Digital Media Mix Optimization Model:  
A Case Study of a Digital Agency promoting its E-Training Services

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ABSTRACT
The paper conceptualized a Digital Media Mix Optimization Model by using Linear Programming mathematical technique. The optimization model is first of its type since it uses media effectiveness, media efficiency and media Impact Quality as three dimensions of the optimization model. The concept presented also take into account complete cost of digital media that includes media space cost, production cost and designing cost. The other model limits cost to media space cost and usually account for production cost and design cost as a common marketing agency cost which may consume a large portion of resources and cost.

This case will be best suited to company and digital agency professionals who are involved in allocation of limited budget, materials, systems and people resources as the situation present in the case is almost similar to one faced by many other companies. Management students could get an insight as to how dynamic is the real life situation and how companies would need to be alert to changing assumptions and constraints. The decision case presented here will specifically help in assimilating:

- How Linear Programming Optimization Model can be applied to real life decision situations and how alternative decision can be mathematically evaluated.
- How a company can enhance its digital media efficiency and effectiveness by improving upon constraints through qualitative changes.
- How Digital Media Mix decisions should be evaluated on both quantitative and qualitative parameters, and under different market conditions and company constraints.

BACKGROUND

Digital Media decisions, unlike Traditional media approach, are metrics based that are evaluated for outcomes before it is implemented. The primary objective and therefore the primary metrics for a Digital Media Advertising is to increase customer awareness measured as Impressions and create interest measured through number of customer Clicks. The concept is explained by consumer response model AIDA (Awareness, Interest, Desire and Action). The metrics stated here is Ad Effectiveness measure of a digital advertising. The second metrics is to achieve cost efficiencies in budget allocations to each media under given resource and market constraints. Third important metrics is to measure Impact Quality of any Digital Media in terms of audience Clicks per unit Impressions. An optimized digital media mix is achieved through consideration of these three metrics of Media Effectiveness, Media Efficiency and Media Impact Quality.

As we increase the budgets for any media on each of the digital media of Email Ads, Search ads, Display ads and YouTube ads the customer clicks in numbers increases in proportion. The paper presents a case of a digital media agency developing an optimization model for developing a digital media mix strategy for promotion of its consulting and training services using popular digital ads – Display Ads, Search Ads, Gmail Ads and YouTube Ads.

THE COMPANY

GraphMatrix Digital Solutions is a Digital Agency Start-Up with offices in Kolkata and Hyderabad. Company is involved in imparting Training and Consulting services in Digital Domain to Advertising Agencies, Consulting Firms, Online Publishers, SMEs, Corporates and Educational Institutions. The company provides Training and Consulting services for domains Digital Sales & Marketing, Digital Project Management, Digital Product Development and Digital Content Development services as in-company location based model as well as off-location outsourcing model. The company is currently focused on developing proprietary models in digital domain which is the platform on which company intends to position itself and differentiate from other digital company offering similar services.

DIGITAL MEDIA MIX OPTIMIZATION

The problem data was obtained through Depth Interview qualitative research tool of company management team. For each digital media option the budget for these operations is shown below (in $ per unit) in Table 1. One unit of digital media product is defined as 1000 Customer Impressions. Therefore, the Media Cost, Production Cost, Design Cost, Total Ad Cost and Customer Clicks are all calculated on per unit basis i.e. per 1000 impressions.
Table 1: Digital Media Operations Costs and Customer Clicks

<table>
<thead>
<tr>
<th>Digital Media</th>
<th>Media Cost (in $)</th>
<th>Production Cost (in $)</th>
<th>Design Cost (in $)</th>
<th>Total Ad Cost (in $)</th>
<th>Customer Clicks (#)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Display Ads</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>150</td>
</tr>
<tr>
<td>2. Gmail Ads</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>9</td>
<td>250</td>
</tr>
<tr>
<td>3. Search Ads</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>10</td>
<td>300</td>
</tr>
<tr>
<td>4. YouTube Ads</td>
<td>7</td>
<td>4</td>
<td>5</td>
<td>16</td>
<td>450</td>
</tr>
</tbody>
</table>

Given the current state of the digital media budget situation in the company, the company management decided that, for each year, for total allowable budget of $210000, they will have $100000 as Media Budget, $50000 of Ad Production budget and $60000 of Ad Design budget. Company wanted to estimate how much of each of the digital ads company needs to spend per year so as to maximize the yearly customer clicks.

The company management also had a second situation to decide how much budget to devote to each of the three operations (Designing, Production and Media) by changing the composition of the work force by additional training, within the total allowable budget amount of $210000 (= $100000 + $50000 + $60000). Company also wanted to estimate how much of each digital media the company needs to spend per year in this case so as to maximize the yearly customer clicks.

