed) if $cl_\theta \leq U$ whenever $A \leq U$ and U is fuzzy super open in (X, τ).
23. Fuzzy δ-generalized super closed set (briefly fuzzy δg super closed) if $cl_\delta(A) \leq U$ whenever $A \leq U$ and U is fuzzy super open in (X, τ).

24. Fuzzy g* -super closed set if cl(A) ≤ U whenever A ≤ U and U is fuzzy g-super open in (X, τ).

Remark 1.1[5]:- Every fuzzy closed set is fuzzy super closed but the converses may not be true.

Remark 1.2[5]:- Let A and B are two fuzzy super closed sets in a fuzzy topological space (X,$\Im$), then A ∪ B is fuzzy super closed.

Remark 1.3[5]:- The intersection of two fuzzy super closed sets in a fuzzy topological space (X,$\Im$) may not be fuzzy super closed.

2 Fuzzy Strongly g*-super closed sets

In this section we have introduce the concept of fuzzy strongly g*-Super closed sets in topological space and we investigate the group of structure of the set of all fuzzy strongly g*-super closed sets.

Definition 2.1: Let (X,τ) be a fuzzy topological space and A be its subset , then A is fuzzy strongly g*-super closed set if cl(int(A)) ≤ U whenever A ≤ U and U is fuzzy g-Super open.

Theorem 2.1: Every super closed set is Fuzzy strongly g*-super closed set.

Proof. The proof is immediate from the definition of super closed set.

Proof: Suppose A is fuzzy g*-super closed in X. Let G be a fuzzy super open set containing A in X. Then G contains cl(A). Now G ≥cl(A) ≥cl(int(A)). Thus A is fuzzy strongly g*-super closed in X.

Theorem 2.3: If A is a subset of a fuzzy topological space X is super open and fuzzy strongly g*-super closed then it is fuzzy super closed.

Proof: Suppose a subset A of X is both fuzzy super open and Fuzzy strongly g*-super closed. Now A ≥cl(int(A)) ≥ cl(A). Therefore A ≥cl(A). Since cl(A) ≥A. We have A ≥cl(A). Thus A is fuzzy super closed in X.

Corollary 2.1: If A is both super open and fuzzy strongly g*-super closed in X then it is both fuzzy regular super open and fuzzy regular super closed in X.

Proof: As A is Super open A = int(A) = int(cl(A)), since A is fuzzy Super closed . Thus A is fuzzy regular super open . Again A is fuzzy super open in X, cl(int(A))=cl(A). As A is fuzzy super closed cl(int(A))=A. Thus A is fuzzy regular super closed.

Corollary 2.2: If A is both fuzzy Super open and Fuzzy strongly g*-super closed then it is fuzzy rg-super closed.

Theorem 3.4: If a subset A of a fuzzy topological space X is both fuzzy strongly g*-super closed and fuzzy semi super open then it is fuzzy g*-super closed.

Proof: Suppose A is both fuzzy strongly g*-Super closed and fuzzy semi super open in X, Let G be a fuzzy super open set containing A. As A is fuzzy strongly g*-super closed, G ≥ cl(int(A)). Now G ≥ cl(A). Since A is fuzzy semi super open. Thus A is fuzzy g*-super closed in X.

Corollary 3.3: If A subset A of a fuzzy topological space X is both fuzzy strongly g*-super closed and fuzzy super open then it is fuzzy g*-super closed set.

Proof: As every super open set is semi super open by the above theorem the proof follows.

Theorem 2.5: A set A is fuzzy strongly g*-super closed iff cl(int(A))−A contains no non empty fuzzy super closed set.

