MATHEMATICAL MODELING AND AVAILABILITY ANALYSIS OF A CRYSTALLIZATION SYSTEM USING MARKOV PROCESS

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ABSTRACT

This paper deals with the mathematical modeling and availability analysis of crystallization system of a sugar plant. The crystallization system of sugar plant consists of three subsystems with three possible states: full working, reduced capacity working and failure. Failure and Repair rates for all the subsystems are assumed to be constant. Formulation of the problem is carried out using Markov Birth-Death process using probabilistic approach and a transition diagram represents the operational behavior of the system. Interrelationship among the full working and reduced working states has been developed. A probabilistic model has been developed, considering some assumptions. Data in feasible range has been selected from a survey of sugar plant and the effect of each subsystem on the system availability is tabulated in the form of availability matrices, which provides various availability levels for different combinations of failure and repair rates of all subsystems.

***Keywords**: Availability evaluation, probabilistic approach, transition diagram, availability matrices.*

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I. INTRODUCTION

The sugar industry is becoming quite complex with large capital investment on process automation to enhance the reliability of system. Invariably, the proper maintenance of such systems and the frequency of maintenance are some of the issues that are gaining importance in industry. The production suffers due to failure of any intermediate system even for small interval of time. The cause of failure may be due to poor design, system complexity, and poor maintenance, lack of communication and coordination, defective planning, lack of experience. Thus, to run a process plant highly experienced maintenance personnel are required. The failed subsystem can however be inducted back into service after repairs/replacements. The rate of failure of the subsystems in the particular system depends upon the operating conditions and repair policies used. A probabilistic analysis of the system under given operative conditions is helpful in forecasting the equipment behavior which further helps in design to achieve minimum failure in the system i.e. to optimize the system working. A sugar plant is a complex system comprising of various systems: feeding, crushing, refining, evaporating, sulphonation, crystallization etc. These systems are connected in complex configuration. One of the most important functionaries of a sugar plant is crystallization system. The optimization of each system in relation to one another is imperative to make the plant profitable for operation. Effectiveness of sugar plant is mainly influenced by the availability of the plant, and its capability to perform as expected. The present paper provides a probabilistic model to achieve the maximum availability.

II. LITERATURE REVIEW

The available literature reflects that several approaches have been used to analyze the system performance in terms of reliability. **W.R. Murthy et al. (2003)** dealt with the case studies in reliability and maintenance. **P.O. Rafael et al. (2004)** dealt with the multiple system governed by a quasi-birth –and-death process. **M.Samrout et al. (2005)** dealt with the methods to minimize the preventive maintenance cost of series-parallel systems using ant colony optimization. **R. Khanduja et al. (2008)** dealt with the availability analysis of the bleaching system in a paper plant. **S.Gupta et al. (2009)** dealt with the development of a performance model of power generation system of a thermal plant for performance evaluation using Markov technique and probabilistic approach. The study covers two areas: development of a predictive model and evaluation of performance with the help of developed model. The present system of thermal plant under study consists of four subsystems with three possible states: full working, reduced capacity working and failed. Failure and repair rates for all the subsystems are assumed to be constant. A transition diagram represents the operational behavior of the system. A probabilistic model has been developed, considering some assumptions. Availability matrix, which provides various performance/availability levels for different combinations of failure and repair rates of all subsystems. On the basis of this study, performance of power generation system is evaluated. **S. Shankuntla et al. (2011)** dealt with availability analysis of a system can benefit the industry in terms of higher productivity and low maintenance cost. It is possible to improve the availability of the plant with proper maintenance planning and monitoring. Reliability analysis helps us to obtain the necessary information about the control of various parameters. **R. Khanduja et al. (2011)** dealt with the steady state behavior and maintenance planning of the bleaching system in a paper plant. The paper plant comprises of various systems including feeding, chipping, digesting, washing, bleaching, screening, stock preparation and paper making, etc. One of the most important functionaries of a paper plant, on which quality of paper depends, is the bleaching system, where removal of coloring constituents is done to obtain the desired degree of brightness. **P C Tewari et al. (2012)** discussed the performance enhancement for crystallization unit of a sugar plant using genetic algorithm. The crystallization unit of a sugar industry has three main subsystems arranged in series. Considering exponential distribution for the probable failures and repairs, the mathematical formulation of the problem is done using probabilistic approach, and differential equations are developed on the basis of Markov birth-death process. These equations are then solved using normalizing conditions so as to determine the steady-state availability of the crystallization unit performance. **P.C. Tewari et al. (July-2012)** discussed the development of availability model and performance analysis for Steam Generating system of a Thermal Power Plant. The system comprises of three subsystems viz. High Pressure Heaters, Economizer and Boiler Drum, which are connected in series. The availability model of Steam Generating system has been developed on the basis of probabilistic approach using Markov Birth – Death Process. The Chapman–Kolmogorov equations developed are further solved recursively in order to develop the Steady State Availability i.e. performance index. The system performance has been analyzed in terms of availability levels for different combinations of failures and repair rates. **H.Garg et al.(24January2012)** dealt with the starting point of reliability theory is that life time of systems or components cannot be predicted reliably because failures are random. Resulting effects vary from minor inconvenience to loss of service time and sometimes to loss of material, equipments and even life.