The first step, in order to arrive at the best possible solution the company decided to formulate the problem into a linear programming model of the digital media operations variables and constraints for both the situations.

In formulating the mathematical programming model company had to establish the variables, constraints and parameters on which the model will be based. The variables include the decision variables and output variables.

**The variables are:**

- $X_i$ be the number of units of digital media $i$ $(i=1,2,3,4)$ made per year
- $M_m$ be the amount of Digital Media Budget spent in Media Space per year
- $M_p$ be the amount of Digital Media Budget spent in Ad Production per year
- $M_d$ be the amount of Digital Media Budget spent in Ad Designing per year

Where, $X_i >= 0$ for $i=1,2,3,4$ and $M_m, M_p, M_d >= 0$
The constraints are:

(a) Media Operations Budget definition

\[ M_m = 2X_1 + 4X_2 + 3X_3 + 7X_4 \text{ (Media)} \]
\[ M_p = 3X_1 + 2X_2 + 3X_3 + 4X_4 \text{ (Production)} \]
\[ M_d = 2X_1 + 3X_2 + 2X_3 + 5X_4 \text{ (Design)} \]

(b) Media Operations Budget limits

The media operations budget limits depend upon the situation being considered. In the first situation, where the maximum budget that can be spent on each operation is specified, we simply have:

\[ M_m \leq 100000 \text{ (Media Space)} \]
\[ M_p \leq 50000 \text{ (Production)} \]
\[ M_d \leq 60000 \text{ (Design)} \]

In the second situation, where the only limitation is on the total budget spent on all operations, we simply have:

\[ M_m + M_p + M_d \leq 210000 \text{ (Total Budget)} \]

The Objective function is:

Maximise Customer Clicks, \( C \)

Hence, \( \text{Clicks, } C = 150X_1 + 250X_2 + 300X_3 + 450X_4 \)

Which gives us the complete formulation of the problem.

The solution to this function was calculated using Linear Programming Tool LINDO software. The optimal solution to the LP for the first situation has value as follows:

\( C = 5800000 \text{ and that } M_m = 82000, M_p = 50000, M_d = 60000, X_1 = 0, X_2 = 16000, X_3 = 6000 \text{ and } X_4 = 0. \) That is \( C = 5.8 \) million clicks, \( X_1 = 0, X_2 = 16 \) million impressions, \( X_3 = 6 \) million impression and \( X_4 = 0. \)

The solution presented interesting direction to the company, not to spend any budget on media 1 and media 4. The company management was totally surprised with the findings and needed to explain the following?

- How can you explain the fact that it appears that the best thing to do is not to spend any budget on the media 1 with the lowest Media Cost per unit impressions?
How can you explain the fact that it appears that the best thing to do is not to spend any budget on the Media 4 with the highest Customer Clicks per unit impressions?

Referring back to the present situation company management came up with one observation. We see that at the LP optimal we have $18,000 of media space budget that is not used (M_m = $82000 compared with a maximum media space budget of $100000) but all of the production and design budget is used. For each constraint in the LP problem one can have a "Slack or Surplus" situation. This tells us that, for a particular constraint, the difference between the left-hand side of the constraint when evaluated at the LP optimal (i.e. when evaluated with X_1, X_2, X_3 and X_4 taking the values given above) and the right-hand side of the constraint. Constraints with a "Slack or Surplus" value of zero are said to be *tight* or *binding* in that they are satisfied with equality at the LP optimal. Constraints that are not tight are called *loose*.

After this understanding of the problem, the company decided to formulate an LP for the second situation with only limitation for the total operating budget spent on all operations in a year. The solution to this situation was also obtained using mathematical software LINDO for LP.

The optimal solution to the LP for second situation has value C=7875000 (7.8 million) and that M_m=78750, M_p=78750, M_d=52500, X_1=0, X_2=0, X_3=26250 and X_4=0. This implies that company only produces variant 3. Management here noted that the associated customer click is higher than before (7875000 compared with 5800000, an increase of 36%). This indicates that the budget allocation given in situation one of $100,000, $50,000 and $60,000 for Media Space, Production and Designing respectively by the company management was not an appropriate decision!

**DIGITAL MEDIA MIX EVALUATION**

After seeing the improvement in the turnover of the company on changing the constraint, the company management wanted to know as to how the solution changes as the constants such as media budget, production budget and designing budget changes. Management found that it was necessary to be absolutely sure of their digital media strategy before implementation. The following three issues was identified by management as important for evaluating the digital media mix strategy:
Robustness

Since the cost data is not completely accurate as it is based on an estimated value from many observations and so management would like some confidence that their proposed course of action is relatively insensitive (robust) with respect to data inaccuracies. For example, management considered that the designing budget consumed by variant 3 is not accurate. It is currently set to exactly $2.0. But in reality it is likely to change as $2.1 or $1.9. What is the likely effect of this on the decision?

