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Static Charged Perfect Fluid Sphere in Higher Dimensional Space-Time

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Abstract

In this work, we have obtained the higher dimensional analytical solutions of Einstein- Maxwell field equations for a static charged perfect fluid sphere. The various physical properties of the solutions are also discussed in the framework of higher dimensional space-time.

Keywords: Higher dimensional space-time, static, perfect fluid, charged sphere, Einstein-Maxwell equations.

INTRODUCTION

In view of the advent of the superstring theory in which the space-time is considered to be of dimensions higher than four, the studies in higher dimensions have attend new importance. Myer and Perry (1986) obtained higher dimensional analog of Schwarzs child, Reissner-Nordstrom and Kerr metrices. Xu Dianyn (1988) solved Einstein-Maxwell equations in higher dimensional space-time and obtained higher dimensional analog of Schwarzschild de-Sitter's metric, Reissner-Nordstrom de-Sitter's metric and Kerr de-Sitter's metric. Joshi and Khadekar (1999) studied the higher dimensional interior solution of Einstein-Maxwell equation for a charged static fluid sphere. Gao and Zhang (2006) extended their work of 2004 in which the McVittie (1933) solution is generalized to the four-dimensional charged black hole to higher dimensions. Hassaine and Martinez (2008) obtained charged black hole solution of Einstein-Maxwell equations in arbitrary higher dimensions with non-linear electro-dynamic sources. Khadekar and Shobhane (2008) generalized the technique used by Hajj-Boutros and Sfeila (1986) in the frame work of higher dimensional space-time and obtained exact solutions of Einstein-Maxwell field equations for static spherically symmetric distribution of charged perfect fluid. Khadekar and Wanjari (2012) have discussed the geometry of quark and strange quark matters in higher dimensional space-time. The work of the above authors motivates one to consider the further work in the context of higher dimensional space-time.

The problem of determination of exact solution of coupled Einstein-Maxwell equations for static spherical distributions of charged matter has attracted wide attention. These distributions constitute possible sources for the Reissner-Nordstrom metric which uniquely describes the exterior field of a spherically symmetric charged distribution of matter. As the field equations do not completely determine the system, different solutions were obtained by many authors by using different conditions to supplement the field equations [11-19]. The supplementary conditions were used partly to specify the physical model and partly to simplify the mathematics. Many researchers have therefore devoted considerable attention to the problem of finding analytical interior solutions of the Einstein-Maxwell equation corresponding to charged perfect fluid spheres in equilibrium. Kyle and Martin (1967) have studied charged fluid spheres of uniform density. Mehra and Bohra (1979) obtained solution for a static charged fluid sphere of radius R having constant mass density and variable charge density. Dionysiou (1982) developed Einstein-Maxwell equations for a static charged perfect fluid sphere in such a manner to generalize the Oppenheimer and Volkoff hydrostatic equilibrium equation. Dionysiou and Kostakis (1983) gave the new mathematical and physical method of finding explicit analytical interior solutions of the Einstein-Maxwell field equations of static perfect fluid sphere with charge.

In this paper, we have obtained higher dimensional exact solutions of the Einstein-Maxwell field equations for a static charged perfect fluid sphere by using the method used earlier by Dionysiou and Kostakis (1983). We consider the static, spherically symmetric distribution of a charged perfect fluid inside the sphere of radius r = R, then the most general static line element with spherical symmetry in (n+2) -dimensions is given by

$$ds^2=e^{\nu(r)}dt^2-e^{\lambda(r)}dr^2-r^2(\chi_n{}^2), \tag{1}$$

where

$$\chi_n{}^2=d\theta_1{}^2+\sin^2\theta_1 d\theta_2{}^2+\sin^2\theta_1\sin^2\theta_2 d\theta_3{}^2+\ldots+\left(\prod_{i=1}^{n-1}\sin^2\theta_i\right)d\theta_n{}^2$$

and the coordinates are

$x_0=t,\ x_1=r,\ x_2=\theta_1,\ x_3=\theta_2,\ldots,\ x_{n+1}=\theta_n.$

The Einstein -Maxwell field equations (using the geometric units G=c=1)are given by