Proof: Necessary: Suppose that F is non empty fuzzy super closed subset of cl(int(A)). Now F ≤ cl(int(A)) − A implies F ≤cl(int(A)) $\cap$ A^c, since cl(int(A)) − A = cl(int(A)) $\cap$ A^c. Thus F ≤ cl(int(A)). Now F ≤ A^c implies A ≤ F^c. Here F^c is fuzzy g-Super open and A is fuzzy strongly g*-super closed, we have cl(int(A)) _ F^c. Thus F ≤ $(cl(int(A)))^c$. Hence F ≤ (cl(int(A))) $\cap$ $(cl(int(A)))^c$ = ϕ Therefore F = ≤(cl(int(A)) − A contains no non empty fuzzy super closed sets.

Sufficient: Let A ≤ G, G is fuzzy g-Super open. suppose that cl(int(A)) is not contained in G then $(cl(int(A)))^c$ is a non empty fuzzy super closed set of cl(int(A)) −A which is a contradiction. Therefore cl(int(A)) ≤ G and hence A is fuzzy strongly g* -super closed .

Corollary 2.4: A fuzzy strongly g*-Super closed set A is fuzzy regular super closed iff cl(int(A)) − A is fuzzy super closed and cl(int(A)) ≤ A.

Proof: Assume A that A is fuzzy regular super closed. Since cl(int(A)) = A, cl(int(A)) −A = ϕ is fuzzy regular super closed and hence fuzzy super closed . Conversely assume that cl(int(A))−A is fuzzy super closed . By the above theorem cl(int(A))−A contains no nonempty fuzzy super closed set .Therefore cl(int(A)) − A = ϕ.Thus A is fuzzy regular super closed .

Theorem 2.6: Suppose that B ≤ A ≤ X, B is fuzzy strongly g*-super closed set relative to A and that both fuzzy super open and fuzzy strongly g * -super closed subset of X then B is fuzzy strongly g* -super closed set relative to X.

Proof: Let B ≤ G and G be a fuzzy super open set in X. But given that B ≤ A ≤ X, therefore B ≤ A and B ≤G. This implies B ≤ A $\cap$ G. Since B is fuzzy strongly g*- super closed relative to A, cl(int(B))≤A$\cap$G.(ie)A$\cap$cl(int(B))≤A$\cap$G.This implies A$\cap$(cl(int(B)))≤G.Thus (A$\cap$(cl(int(B)))) $\cup$ $(cl(int(B)))^c$ ≤ G$\cup$$(cl(int(B)))^c$ implies A $\cup$ $(cl(int(B)))^c$ ≤ G ($(cl(int(B)))^c$. since A is fuzzy strongly g*-super closed in X, we have (cl(int(A))) ≤ G $\cup$ $(cl(int(B)))^c$. Also B ≤ A $\Rightarrow$ cl(int(B)) ≤ cl(int(A)). Thus cl(int(B)) ≤ cl(int(A)) ≤ G ≤ $(cl(int(B)))^c$. Therefore B is fuzzy strongly g* -super closed set relative to X.

Corollary 2.5: Corollary 3.14: Let A be fuzzy strongly g* -super closed and suppose that F is fuzzy super closed then A $\cap$ F is fuzzy strongly g* -super closed set.

Proof: To show that $A \cap F$ is fuzzy strongly g*-Super closed, we have to show cl(int(A\F)) ≤ G whenever $A \cap F \leq G$ and G is fuzzy g-super open. $A \cap F$ is fuzzy super closed in A so it is fuzzy strongly g* -super closed in B. By the above theorem $A \cap F$ is fuzzy strongly g* -super closed in X. Since $A \cap F \leq A \leq X$.

Theorem 2.7: Theorem 3.15: If A is fuzzy strongly g* Super closed and A ≤ B≤cl(int(A)) then B is fuzzy strongly g *-super closed .

Proof: Given that B ≤cl(int(A)) then cl(int(B)) ≤ cl(int(A)), cl(int(B)) −B ≤ cl(int(A)) − A .Since A ≤ B. As A is fuzzy Strongly g* -super closed by the above theorem cl(int(A)) − A contains no non empty fuzzy super closed set, cl(int(B)) − B contains no empty fuzzy super closed set. Again by theorem 3.13, B is fuzzy strongly g*-super closed set.