What company management was interested here to know is "the outcome of the strategy" rather than the specific numeric values. The click number of 5800000 gave the outcome of the strategy as "none of media 1 or 4, lots of media 2 and a reasonable amount on media 3". The click number of 7875000 gave the outcome of the strategy as "only of variant 3". What management wanted to know was when the designing budget of $2 for variant 3, is replaced by $2.1 or $1.9, the general outcome of strategy remains the same or it changes, as was the case with change in operations budget constraints.

If the general outcome of the strategy remains essentially the same under small data changes we say that the strategy is robust.

Planning Optimality

With regard to planning, management is interested in seeing how the total clicks changes as the constant data changes. For example for the media mix planning how would improving the output clicks per unit on media 4 (e.g. by 10% to 495 by raising the media impact) influence upon the optimal strategy.

Sensitivity Analysis

The company management decided upon to perform sensitivity analysis to evaluate various decision alternatives. Company first decided to deal with each of constants and constraints in turn, that is noted the figures obtained for a single change. The case of two or more things change at the same time was avoided in this study, as it will effectively need to resolve the LP Problem again. The management decided to do the following sensitivity analysis as they concluded that analysis of no other constants and constraints are needed to draw conclusions.
(a) Changing the objective function coefficient (Customer Clicks) for a variable:

Company decided to vary the coefficient of \( X_2 \) in the objective function. How will the LP optimal solution change?

Currently \( X_1=0, X_2=16000, X_3=6000 \) and \( X_4=0 \). The sensitivity analysis tells us that, provided the coefficient of \( X_2 \) in the objective function lies between \( 235.71 \) and \( 450 \), the values of the variables in the optimal LP solution will remain unchanged, though the actual optimal solution value will change.

In terms of the original problem we are effectively saying that the decision to produce 16000 of variant 2 and 6000 of variant 3 remains optimal even if the clicks per unit on variant 2 is not actually 250 (but lies in the range \( 235.71 \) to \( 450 \)).

In terms of the interpreting simplex algorithm this arises because the current simplex basic solution (vertex of the feasible region) remains optimal provided the coefficient of \( X_2 \) in the objective function lies between \( 235.71 \) and \( 450 \).

(b) Forcing a variable which is currently zero to be non-zero

For the variables, the Reduced Cost gave the management, for each variable, which is currently zero (\( X_1 \) and \( X_4 \)), an estimate of how much the objective function will change if they make that variable non-zero.

Hence they have the data as:

<table>
<thead>
<tr>
<th>Variable</th>
<th>( X_1 )</th>
<th>( X_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opportunity/Reduced Cost</td>
<td>1.5</td>
<td>0.2</td>
</tr>
<tr>
<td>New value (= or ( \geq ))</td>
<td>( X_1=A ) ( X_4=B )</td>
<td>( X_1\geq A ) ( X_4\geq B )</td>
</tr>
<tr>
<td>Estimated objective function change</td>
<td>1.5A</td>
<td>0.2B</td>
</tr>
</tbody>
</table>

The objective function will always get worse (go down if we have a maximisation problem, go up if we have a minimisation problem) by at least this estimate. The larger \( A \) or \( B \) are the more inaccurate this estimate is of the exact change that would occur if we were to resolve the LP with the corresponding constraint for the new value of \( X_1 \) or \( X_4 \) added. Note here that the value in the "Reduced Cost" column for a variable is often called the "opportunity cost" for the variable. The other alternative interpretation of the reduced cost is the amount by which the objective function coefficient for a variable needs to change before that variable will become non-zero.
Hence for variable $X_1$ the objective function needs to change by 1.5 (increase since company is maximising) before that variable becomes non-zero. In other words, referring back to original situation, the clicks per unit on media 1 would need to need to increase by 1.5 before it would be profitable to spend any budget on media 1. Similarly the click per unit on media 4 would need to increase by 0.2 before it would be effective to spend any budget on media 4.

(c) Changing the right-hand side of a constraint.

