AN OPTIMIZATION OF CO-ORDINATED VENDOR-BUYER REPAIRABLE INVENTORY MODEL

W.Ritha[1],
P.Bharathi[2]
Holy Cross College (Autonomous) ,Trichirapalli – 620 002
Government Arts College, Thiruverumbur, Trichy – 620 022

ABSTRACT

In this paper, we develop an incorporated stock model for vendor-buyer co-ordination under a flawed creation process. The extent of imperfect things in every creation part is thought to be stochastic and takes after a known probability density function. The merchant reviews the things while they are being created and conveys great quality things to the purchaser in parcels over different shipments. In order to reduce the total cost and improve the profit, the defective items are repaired. The expected annual integrated total cost is derived to find the optimal solution. This paper helps the enterprises to improve their quality. Numerical illustrations demonstrates that the co-ordinated model gives a noteworthy cost decrease in integrated inventory model compared to buyer's independent model.

Keywords : Integrated model, imperfect, backordering, repair, inventory.

INTRODUCTION

Incorporated inventory management has as of in consideration recently. Numerous popular firms, for example, P&G, Wal-mart, Campbell soup, Johnson and Johnson, have been endeavouring to accomplish more noteworthy benefits of their supply chain network in the previous couple of decades. For instance, P&G and Wal-Mart have accomplished a win-win arrangement through production and advertising collusion. All the more particularly, the inventory oriented favourable circumstances can be defined from the five viewpoints. Process productivity, offering adaptability, business cooperative strategy, quality and development. Cao and Zhang [3] showed us the inventory management from manufacturer's point of view. Goyal [7] considered the joint optimization problem of a single vendor and a single buyer, in which he assumed that the vendor's production rate is infinite. Banerjee [1] assumed finite rate of production and developed a joint economic lot-size model for the product with a lot-for-lot shipment policy. Hoque and Goyal [8] developed an optimal policy for the single vendor – single buyer integrated production inventory system under the capacity constraint of the transport equipment with both equal and unequal size from the vendor to the buyer. Hill [10] determined the optimal production and inventory policy for a vendor manufacturing to supply for a single buyer.

Ha and Kim [9] developed an integrated just-in-time lot-splitting model of facilitating multiple shipments in small lots of a single vendor-single buyer system under deterministic conditions. Ertogral et al. [6] incorporated transportation cost explicitly into a joint vendor-buyer inventory model under equal shipment policy. Every one of these papers shows that the seller and the purchase ought to cooperate in an agreeable way towards amplifying their shaded advantages or minimizing their common expenses.

The traditional economic order quantity model is based on that things are created of impeccable quality, which is normally not the situation in genuine generation. Porteus [13] presented a model that demonstrated a noteworthy relationship in the quality and the order quantity. Schwaller [16] extended the EOQ model by adding the assumptions that defective items of a known proportion were present in incoming lots. Ben Daya and Haringa [2] studied the effect of imperfect production processes on the economic lot sizing policy. They developed a mathematical model of ELSP, taking into account the effect of imperfect quality and process restoration Salameh and Jaber [14] developed an economic order quantity model when a random proportion of the items in a lot are defective. Cardenas-Baron [4] developed an economic production quantity inventory model with planned backorders under imperfect items.

Unlike the above mentioned papers the purpose of this paper is to develop a co-ordinated vendor-buyer inventory model with imperfect production process and consideation of repair shop. The rest of this paper is organized as follows. In section 2, the notation and assumptions used in this paper are introduced. In section 3, development of a mathematical model, that integrates the vendor's and the buyers. Section 4 provides numerical example. Finally, section 5 concludes the paper.

2. ASSUMPTIONS

(i) The demand rate is known and constant.

(ii) Lead time is constant.

(iii) The production process is imperfect and may produce defective items. The defective percentage is γ.

(iv) Once an item is produced, it is inspected with an inspection cost of C_i.