III. SYSTEM DECRIPTION

Crystallization unit consists of three subsystems in series configuration with the following description:

- Subsystem Ai (i = 1 to 3): It consists of three crystallizer units connected in parallel. The failure of any one reduces the capacity of the system and, hence, loss in production. Complete failure occurs when more than one unit fail at a time.
- Subsystem Bj (j = 1 to 5): It consists of five centrifuge units connected in parallel. Complete failure occurs when more than two units fail at a time.
- Subsystem Ck (k = 1 to 3): It consists of three sugar grader units connected in series. The failure of any one causes the complete failure of the system.

IV. ASSUMPTIONS

The assumptions used in the probabilistic model are as follows:

1. Failure/repair rates are constant over time and statistically independent.
2. A repaired unit is as good as new and performance wise for a specified duration.
3. Sufficient repair facilities are provided.
4. Standby units (if any) are of the same nature and capacity as the active units.
5. System failure/repair follows exponential distribution.
6. System may work at a reduced capacity/efficiency.
7. There is no simultaneous failure among the system.

V. NOTATIONS

The following notations are addressed for the purpose of mathematical analysis of the system:

A¡ , Bj ,Ck : represent good working states of respective crystallizer, centrifuges and sugar grading units

a,b,c : represent failed states of respective crystallizer, centrifuges and sugar grading units

Φ_4, Φ_5, Φ_6 : respective mean constant failure rates of A¡ , Bj ,Ck

$\lambda_4, \lambda_5, \lambda_6$: respective mean constant repair rates of A¡ , Bj ,Ck

d/dt : represents derivative w.r.t ‘t’

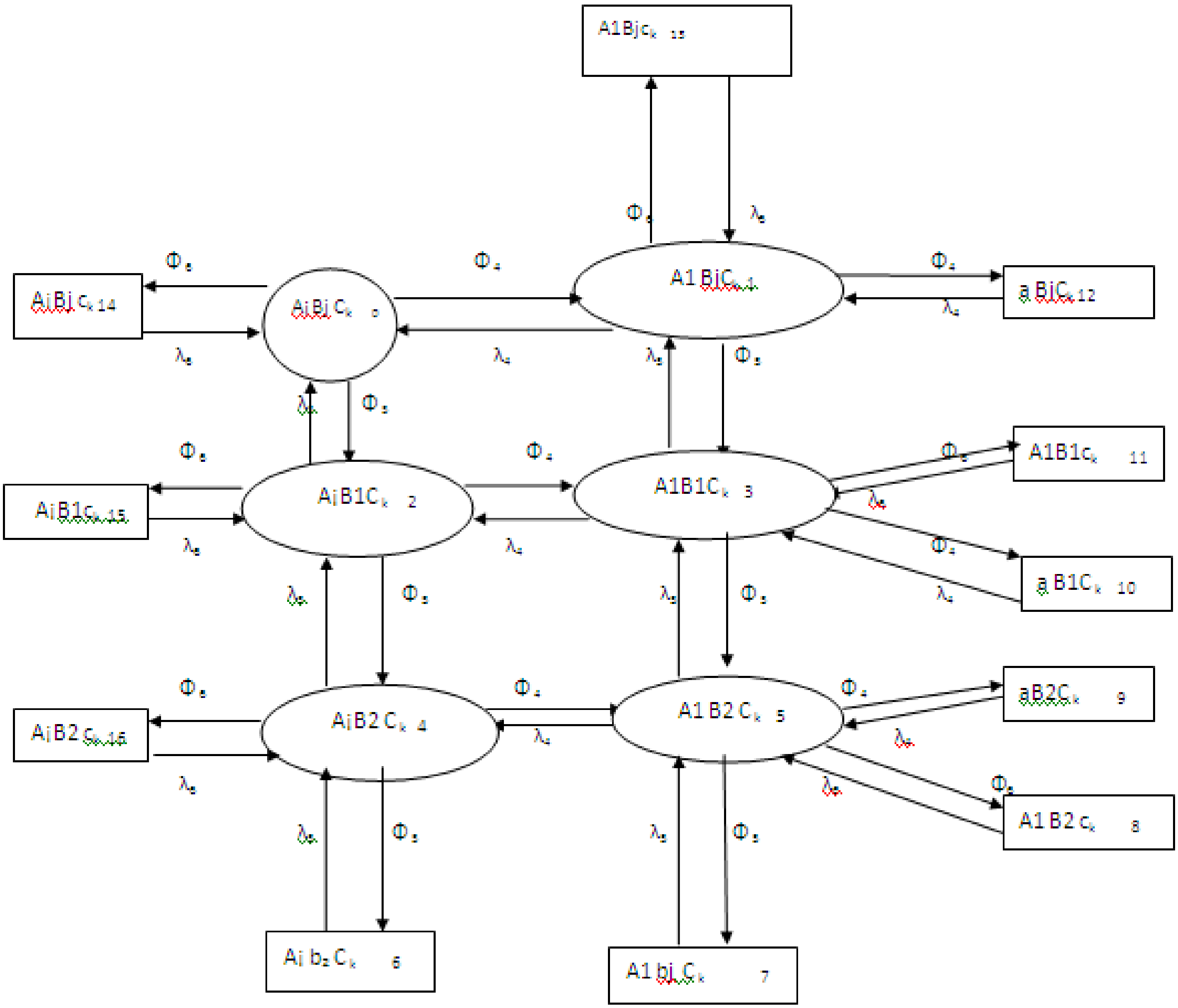
$P_i(t)$:state probability that the system is in ith state at time t .