$$G_j^i \equiv R_j^i-\frac{1}{2}\delta_j^i R=8\pi T_j^i,\ \ i=j=0,1,2,\ldots,(n+1) \tag{2}$$

and the energy- momentum tensor T_j^i is given by

$$T_j^i=(\delta+p)u_j u^i-p\delta_j^i+\frac{1}{4\pi}\left(-F^{ik}F_{jk}+\frac{1}{4}\delta_j^i F_{kl}F^{kl}\right), \tag{3}$$

where $u_i=(u_0,0,0,\ldots,0(n+1)-\text{times})$ is the (n + 2)-velocity, $\delta(r)$ is the mass density and $p(r)$ is the pressure. The anti-symmetric electromagnetic tensor

$$F_{ik}=\frac{\partial A_k}{\partial x^i}-\frac{\partial A_i}{\partial x^k},\qquad (i,k=0,1,2,\ldots,(n+1)) \tag{4}$$

satisfies the Maxwell's equations

$$\frac{\partial}{\partial x^k}\left(\sqrt{(-1)^{n+1}g}\,F^{ik}\right)=-4\pi\sqrt{(-1)^{n+1}g}\,J^i$$

and

$$F_{ik,l}+F_{kl,i}+f_{li,k}=0,$$

where A_i is the(n+2)-potential, J^i is the (n+2)-current vector and comma denotes ordinary differentiation. For the(n+2)-potential, let $A_0=\Psi(r)$ be the electrostatic potential, the remaining $A_1, A_2, A_3,\ldots, A_{n+1}$ being zero, that is,

$A_i=(\Psi(r),0,0,\ldots,0(n+1)\text{-times})$.

Using equation (4), the only non-vanishing electro-magnetic field tensor is F^{01} which is given by

$$F^{01} = -e^{-(\upsilon+\lambda)/2}\frac{q(r)}{r^n}, \tag{5}$$

where

$$q(r) = 4\pi\int_0^r e^{(\upsilon+\lambda)/2}\, r^n J^0 dr. \tag{6}$$

The (n+2) - current vector is defined by expression

$$J^i = \frac{\rho}{\sqrt{g_{00}}}\frac{dx^i}{dx^0}, \tag{7}$$

where the continuous function $\rho=\rho(r)$ is the charge density. Using equation (7), we obtain

$$J^0 = e^{-\nu/2}\rho.$$

Therefore, using equation(6), the static spherically symmetric distribution of charge is given by

$$q(r) = 4\pi\int_0^r \rho e^{\lambda/2}\, r^n dr. \tag{8}$$

The function $q(r)$ is continuous across the surface of the sphere of radius R.

FIELD EQUATIONS

The functions $\lambda = \lambda(r)$ and $\nu = \nu(r)$ are to be determined by the field equations (2) under the boundary conditions, which demands the continuity of e^{λ}, e^{ν} and $e^{\nu}\,\nu'$ across the surface of the sphere. We have

$$u^2 = u_i u^i = u_0 u^0 = 1,\; g_{00}u^2 = g_{00}\, u_0 u^0 = u_0^2$$

and hence

$$u^2 = e^{-\nu}{u_0}^2 = 1.$$

Then the field equations (2) for the metric (1) take the following form:

$$G_0^0 \equiv -e^{-\lambda}\left[\frac{n(n-1)}{2r^2} - \frac{n\lambda'}{2r}\right] + \frac{n(n-1)}{2r^2} = 8\pi\delta + \frac{q^2(r)}{r^{2n}} \tag{9}$$

$$G_1^1 \equiv -e^{-\lambda}\left[\frac{n\nu'}{2r}+\frac{n(n-1)}{2r^2}\right]+\frac{n(n-1)}{2r^2} = -8\pi p + \frac{q^2(r)}{r^{2n}} \tag{10}$$

$$G_2^2 \equiv -e^{-\lambda}\left[\frac{\nu''}{2}+\frac{\nu'^2}{4}+\frac{(n-1)(\nu'-\lambda')}{2r}-\frac{\lambda'\nu'}{4}+\frac{(n-1)(n-2)}{2r^2}\right] + \frac{(n-1)(n-2)}{2r^2} = -8\pi p - \frac{q^2(r)}{r^{2n}}, \tag{11}$$

and

$$G_3^3 = G_4^4 = \cdots = G_{n+1}^{n+1} = G_2^2,$$

where the prime denotes the differentiation with respect to r. The other components of equations (2) vanish identically.