(v) Shortage is completely backordered.

(vi) Single product is considered.

(vii) There is a single vendor and single buyer.

(viii) Defective items are taken into the repairing process.

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2.1. NOTATIONS

Q	-	The size of the shipments from the vendor to the buyer.
B	-	Backorer quantity.
n	-	Number of shipments per cycle.
D	-	Annual Demand rate
P	-	Annual Production rate
S_v	-	Setup cost
S_B	-	Ordering cost
b	-	backordering cost per unit per year
h_v	-	holding cost for the vendor
h_B	-	holding cost for the buyer
F	-	Transportation cost (fixed)
R	-	Repair rate
S	-	Repair setup cost
C_1	-	Material and Labour cost to repair an item
C_T	-	Unit transportation cost
h'	-	holding cost at the repair shop
t_T	-	Total transport time of γQ units from the inventory system to repair shop and back to the system
γ	-	The probability that an item produced is defective
$f(\gamma)$	-	The probability density function of γ.
C_i	-	Vendor's inspection cost
T_c	-	Total Cycle Time

3. MATHEMATICAL MODEL

a) Buyer's cost per cycle

The buyer's cost consists of ordering, transportation, holding and backordering costs. It is notated as :

$$TC_B(n, Q, B) = S_B + nF + nh_B \frac{(Q-B)^2}{2D} + \frac{nbB^2}{2D} \quad \ldots (1)$$

b) Vendor's cost per cycle

Vendor has two holding costs namely holding cost for non-defective items and holding cost for defective items.

Holding cost good quality items is calculated as :

$$h_v \left\{ \left(nQ\left(\frac{Q}{P(1-\gamma)} \right) + (n-1)T \right) - \frac{nQ(nQ)}{2P(1-\gamma)} - T(Q + 2Q + \ldots + (n-1)Q) \right\}$$

$$= h_v \left\{ \frac{nQ^2}{P(1-\gamma)}\left(1-\frac{n}{2}\right) + \frac{n(n-1)Q^2}{2D} \right\}$$

The number of defective items produced is $\frac{nQ\gamma}{(1-\gamma)}$. Hence the holding cost per cycle for defective items is $h_v \left(\frac{nQ\gamma}{2(1-\gamma)} \right)\left(\frac{nQ}{P(1-\gamma)} \right)$.

The total cost of vendor is

$$TC_v(n, Q) = S_v + C_i \frac{nQ}{(1-\gamma)} + h_v \left\{ \frac{nQ^2}{P(1-\gamma)}\left(1-\frac{n}{2}\right) + \frac{n(n-1)Q^2}{2D} + \frac{n^2Q^2\gamma}{2P(1-\gamma)^2} \right\}$$

$$+ (1 + m)\left(\frac{S+2F}{\gamma} + C_1 + 2C_T + h't_R \right)$$

We added the cost of setup, inspection, holding and repaired item.

$$TC_v(n, Q) = S_v + C_i \frac{nQ}{(1-\gamma)} + h_v \left\{ \frac{nQ^2}{P(1-\gamma)}\left(1-\frac{n}{2}\right) + \frac{n(n-1)Q^2}{2D} + \frac{n^2Q^2\gamma}{2P(1-\gamma)^2} \right\}$$

$$+ (1 + m)\left(\frac{S+2F}{\gamma} + C_1 + 2C_T + h't_R \right) \quad \ldots (2)$$

c) The coordinated vendor-buyer model

$$TC_c(n, Q, B) = TC_v(n, Q) + TC_B(n, Q, B)$$

$$= S_v + S_B + nF + C_i \frac{nQ}{(1-\gamma)} + \frac{h_B}{2} \frac{n(Q-B)^2}{D}$$

$$+ h_v \left\{ \frac{nQ^2}{P(1-\gamma)}\left(1-\frac{n}{2}\right) + \frac{n(n-1)Q^2}{2D} + \frac{n^2Q^2\gamma}{2P(1-\gamma)^2} \right\}$$

$$+\frac{1}{2}bn\frac{B^2}{D}+(1+m)\left(\frac{S+2F}{\gamma}+C_1+2C_T+h't_R\right) \quad \ldots(3)$$