For each constraint the column headed Shadow Price told the company management exactly how much the objective function will change if there is change in the right-hand side of the corresponding constraint within the limits given in the Allowable Min/Max RHS column. The values are presented in Table 2 below:

**Table 2: Analysis of Constraints**

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Media</th>
<th>Production</th>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opportunity Cost (ignore sign)</td>
<td>0</td>
<td>0.80</td>
<td>0.30</td>
</tr>
<tr>
<td>Change in right-hand side</td>
<td>a</td>
<td>b</td>
<td>c</td>
</tr>
<tr>
<td>Objective function change</td>
<td>0a</td>
<td>0.80b</td>
<td>0.30c</td>
</tr>
<tr>
<td>Lower limit for right-hand side</td>
<td>82000</td>
<td>40000</td>
<td>33333.34</td>
</tr>
<tr>
<td>Current value for right-hand side</td>
<td>100000</td>
<td>50000</td>
<td>60000</td>
</tr>
<tr>
<td>Upper limit for right-hand side</td>
<td>150000</td>
<td>90000</td>
<td>75000</td>
</tr>
</tbody>
</table>

For example for the production budget constraint, provided the right-hand side of that constraint remains between 40000 and 90000 the objective function change will be exactly 0.80[change in right-hand side from 50000].

The direction of the change in the objective function (up or down) depends upon the direction of the change in the right-hand side of the constraint and the nature of the objective (maximise or minimise).

To decide whether the objective function will go up or down the following will give the answer:

(a) Constraint more restrictive after change in right-hand side implies objective function worse and constraint less restrictive after change in right-hand side implies objective function better.

(b) Since objective is maximize customer clicks then worse means down, better means up.

Hence, the value in the column headed Shadow Price for a constraint is often called the "marginal value" or "dual value" for that constraint. It is noted that, as would seem logical, if the constraint is loose the shadow
price is zero (as if the constraint is loose a small change in the right-hand side cannot alter the optimal solution).

**MANAGERIAL APPLICATIONS**

- In fact the approach taken both for robustness and planning issues is identical and interrelated, it is often best to study it together under **sensitivity analysis**.
- Given the LP package it is a simple matter to change the data and resolve to see how the solution changes (if at all) as certain key data items change before actually implementing the change in real life situation.
- In fact, as a by-product of using the simplex algorithm, we automatically get sensitivity information (e.g. the reduced cost information given on the LP output for the media mix planning problem). Therefore, it is worthwhile to evaluate the decision alternatives.
- For the variables, the Reduced Cost (also known as Opportunity Cost) column gives us, for each variable, which is currently zero, an estimate of how much the objective function will change if we make that variable non-zero. This is often called the "reduced cost" for the variable. This gives an idea into effectiveness of the media for digital mix strategy.
- For each constraint the column headed Shadow Price tells us by how much the objective function will change if we change the right-hand side of the corresponding constraint. This is often called the "marginal value" or "dual value" for the constraint. Therefore, managers can concentrate on removing constraints, which will give maximum customer clicks.
- This sensitivity information gives manager a measure of how robust the solution is i.e. how sensitive it is to changes in input data that enables manager in scenario building from Optimistic to Pessimistic and, also evaluates the outcome before implementation.

**READER QUESTIONS**

1. Perform the Linear Programming Problem calculations in the case using Simplex Method? Is there any other method that you recommend which can give better way to get the solution to the management problem?

2. What type of sensitivity analysis will you perform to assist the company management to find the (a) Robustness and (b) Planning Optimality of the decision?
3. Suppose management were to vary the coefficient of $X_1, X_3 \& X_4$ in the objective function. How will the LP optimal Revenue change for each case?

4. If exactly 10000 units of media 1 need to be achieved because of market requirement what would be your estimate of the new objective function value?

5. If you had an extra $10000 dollars to which operation would you assign it? If you had to take $5000 away from designing or production budget which one would you choose? What would the new objective function value be in these two cases?

TEACHING NOTE

Overview

In the above case we have discussed how the company had developed the digital marketing & media mix strategy to increase its media efficiency and effectiveness. Because of company management’s willingness to evaluate all decision alternatives, the company had an increase in customer clicks as well as cost efficiency in digital media mix. Company was first able to identify the optimum digital media mix for the company. It was also able to evaluate all constraints and also identify the constraints, which on improvement lead to greater contribution to customer clicks.

Application

This case will be best suited to company and digital agency professionals who are involved in allocation of limited budget, materials, systems and people resources. As the situation faced by the company in the case is almost similar to one faced by many other companies in India, management students could get an insight as to how dynamic is the real life situation and how companies would need to be alert to changing assumptions and constraints.

Objective of the Case

The case is a type of Decision case. This case will present the readers an insight of:

1) How to formulate Linear Programming Model to real life decision situations and how alternative decision can be mathematically evaluated.
2) Force the student to explore alternative solutions and evaluate it using various approaches as per the case.

3) How a company can enhance its digital media efficiency and effectiveness by improving upon constraints through qualitative changes.

**BIBLIOGRAPHY**


3. Problem Data (March 2017) on Digital Media through Depth Interview with company management of “GraphMatrix Digital Solutions”, Hyderabad (India)