Theorem 2.8: Theorem 3.16: Let A ≤ Y ≤ X and suppose that A is fuzzy strongly g* -super closed in X then A is fuzzy strongly g* -super closed relative to Y.

Proof: Given that A ≤ Y ≤ X and A is fuzzy strongly g*-super closed in X. To show that A is fuzzy strongly g*–super closed relative to Y, let $A \leq Y \cap G$, where G is fuzzy g-super open in X. Since A is fuzzy strongly g*-super closed in X,A ≤ G implies cl(int(A))≤G. (ie) $Y \cap$ cl(int(A)) ≤ $Y \cap G$, where $Y \cap$ cl(int(A)) is closure of interior of A in Y. Thus A is fuzzy strongly g* -super closed relative to Y.

Theorem 2.9: If a subset A of a fuzzy topological space X is gsp-super closed then it is fuzzy strongly g*-super closed but not conversely.

Proof: Suppose that A is fuzzy gsp- super closed set in X, let G be fuzzy super open set containing A. Then G ≥spcl(A),$A \cup G \geq A$ ((int(cl(int(A)))) which implies G ≥ int(cl(int(A))) as G is fuzzy super open. (ie)G ≥ cl(int(A)) − A is fuzzy strongly g* -super closed set in X.

Theorem 2.10: Theorem 3.19: Every fuzzy $\delta-$ super closed set is a fuzzy strongly g*-super closed set.

Proof: The Proof of the theorem is immediate from the definition.

Theorem 2.11: Every fuzzy δ -super closed set is a fuzzy strongly g*-super closed set.

Proof: The Proof of the theorem is immediate from the definition.

Theorem 2.12: Every fuzzy strongly g*-super closed set in an fuzzy α g-super closed set and hence fuzzy gs -super closed , fuzzy gsp-super closed , fuzzy gp-super closed , fuzzy gpr -super closed set and fuzzy rg -super closed set but not conversely.

Proof: Let A be a fuzzy strongly g*-super closed set of (X,τ). By above theorem, A is fuzzy g-super closed. A is fuzzy αg-super closed. We know that every fuzzy g-super closed set is fuzzy gs-super closed, fuzzy gsp-super closed, fuzzy gp-super closed, fuzzy gpr-super closed and fuzzy rg-super closed. By above theorem every fuzzy strongly g*-super closed set is fuzzy gs-super closed, fuzzy gsp -super closed and fuzzy rg -super closed.

Remark 2.1: The following are the implications of fuzzy strongly g*-Super closed set.