Since the cycle length $T_c = \frac{nQ}{D}$, the expected total annual cost is

$$ETC(n, Q, B) = \frac{E(TC_c(n, Q, B))}{T_c}$$

$$= \frac{(S_v+S_B)D}{nQ}+\frac{FD}{Q}+C_iDE\left(\frac{1}{1-\gamma}\right)$$

$$+h_v\left\{\frac{QD}{P}\left(1-\frac{n}{2}\right)E\left(\frac{1}{1-\gamma}\right)+\frac{(n-1)Q}{2}+\frac{nQD}{2P}E\left(\frac{\gamma}{(1-\gamma)^2}\right)\right\}$$

$$+\frac{1}{2}h_B\frac{(Q-B)^2}{Q}+\frac{1}{2}\frac{bB^2}{Q}+\frac{(C_1+2C_T)D}{nQ}$$

$$+(1+m)\left(\frac{(S+2F)D}{nQ}\right)E\left(\frac{1}{\gamma}\right)+h'\left(\frac{D}{n(R+t_T)}\right)E(\gamma) \quad \ldots(4)$$

Here E(x) denotes the expected value.

On calculating the first derivative with respect to Q and B of (4) we get,

$$\frac{\partial ETC(n, Q, B)}{\partial Q} = \frac{-(S_v+S_B+nF)D}{nQ^2}+h_v\left\{\frac{D}{P}\left(1-\frac{n}{2}\right)E\left(\frac{1}{1-\gamma}\right)+\frac{(n-1)Q}{2}+\frac{nD}{2P}E\left(\frac{\gamma}{(1-\gamma)^2}\right)\right\}$$

$$+\frac{h_B}{2}\left\{1-\frac{B^2}{Q^2}\right\}-\frac{b}{2}\frac{B^2}{Q^2}-\frac{(1+m)(S+2F)D}{nQ^2}E\left(\frac{1}{\gamma}\right)-\frac{C_1+2C_T)D}{nQ^2} \quad \ldots(5)$$

Now taking the first derivative for B,

$$\frac{\partial ETC(n, Q, B)}{\partial B} = h_B\left\{\frac{B}{Q}-1\right\}+\frac{bB}{Q} \quad \ldots(6)$$

For the optimal solution, now we have to check the convex property. For that, taking the second derivative, we have

$$\frac{\partial ETC(n, Q, B)}{\partial Q^2} = \frac{2(S_v+S_B+nF)D}{nQ^3}+\frac{h_BB^2}{Q^3}+\frac{bB^2}{Q^3}+\frac{2(1+m)(S+2F)D}{nQ^3}E\left(\frac{1}{\gamma}\right)+\frac{2(C_1+2C_T)D}{nQ^3}$$

and

$$\frac{\partial ETC(n, Q, B)}{\partial B^2} = \frac{h_B}{Q}+\frac{b}{Q}$$

Also calculating, $\frac{\partial ETC(n, Q, B)}{\partial Q \partial B}$, we get

$$\frac{\partial ETC(n, Q, B)}{\partial Q \partial B} = \frac{-(h_B + b)B}{Q^2}$$

Finally,

$$\left(\frac{\partial ETC(n, Q, B)}{\partial Q^2}\right)\left(\frac{\partial ETC(n, Q, B)}{\partial B^2}\right) - \left(\frac{\partial ETC(n, Q, B)}{\partial Q \partial B}\right)^2$$

$$= \frac{2\left\{(S_v + S_B + nF)(1+m)(S+2F)(C_1+2C_T)E\left(\frac{1}{\gamma}\right)\right\}D(h_B+b)}{nQ^4}$$