This system consists of 17 states as:

State 0- Full capacity working with no standby

State 1to5- Reduced capacity working

States 6 to 16- System in failed state due to complete failure of one or other unit of the system



Transition Diagram

VI. MATHEMATICAL MODELING

Simple probabilistic approach gives the following differential equations, associated with the transition diagram of crystallization system

P0 (t) (d/dt+Φ4+ Φ5+ Φ6) =P1 (t) λ4+P2 (t) λ5+P14 (t) λ6 …...1

P1 (t) (d/dt+ λ4+ Φ4+ Φ5+ Φ6) =P0 (t) Φ4+P12 (t) λ4+P3 (t) λ5+P13 (t) λ6………………….2

P2 (t) (d/dt+ λ5 +Φ4+ Φ5+ Φ6) =P0 (t) Φ5+P3 (t) λ4+P4 (t) λ5+P15 (t) λ6…………….…...3

P3 (t) (d/dt+ λ4+ λ5+Φ4+ Φ5+ Φ6) =P2 (t) Φ4+P1 (t) Φ5+P10 (t) λ4+P5 (t) λ5+P11 (t) λ6…..4

P4 (t) (d/dt+ λ5+Φ4+ Φ5+ Φ6) =P2 (t) Φ5+P5 (t) λ4+P6 (t) λ5+P16 (t) λ6……………………5

P5 (t) (d/dt+ λ4+ λ5+Φ4+ Φ5+ Φ6) =P4 (t) Φ4+P3 (t) Φ5+P9 (t) λ4+P7 (t) λ5+P8 (t) λ6…......6

P6 (t) (d/dt+ λ5) = P4 (t) Φ5…………………………………………………………......7

P7 (t) (d/dt+ λ5) = P5 (t) Φ5……………………………………………………………..8

P8 (t) (d/dt+ λ6) = P5 (t) Φ6……………………………………………………………..9

P9 (t) (d/dt+ λ4) = P5 (t) Φ4 ……………………………………………………………10

P10 (t) (d/dt+ λ4) = P3 (t) Φ4 …………………………………………………………..11

P11 (t) (d/dt+ λ6) = P3 (t) Φ6…………………………………………………………...12

P12 (t) (d/dt+ λ4) = P1 (t) Φ4 …………………………………………………………..13

P13 (t) (d/dt+ λ6) = P1 (t) Φ6 …………………………………………………………..14

P14 (t) (d/dt+ λ6) = P0 (t) Φ6 …………………………………………………………..15

P15 (t) (d/dt+ λ6) = P2 (t) Φ6 …………………………………………………………..16

P16 (t) (d/dt+ λ6) = P4 (t) Φ6 …………………………………………………………..17

VII. STEADY STATE ANALYSIS

The steady state behavior of the system can be analyzed by setting t → ∞ and d/dt→ 0; The limiting probabilities from equations (1) – (17) and Solving these equations recursively, we get:

P0 = (P1 λ4+ P2 λ5+ P14 λ6) / (Φ4+ Φ5+ Φ6)

P1= P0 X3P2= P0 X4

P3= Φ5 X 3 P0 / λ5

P4= P0 X5 P0

P5= X3 P0 (Φ5 / λ5) ^3

P6 = P0 X5 Φ5 / λ5

P7 = P0 X3(Φ5 / λ5) ^2

P8= P0 Φ6 X3 (Φ5) ^2 / λ6 (λ5) ^2

P9 = P0 Φ4 X3 (Φ5) ^2/ (λ4 (λ5) ^2)

P10 = (P0 X3 Φ4 Φ5) / λ4 λ5

P11= P0 Φ4 X3 / λ4

P12 = P0 Φ6 Φ5 X3 / λ6 λ5

P13 = P0 Φ6 X3 / λ6

$P14 = P0\ \Phi6 / \lambda6$

$P16 = P0\ \Phi6\ X5 / \lambda6$

$P15 = P0\ \Phi6\ X4 / \lambda6$

Let the values are to be as:

$X1 = \Phi4 + \Phi5 - [(\Phi5\ \lambda5)/(\lambda5+\Phi4)]$

$X2 = \lambda4 + [(\lambda4\ \lambda5)/(\lambda5+\Phi4)]$

$X3 = \lambda5 + [(\lambda4\ \lambda5)/(\Phi5+\lambda4)]$

$X4 = (\Phi4 + \Phi5 - \lambda4\ X3)/\lambda5$

$X5 = \lambda5\ (\Phi5/\lambda5)^2 + \Phi5\ X4$

The probability of full capacity working P0 is determined by using the normalizing condition

$$\sum_{i=0}^{16} \mathbf{Pi} = \mathbf{1}$$

$P0\ [1 + X3 + X4 + (\Phi5/\lambda5)X3 + X5 + (\Phi5/\lambda5)^3 X3 + (\Phi5/\lambda5)X5 + (\Phi5/\lambda5)^2 X3 + (\Phi5/\lambda5)^2 X3 (\Phi6/\lambda6) + (\Phi5/\lambda5)^2 X3 (\Phi4/\lambda4) + (\Phi5/\lambda5) X3 (\Phi4/\lambda4) + (\Phi4/\lambda4) X3 + (\Phi6/\lambda6)(\Phi5/\lambda5) X3 + (\Phi6/\lambda6) X3 + (\Phi6/\lambda6) + (\Phi6/\lambda6) X4 + (\Phi6/\lambda6) X5] = 1$

$P0 = 1 / [1 + X3 + X4 + (\Phi5/\lambda5)X3 + X5 + (\Phi5/\lambda5)^3 X3 + (\Phi5/\lambda5)X5 + (\Phi5/\lambda5)^2 X3 + (\Phi5/\lambda5)^2 X3 (\Phi6/\lambda6) + (\Phi5/\lambda5)^2 X3 (\Phi4/\lambda4) + (\Phi5/\lambda5) X3 (\Phi4/\lambda4) + (\Phi4/\lambda4) X3 + (\Phi6/\lambda6)(\Phi5/\lambda5) X3 + (\Phi6/\lambda6) X3 + (\Phi6/\lambda6) + (\Phi6/\lambda6) X4 + (\Phi6/\lambda6) X5]$

AV$= [1 + (\Phi4 + \Phi5 - \lambda4\ X3)/\lambda5 + (\Phi5/\lambda5)^2\ \lambda5 + \Phi5\ X4 + X3\ (1 + (\Phi5/\lambda5) + (\Phi5/\lambda5)^2)] / [1 + X3 + X4 + (\Phi5/\lambda5)X3 + X5 + (\Phi5/\lambda5)^3 X3 + (\Phi5/\lambda5)X5 + (\Phi5/\lambda5)^2 X3 + (\Phi5/\lambda5)^2 X3 (\Phi6/\lambda6) + (\Phi5/\lambda5)^2 X3 (\Phi4/\lambda4) + (\Phi5/\lambda5) X3 (\Phi4/\lambda4) + (\Phi4/\lambda4) X3 + (\Phi6/\lambda6)(\Phi5/\lambda5) X3 + (\Phi6/\lambda6) X3 + (\Phi6/\lambda6) + (\Phi6/\lambda6) X4 + (\Phi6/\lambda6) X5]$