References

[1]. A. Bhattacharya and M.N. Mukherjee, on fuzzy δ-almost continuity and δ^*-almost continuous functions. Jour. Tripura Math. Soc.2(2000) 45-58.
[2]. A.S. Bin Shahana, On fuzzy strong semi continuity and fuzzy pre continuity. Fuzzy Sets and Systems. 44(1991), 303-308.
[3]. Axad K.K. On fuzzy semi continuity fuzzy almost continuity and fuzzy weakly continuity , J.Math. Anal. Appl. 82(1981),14, 14-32.
[4]. Bin Sahana A. S. Mapping in fuzzy topological spaces fuzzy sets and systems,61(1994),209-213.
[5]. Bin Sahana A. S. on fuzzy strongly semi continuity and fuzzy pre continuity, fuzzy sets and systems 44(1991), 303-308.
[6]. C.L. Chang, Fuzzy topological spaces J. Math. Anal. Appl. 24(1968), 182-190.
[7]. Chang C.L. Fuzzy topological spaces .J. Math Anal. Appl..24(1968), 182-190.
[8]. D.Andrijevic, Semi preopen sets, Mat. Vesnik 38(1986),24-32.
[9]. George J. Klir and Bo Yuan, Fuzzy sets and fuzzy logic theory and applications Prentice Hall of India New Delhi 2003.
[10]. K.K. Azad, On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity. J. Math. Anal. Appl. 82(1981), 14-32.
[11]. L.A. Zadeh, Fuzzy Sets, Inform and control. 8(1965), 338-35.
[12]. Lin.Y. M and Lou. K.M., Fuzzy topology, World Scientific Publication Singapore(1997)
[13]. Livine N. Generalized closed sets in topology Rand. Circ Mat. Palermo, 19(2)(1970) ,571-599.
[14]. Livine N. Semi Open Sets and semi continuity in topological spaces Amer. Math. Monthly,70(1963),36-41.
[15]. Lowen R. A comparison of different compactness notion on fuzzy topological spaces, J. Math. Anal. Appl. 64(1978), 446-454.
[16]. Lowen R. Fuzzy topological Spaces and fuzzy compactness in fuzzy J. Math,Anal. Appl. 56(1976),621-633.
[17]. Mahashwari S. N. and Prasad R. on s-regular spaces Glasnik Mat. Ser.III 10(3),(1975),347-350.
[18]. Mahashwari S. N. and Prasad R. Some new separation axioms Ann. Soc. Sci. Bruxelles T.-89III(1975),395-402.
[19]. Mashour A. S. , M.F. Abd. Monsef. EI.,Deeb S.N. on ptre continuous and weak precontinious mappings,Proc. Math and Phys. Soc, Egypt53(1982),47-53.
[20]. Mishra M.K. ,et all on " Fuzzy super closed set" International Journal International Journal of Mathematics and applied Statistics.

[21]. Mishra M. K. ,et all on " Fuzzy super continuity" International Review in Fuzzy Mathematics ISSN : 0973-4392July –December2012.

[22]. Mishra M.K., Shukla M. "Fuzzy Regular Generalized Super Closed Set" International Journal of Scientific and Research Publication ISSN2250-3153. July December 2012.

[23]. Mukerjee M.N. And Sinha S.P. Almost compact fuzzy topological spaces Mat Vasnik 41(1989),89-97.

[24]. Nanda S. On fuzzy topological Spaces fuzzy sets and systems 19(2),(1986),193-197.

[25]. Nanda S. Principal super connected fuzzy topological spaces fuzzy sets and systems 35,1990),397-399.

[26]. Nanda S. Strongly compact fuzzy topological spaces fuzzy sets and systems 42,(1990),259-262.

[27]. P.M. Pu and Y.M. Liu, Fuzzy topology I. Neighbourhood structure of a fuzzy point and moore smith convergence. J.Math. Anal. Appl. 76(1980),571-599.

[28]. Palaniappan n. and Rao K.C. Regular Generalized closed sets Kyungpook Math. J.33(2),1993,211-219.

[29]. Prasad R. ,Thakur S.S. and Saraf R.K. Fuzzy α-irresoulate mapping J. Fuzzy Math 2(1994),No,2,335-339.

[30]. Pu. P.M. and Lin.Y.M., Fuzzy topology II. Product Quotient spaces. J.Math. Anal. Appl. 77(1980)20-27

[31]. Pu.P.M. and Lin.Y.M., Fuzzy topology, I.Neighbourhood structure of a Fuzzy point Moore Smith convergence.J.Math.Anal.Appl.76(1980)571-599.

[32]. S.Ganguli and S. Saha, On separation axioms and separations of connected sets in fuzzy topological spaces. Bull. Cal. Math. 79(1987),215-225.

[33]. T.H.Yalvac, Semi interior, semi closure of a fuzzy set. J. Math. Anal. Appl. 132(1988),356-364.

[34]. Wong C.K on fuzzy points and local properties of fuzzy topology J. Math Anal. Appl 46(1974)316-328.

[35]. Yalvac T.H.Fuzzy Sets and functions in fuzzy Spaces J. Math Anal. Appl 126 (1987),409-423.

[36]. Zadeh I.H on Fuzzy sets information and control 8(1965),38-353.