We get $\frac{\partial ETC(n, Q, B)}{\partial Q^2} > 0$, $\frac{\partial ETC(n, Q, B)}{\partial B^2} > 0$

and $\left(\frac{\partial ETC(n, Q, B)}{\partial Q^2}\right)\left(\frac{\partial ETC(n, Q, B)}{\partial B^2}\right) - \left(\frac{\partial ETC(n, Q, B)}{\partial Q \partial B}\right)^2 > 0$

This implies the total cost is a convex function and there exists optimal solution. On equating the equation (5) to zero we get the optimal order quantity as,

$$Q^*(n) = \sqrt{\frac{2D\left[(S_v + S_B + nF) + \frac{(m+1)}{2}(S+2F)E\left(\frac{1}{\gamma}\right) + C_T + \frac{C_1}{2}\right]}{n\left\{h_v\left(\frac{D}{P}(2-n)\right)E\left(\frac{1}{1-\gamma}\right) + (n-1) + \frac{nD}{P}E\left(\frac{\gamma}{(1-\gamma)^2}\right) + \frac{h_B b}{h_B + b}\right\}}} \quad \ldots (7)$$

Now equating the equation (6) we get

$$B^*(n) = \frac{h_B}{(h_B + b)} Q^*(n)$$

Buyer's Independent Optimal Solution

This case is for vendor-buyer is not jointly optimized. If the buyer takes his own decision the optimal solution and its effects are as follows.

$$ETC_B(Q, B) = \frac{S_B D}{Q} + \frac{FD}{Q} + \frac{h_B (Q-B)^2}{2Q} + \frac{bB^2}{2Q}$$

Obtaining optimal solution by the similar procedure we get

$$Q_B^* = \sqrt{\frac{2(S_B+F)D(h_B+b)}{bh_B}} \quad \& \quad B_B^* = \frac{h_B}{(h_B+b)} Q_B^*$$

The vendors expected annual cost is

$$ETC_v(Q) = \frac{S_v D}{Q} + C_i DE\left\{\frac{1}{1-\gamma}\right\} + h_v\left\{\frac{QD}{2P} E\left(\frac{1}{1-\gamma}\right) + \frac{QD}{2P} E\left(\frac{\gamma}{(1-\gamma)^2}\right)\right\}$$

$$+(1+m)\left(\frac{(S+2F)D}{nQ}\right) E\left(\frac{1}{\gamma}\right) + \frac{(C_1+2C_T)D}{nQ} + h'\left(\frac{D}{nCR+t_T}\right) E(\gamma)$$

4. NUMERICAL EXAMPLE

Consider the following data,

$P = 160000$, $S_v = 300$, $S_B = 100$, $h_v = 2$, $D = 50000$, $h_B = 5$, $C_i = 0.5$, $b = 10$, $\gamma = 0.04$,

$S = 100$, $A = 200$, $C_T = 2$, $C_1 = 5$, $h' = 4$, $R = 50000$, $t_T = 2/200$, $h_R = 6$, $M = 20\%$

Buyer's independent solution

Using the equations and substituting values

$Q_B^* = 2654.61$, $B_B^* = 436.44$

$ETC_B = 4289$, $ETC_v = 61,904.16$

Integrated Model

$Q^* = 1692.9$, $B^* = 284.58$, $ETC = 63.104.77$

Comparing both the solutions the solutions there is a cost reduction of Rs.3028.39 in the integrated model.

5. CONCLUSION

In this paper, we consider an incorporated vendor-buyer inventory approach with blemished generation and shortage backordering. In order to increase the profit the defective items are repaired and resold. The goal is to minimize the aggregate joint yearly expenses acquired by the vendor and buyer. Deficiencies are allowed and are totally put in backorder. Numerical illustrations demonstrate that the coordinated model gives a noteworthy cost diminishment compared to buyer's independent model. This model is beneficial to the organizations.

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