INTERIOR SOLUTIONS

Equation (9) can be written in the form

$$\frac{d}{dr}\left[r^{n-1}e^{-\lambda}\right] = (n-1)r^{n-2} - \frac{2}{n}\left[8\pi\delta r^n + \frac{q^2}{r^n}\right] \tag{12}$$

On integration equation (12) leads to

$$r^{(n-1)}e^{-\lambda} = r^{(n-1)} - \frac{2}{n}\int_0^r\left(8\pi\delta r^n + \frac{q^2}{r^n}\right)dr + C,$$

where C is an arbitrary constant. In order to avoid a singularity at the origin, we put C = 0, hence

$$e^{-\lambda} = 1 - \frac{2}{nr^{n-1}}\int_0^r\left(8\pi\delta r^n + \frac{q^2}{r^n}\right)dr. \tag{13}$$

or otherwise

$$e^{-\lambda} = 1 - \frac{2\mu}{r^{n-1}} + \frac{2q^2}{n(n-1)r^{2(n-1)}}, \tag{14}$$

where

$$\mu(r) = \frac{8\pi}{n}\int_0^r \delta r^n dr + \frac{1}{n}\int_0^r \frac{q^2}{r^n}dr + \frac{q^2}{n(n-1)r^{n-1}}. \tag{15}$$

It may be noted here that the function $\lambda = \lambda(r)$ is not explicitly defined by equations (13) or (14) for the function q(r)is defined in terms of $\lambda(r)$. For the explicit solutions, we have taken the quantity

$$q'(r) = 4\pi\rho\, e^{\lambda/2}\, r^n \tag{16}$$

as the charge density instead of the function $\rho=\rho(r)$ [Florides(1977)].

Further, the gravitational mass of the sphere is defined by

$$\mu(R)=m(say) \tag{17}$$

and the total charge inside the sphere as

$$q(R) = 4\pi \int_0^R \rho e^{\lambda/2} r^n dr = e(say). \tag{18}$$

Thus at the boundary of the sphere $r = R$, equation (14) become

$$e^{-\lambda(R)} = 1 - \frac{2m}{R^{n-1}} + \frac{2e^2}{n(n-1)R^{2(n-1)}} \tag{19}$$

The higher dimensional Reissner-Nordstrom exterior solution [Joshi and Khadekar (1999)] in matter-free and charge-free space-time is

$$ds^2 = \left[1 - \frac{2m}{r^{n-1}} + \frac{2e^2}{n(n-1)r^{2(n-1)}}\right]dt^2 - \left[1 - \frac{2m}{r^{n-1}} + \frac{2e^2}{n(n-1)r^{2(n-1)}}\right]dr^2 - r^2(\chi_n^2), \tag{20}$$

where the gravitational mass m and the total charge e of the source are constants. In order that the interior metric (1) matches with the exterior metric (20) at the boundary $r = R$, we use equations (17) and (18) to define

$$e^{-\lambda(r)} = e^{\nu(r)} = \left[1 - \frac{2m}{r^{n-1}} + \frac{2e^2}{n(n-1)r^{2(n-1)}}\right], r \geq R \tag{21}$$

with assumption $\lambda(0) = \nu(0) = 0$.