AV$=P0\ (1+X3+X4+X3B5+X5+X3B5^3)$

VIII. PERFORMANCE ANALYSIS

Table 1 Availability matrices for 'crystallizer' subsystem of crystallization system

AV →

λ4 / Φ4	0.010	0.015	0.020	0.025	0.030
0.002	.787225	.791074	.792821	.793742	.794261
0.004	.773192	.78152	.785046	.787299	.787733
0.006	.759943	.771684	.777588	.781099	.7834

0.008	.747414	.762642	.770427	.775126	.778252
0.010	.337900	.452413	.483281	.515731	.540467

Constant values are as: Φ5=0.06, λ5=0.10, Φ6=0.02, λ6=0.10

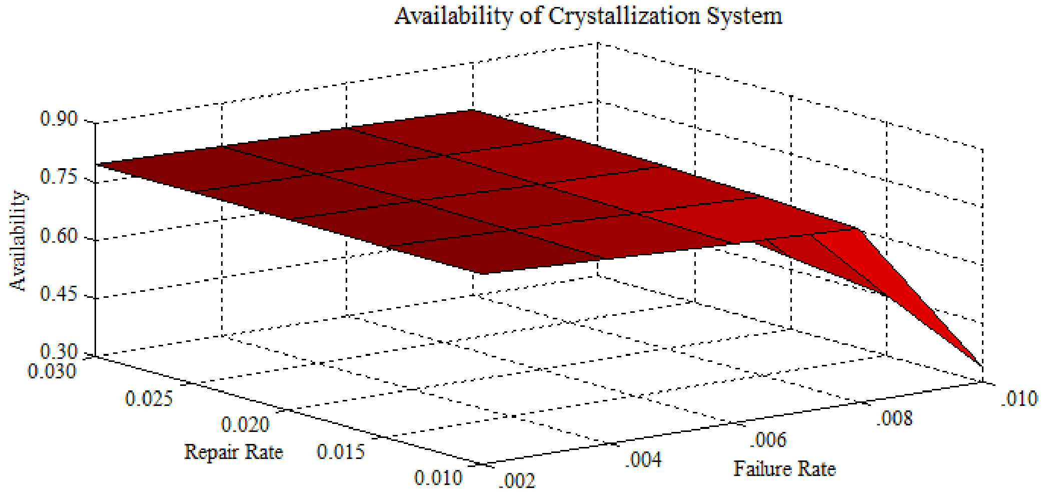


Figure1. Effect of failure and repair rate of 'crystallizer' subsystem on system availability

Table 2 Availability matrices for 'centrifuge' subsystem of crystallization system

AV →

λ5	0.10	0.15	0.20	0.25	0.30
0.06	.777588	.780605	.778724	.775222	.771193
0.08	.756784	.767281	.769577	.7687	.766448
0.10	.73303	.75116	.757975	.760031	.759805
0.12	.70783	.733138	.744528	.749675	.751641
0.14	.395634	.517929	.499322	.530954	.540595

Constant values are as: Φ4=0.006, λ4=0.02, Φ6=0.02, λ6=0.10

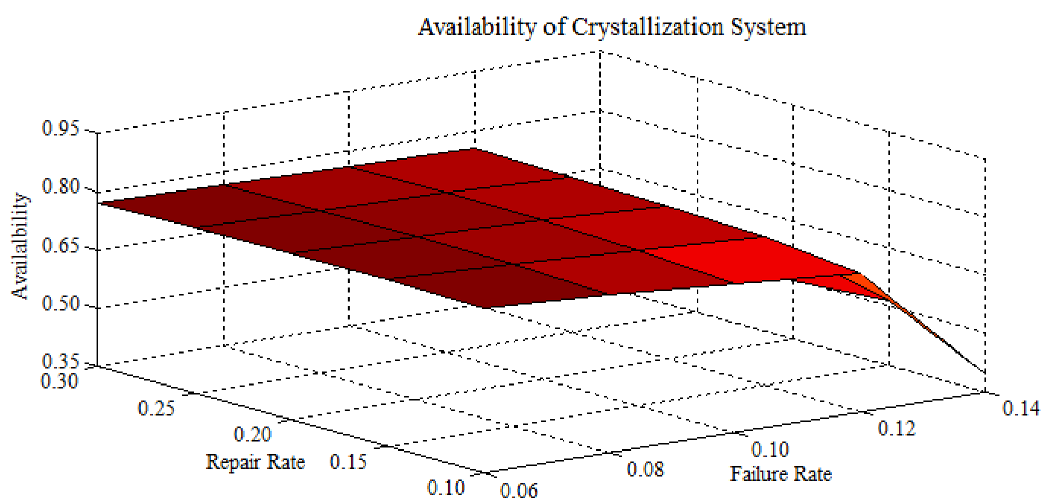


Figure2. Effect of failure and repair rate of 'centrifuge' subsystem on system availability