Now, from equations (10)and (11), we obtain

$$(n-1)\frac{d}{dr}\left(\frac{e^{-\lambda}-1}{r^2}\right) + \frac{d}{dr}\left(\frac{e^{-\lambda}\nu'}{2r}\right) + e^{-(\lambda+\nu)}\frac{d}{dr}\left(\frac{e^{\nu}\nu'}{2r}\right) = \frac{4q^2}{r^{2n+1}}. \tag{22}$$

Now, if we introduce the transformation

$$\nu' = 2r^{n-1}X(r), \quad e^{-\lambda} = Y(r), \tag{23}$$

then equation (22) becomes

$$\frac{dY}{dr} + f(r)\,Y = g(r), \tag{24}$$

where

$$f(r) = \frac{2r^n\left[\frac{dX}{dr} + X^2 r^{n-1} + \frac{(n-2)X}{r} - \frac{(n-1)}{r^{n+1}}\right]}{[(n-1) + Xr^n]} \tag{25}$$

and

$$g(r) = \frac{4q^2 - 2(n-1)r^{2(n-1)}}{r^{2n-1}[(n-1) + Xr^n]}. \tag{26}$$

Solving equation (24), we obtain

$$Y \exp\left[\int f(r)dr\right] = \int g(r) \exp\left[\int f(r)dr\right] dr + C_1 \tag{27}$$

where C_1 is the constant of integration.

Using equation (19),the equation (27)can be rewritten as

$$e^{-\lambda} = \left[1 - \frac{2m}{R^{n-1}} + \frac{2e^2}{n(n-1)R^{2(n-1)}}\right] \exp[-F(r)] + \exp[-F(r)] \int_R^r g(r) \exp[F(r)]\, dr, \tag{28}$$

where

$$F(r) = \int_R^r f(r)dr. \tag{29}$$

On the other hand, using transformation (23), we find that

$$\nu(r) = 2\int r^{n-1} X(r)dr + C_2, \tag{30}$$

where C_2 is the constant of integration.

Using equations (21) at *r=R*, it is always possible to write

$$\nu(r) = 2\int_R^r r^{n-1} X(r)dr + \ln\left[1 - \frac{2m}{R^{n-1}} + \frac{2e^2}{n(n-1)R^{2(n-1)}}\right], \tag{31}$$

or, alternatively

$$e^{\nu(r)} = \left[1 - \frac{2m}{R^{n-1}} + \frac{2e^2}{n(n-1)R^{2(n-1)}}\right] \exp\left[2\int_R^r r^{n-1} X(r)dr\right], \tag{32}$$

The conditions of flatness at r=0 imply that $\lambda(0)=0$ and $\nu(0)=0$.
Now, under the above conditions, equation (32) discloses that, when *r=R*,

$$e^{\nu(R)} = 1 - \frac{2m}{R^{n-1}} + \frac{2e^2}{n(n-1)R^{2(n-1)}}. \tag{33}$$

Differentiating equation (32) ,we obtain

$$e^{\nu(r)}\nu'(r) = 2r^{n-1}X(r)\left[1 - \frac{2m}{R^{n-1}} + \frac{2e^2}{n(n-1)R^{2(n-1)}}\right] \times \exp\left[2\int_R^r r^{n-1}X(r)dr\right], \tag{34}$$

Setting *r* = *R*, we obtain

$$e^{\nu(R)}\nu'(R) = 2R^{n-1}X(R)\left[1 - \frac{2m}{R^{n-1}} + \frac{2e^2}{n(n-1)R^{2(n-1)}}\right] \tag{35}$$

and hence using equation (33), we obtain

$$\nu'(R) = 2R^{n-1}X(R). \tag{36}$$

Comparing equations (23) and (36), we observed that the transformation (23) is satisfied at *r* = *R*. Equations (13) or (14) and (32) give us the explicit analytic interior solutions (using equation(16)), if and only if ,the function X(r) is known.

GENERAL CONSIDERATION

In order to ensure the integrability of equation (28),we consider

$$\frac{r^n\left[\frac{dX}{dr} + X^2r^{n-1} + \frac{(n-2)X}{r} - \frac{(n-1)}{r^{n+1}}\right]}{[(n-1) + Xr^n]} = -\frac{1}{r} + X\phi(r) = \frac{1}{2}f(r), \tag{37}$$

where $\phi(r)$ is to be considered as an arbitrary but known analytic function. Also, we regard the function

$$\frac{1}{2}f(r) = -\frac{1}{r} + X\phi(r)$$

as analytic everywhere except at certain singular points and specially at r=0. Furthermore, equation (37)takes the form

$$\frac{dX}{dr} + \left[\frac{(n-1)}{r} - \frac{(n-1)\phi(r)}{r^n}\right]X = [\phi(r) - r^{n-1}]X^2, \tag{38}$$

which is Bernoulli's differential equation. Solving equation (38), we obtain the function *X(r)* in terms of the function $\phi(r)$,i.e.,

$$X(r) = \frac{\frac{1}{r^{n-1}}\exp\left[\int\frac{(n-1)\phi(r)}{r^n}dr\right]}{\int\left[1 - \frac{\phi(r)}{r^{n-1}}\right]\exp\left[\int\frac{(n-1)\phi(r)}{r^n}dr\right]dr + C_3}, \tag{39}$$

where C_3 is an integration constant . In order to define the function X(r), we use equations (33) and (36) to see that

$$X(R) = \frac{(n-1)[mn(n-1)R^{n-1} - 2e^2}{R^n[n(n-1)R^{2(n-1)} - 2mn(n-1)R^{n-1} + 2e^2]} \tag{40}$$

Then using (39),we arrive at the result disclosing that

$$X(r) = \frac{\frac{1}{r^{n-1}}A(r)}{\int_R^r \left[1 - \frac{\phi(r)}{r^{n-1}}\right] A(r)dr + \frac{R[n(n-1)R^{2(n-1)} - 2mn(n-1)R^{n-1} + 2e^2]}{(n-1)[mn(n-1)R^{n-1} - 2e^2]}}, \tag{41}$$

where

$$A(r) = \exp\left[\int_R^r \frac{(n-1)\phi(r)}{r^n} dr\right].$$

Using equations (32) and (41), we obtain an explicit solution of $\nu(r)$in terms of arbitrary known function$\phi(r)$. With the integrability condition (37) is satisfied, the general solution, i.e., equation (28)is given by

$$\begin{aligned} e^{-\lambda}\exp\left[2\int_R^r \left(-\frac{1}{r} + X\phi(r)\right) dr\right] &= \left[1 - \frac{2m}{R^{n-1}} + \frac{2e^2}{n(n-1)R^{2(n-1)}}\right] \\ &+ \int_R^r \frac{4q^2 - 2(n-1)r^{2(n-1)}}{r^{2n-1}[(n-1) + Xr^n]} \exp\left[2\int_R^r \left(-\frac{1}{r} + X\phi(r)dr\right)\right] dr \end{aligned} \tag{42}$$

which gives a solution of the function $\lambda = \lambda(r)$. The most satisfactory procedure would be to find the value of $\lambda(r)$ (if we put into equation (42), equation (6), (13) or (14), (41)) as a function of known arbitrary function $\phi(r)$. In order to make the above more clearly, we consider the following particular cases:

PARTICULAR SOLUTIONS

Case I. Suppose *p=0* and *q=0*, then from(13), we obtain

$$e^{-\lambda} = 1 - \frac{16\pi}{nr^{n-1}} \int_0^r \delta r^n dr. \tag{43}$$

Also from (10), (15) and (23), we get

$$\frac{\nu'}{2r^{n-1}} = \frac{(n-1)\mu(r)}{r^{2n-1}\left[1 - \frac{2\mu}{r^{n-1}}\right]} = X(r) \tag{44}$$

Using (44), equation (32) can be deduced to

$$e^{\nu(r)} = \left[1 - \frac{2m}{R^{n-1}}\right] exp\left[16\pi(n-1)\int_R^r \frac{\left(\int_0^r \delta r^n\, dr\right)}{r\left(nr^{n-1} - 16\pi \int_0^r \delta r^n dr\right)} dr\right], \tag{45}$$

where there should be no singularity at r = 0. Equations (43), (45) are interior solutions of

Einstein's equations $(r \leq R)$ with the restrictions $r > (2\mu)^{\frac{1}{n-1}}$.