Table 3 Availability matrices for 'sugar grading' subsystem of crystallization system

AV →

λ6 / Φ6	0.10	0.15	0.20	0.25	0.30
0.02	.777588	.820518	.843811	.858433	.868466
0.04	.672094	.738926	.777588	.802789	.820518
0.06	.591804	.672094	.721002	.75392	.777588
0.08	.528651	.616348	.672094	.710659	.738926
0.10	.427067	.564225	.604541	.664647	.667730

Constant values are as: Φ4=0.006, λ4=0.02, Φ5=0.06, λ5=0.10

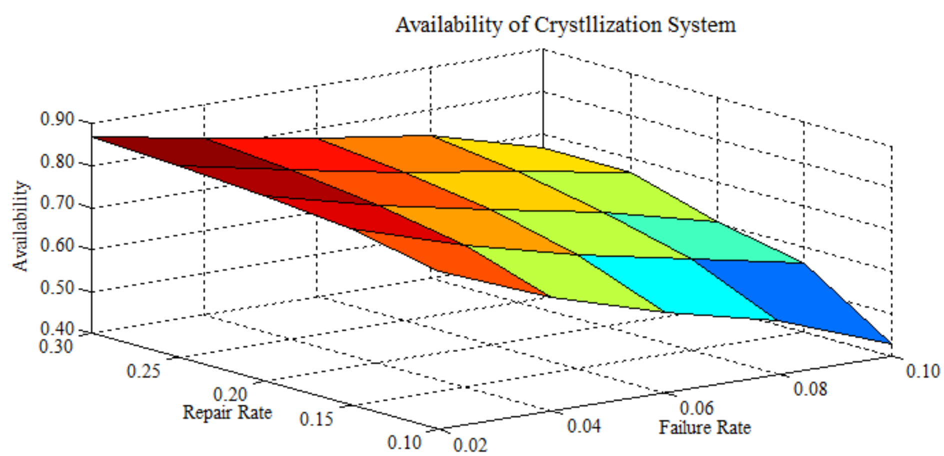


Figure3. Effect of failure and repair rate of 'sugar grading' subsystem on system availability

IX. RESULTS AND DISCUSSION

Table 1 and figure 1 illustrates the variation of system availability with change in failure rate and repair rate of crystallizer subsystem. As failure rate of crystallizer increases from 0.002 (once in 500 hrs) to 0.010(once in 100 hrs), the system availability decreases drastically by 45%. Similarly as repair rate of washer increases from 0.010(once in 100 hrs) to 0.030(once in 33.33hrs), the system availability increases appreciably by 7%. Table 2 and figure 2 illustrates the variation of system availability with change in failure rate and repair rate of centrifuge subsystem. As failure rate of centrifuge increases from 0.06 (once in 16.6 hrs) to 0.14(once in 7.14 hrs), the system availability decreases by 38.2%. Similarly as repair rate of centrifuge increases from 0.10(once in 10 hrs) to 0.30(once in 3.33hrs), the system availability increases by 6%. Table 3 and figure 3 reveals the variation of system availability with change in failure rate and repair rate of sugar grading subsystem. As failure rate of sugar grading increases from 0.02 (once in 50 hrs) to 0.10(once in 10 hrs), the system availability decreases by 66%. Similarly as repair rate of sugar grading increases from 0.10(once in 10 hrs) to 0.30(once in 3.33hrs), the system availability increases appreciably by 9.1%.

X. CONCLUSIONS

Probabilistic models for various subsystems of a sugar plant have been developed and analyzed in real environment. The steady state availability expressions have been derived. The inter-relationships among various working units in the operating environment have been developed. These indicate the critical juncture of profitable operation. Decision matrices have been developed which help in deciding maintenance strategies and performance level. The effect of each unit behavior on the system performance has also been analyzed through decision matrices and availability plots. Desired level of performance has been established and the practical values of states of nature and courses of action have been determined.

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