Case II. If p = 0, then from equation (10), we obtain

$$-e^{-\lambda}\left[\frac{n\nu'}{2r}+\frac{n(n-1)}{2r^2}\right]+\frac{n(n-1)}{2r^2}=\frac{q^2}{r^{2n}}, \tag{46}$$

while from equation (13), we have

$$e^{-\lambda}=1-\frac{2}{nr^{n-1}}\int_0^r\left(8\pi\delta r^n+\frac{q^2}{r^n}\right)dr.$$

Equation (46), together with (13) and (23) discloses that

$$\frac{\nu'}{2r^{n-1}}=\frac{1}{n}\left[\frac{n(n-1)}{2r^n}-\frac{q^2}{r^{3n-2}}\right]e^{\lambda}-\frac{(n-1)}{2r^n}=X(r). \tag{47}$$

With the knowledge of *X(r)* provided by equation (47), equation(32) can be reduced to

$$e^{\nu(r)}=\left[1-\frac{2m}{R^{n-1}}+\frac{2e^2}{n(n-1)R^{2(n-1)}}\right]\times\exp\left[16\pi\int_R^r\frac{\left[(n-1)B(r)-\frac{q^2}{8\pi r^{n-1}}\right]}{r[nr^{n-1}-16\pi B(r)]}dr\right], \tag{48}$$

where

$$B(r)=\int_0^r\left(\delta+\frac{q^2}{8\pi r^{2n}}\right)r^n dr$$

and there should be no singularity at *r = 0*.

The foregoing results from equations (13), (48) are interior solutions of Einstein- Maxwell field equations for a static and spherically symmetric charged fluid sphere. It may be noted here that X(r) defined by equation(47), by using (13) has the form:

$$X(r)=\frac{(n-1)[n(n-1)\mu r^{n-1}-2q^2]}{r^n[n(n-1)r^{2(n-1)}-2n(n-1)\mu r^{n-1}+2q^2]}. \tag{49}$$

The matching at the boundary r = R gives

$$X(R)=\frac{(n-1)[mn(n-1)R^{n-1}-2e^2]}{R^n[n(n-1)R^{2(n-1)}-2mn(n-1)R^{n-1}+2e^2]}$$

which is exactly equation (40).

Case III: If we choose q=0 with $\phi(r) = 0$, then from (37), we obtain

$$X(r) = \frac{1}{r^n + C_4 r^{n-1}}, \tag{50}$$

where C_4 is an integration constant.

Then from equation (32), we obtain

$$e^{\nu(r)} = (a_0 + a_1 r)^2, \tag{51}$$

where a_0, a_1 are the constants.

Moreover by use of general equation(42) with q=0, $\phi(r)$=0,we find that

$$e^{-\lambda(r)} = a_2 r^2 - \frac{2}{a_3} r + \frac{2nr^2}{{a_3}^2} \ln\left(\frac{nr + a_3}{r}\right) + 1, \tag{52}$$

where a_2 , a_3 are constants.

Equations (51) and (52) are interior solutions of Einstein-Maxwell field equations.

CONCLUSION

In this paper, we have found explicit analytical interior solutions of Einstein- Maxwell field equations of a static perfect fluid sphere with charge in the framework of higher dimensional space- time. If X(r) is known, equations (13) or (14) and (32) give us the explicit analytical interior solutions (using equation (16)). The particular solutions are obtained for the cases: (i) p=0 and q=0, (ii) p=0 and (iii) q=0 and $\phi(r)$=0. For the case (i) p = q = 0, there should be no singularity at r=0. Equations (43) and (45) are higher dimensional interior solutions of Einstein-Maxwell equations with the restriction $(2\mu)^{1/(n-1)} < r \leq R$. For the case (ii) p=0,there should be no singularity at r=0. For the case (iii) q = 0 and $\phi(r)$=0, we get equation (51) which is same as that of the result obtained earlier by Dionysiou and Kostakis (1983).

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