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Connection between Frustum of the cone with Jarasandha Numbers and Some Special Numbers

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Abstract: This paper deals with the problem of obtaining the radii of the bottom and top circles of the frustum of the cone and the height 'h' in connection with the Jarasandha number and some special numbers. Few different patterns of integer points satisfying the frustum of the cone under consideration are obtained.

Keywords: Frustum of a Cone, Jarasandha numbers.

Notations:

Volume of the frustum of a cone, $V = \frac{\pi h}{3}\left[R^2 + r^2 + Rr\right]$

where, h =Height of frustum of the cone
r =Radius of the upper circle
R= Radius of the base circle

$Star_h = 6h(h-1)+1$

$CS_h = h^2 + (h-1)^2$

$Obl_h = h^2 + h$

$T_{12,h} = h(5h-4)$

Introduction:

Mathematics is the language of patterns and relationships, and is used to describe anything that can be quantified. The main goal of Number theory is to discover interesting and unexpected relationships. It is devoted primarily to the study of natural numbers and integers. In Number theory, Pythagorean triangles have been a matter of interest to various mathematicians. For an extensive variety of fascinating problems one may refer [1-3]. Apart from the polygonal numbers we have some more fascinating patterns of numbers namely Jarasandha numbers, Nasty numbers and Dhuruva numbers are presented in [4-7].

In [8], special pythagorean triangles with polygonal numbers are obtained. In [9], pythagorean triangles with Jarasandha number are exhibited. In [10-11], rectangles in connection with Jarasandha numbers are given. Recently in [12], connection between triangular number and frustum of the cone has been analysed.

This paper concerns with the problem of obtaining the radii of the bottom and top circles of the frustum and the height 'h' in connection with the Jarasandha number and some special numbers. Few different patterns of integer points satisfying the frustum of the cone under consideration are obtained.

Jarasandha Numbers:

In our indian epic Mahabharatha, we come across a person named 'JARASANDHA'. He had a boon that if he was split into two parts and thrown apart, the parts would rejoin and return to life. In fact, he was given life by the two halves of his body. In the field of mathematics, we have numbers exhibiting the same property as Jarasandha.

Consider a number of the form XC. This may split as two numbers X and C and if these numbers are added and squared we get the same number XC.

(i.e) $XC = (X+C)^2 = XC$

Note: If C is an n-digit number, then $(X+C)^2 = (10^n)(X)+C$

Method of Analysis:

Let 'R', 'r' be the radii of the bottom and top circles and 'h' the height of the frustum of the cone to be determined as follows:

Case 1: For the 2-digit Jarasandha number 81, we have the relation

$$\frac{3V}{\pi h} - CS_h + Star_h - 10\,Obl_h + T_{12,h} = 81 \quad (1)$$

which reduces to

$$R^2 + r^2 + Rr = (h+9)^2 \quad (2)$$

Introducing the linear transformations

$$R = u+v,\ \ r = u-v, \qquad u,v \ge 0 \quad (3)$$

in (2), we have

$$3u^2 + v^2 = z^2 \ , \qquad \text{where} \quad z = h+9$$

Case 2: For the 4-digit Jarasandha number 2025, the relation obtained is,

$$\frac{3V}{\pi h} - CS_h + Star_h - 50\,Obl_h + 9T_{12,h} = 2025 \quad (4)$$

which reduces to

$$3u^2 + v^2 = z^2 \ , \qquad \text{where} \quad z = h+45$$

Case 3: Considering the 4-digit Jarasandha number 3025, we have the relation,

$$\frac{3V}{\pi\, h} + CS_h - Star_h - 62\, Obl_h + 13T_{12,\,h} = 3025 \qquad (5)$$

which reduces to $3u^2 + v^2 = z^2$, where $z = h + 55$

Case 4: For the Jarasandha number 9801, we have the relation

$$\frac{3V}{\pi\, h} - CS_h + Star_h - 110\, Obl_h + 21T_{12,\,h} = 9801 \qquad (6)$$

reduces to $3u^2 + v^2 = z^2$, where $z = h + 99$

Case 5: For the 5-digit Jarasandha number 88209, we have the relation

$$\frac{3V}{\pi\, h} - CS_h + Star_h - 330\, Obl_h + 65T_{12,\,h} = 88209 \qquad (7)$$

which reduces to $3u^2 + v^2 = z^2$, where $z = h + 297$

For all the five cases, all the relations reduces to

$$3u^2 + v^2 = z^2 \qquad (8)$$

In all the five cases the radii of bottom and top circles are same but the height of the frustum is different. We present below 4 different solutions satisfying the frustum:

Set 1:

The most cited solution of $3u^2 + v^2 = z^2$ is

$$u = 2ab$$
$$v = 3a^2 - b^2 \qquad (9)$$
$$z = 3a^2 + b^2$$

Thus the radii of bottom and top circles of the frustum are given by,

$$R = 3a^2 - b^2 + 2ab$$
$$r = 2ab - 3a^2 + b^2 \qquad (10)$$

The heights of the frustum are given by

Case 1	Case 2	Case 3	Case 4	Case 5
$h = 3a^2 + b^2 - 9$	$h = 3a^2 + b^2 - 45$	$h = 3a^2 + b^2 - 55$	$h = 3a^2 + b^2 - 99$	$h = 3a^2 + b^2 - 297$

Set 2:

Assume $z = a^2 + 3b^2$ (11)

where a and b are non-zero integers.

Substituting (11) in (8), and using factorization method,

$$(v+i\sqrt{3}u)\,(v-i\sqrt{3}u) = \left((a+i\sqrt{3}b)^2\,(a-i\sqrt{3}b)^2\right) \qquad (12)$$

Equating the like terms and comparing real and imaginary parts, we get

$$u = 2ab$$
$$v = a^2 - 3b^2 \qquad (13)$$

Thus the radii of bottom and top circles of the frustum are given by,

$$R = a^2 - 3b^2 + 2ab$$
$$r = 2ab - a^2 + 3b^2 \qquad (14)$$

The heights of the frustum are given by

Case 1	Case 2	Case 3	Case 4	Case 5
$h = a^2 + 3b^2 - 9$	$h = a^2 + 3b^2 - 45$	$h = a^2 + 3b^2 - 55$	$h = a^2 + 3b^2 - 99$	$h = a^2 + 3b^2 - 297$

Set 3:

(8) can be written as $v^2 + 3u^2 = 1 * z^2$

(15)

and write $1 = \dfrac{(1+i\sqrt{3})(1-i\sqrt{3})}{4}$ (16)

Substituting (11) & (16) in (15), and using factorization method,

$$u = \frac{1}{2}(a^2 - 3b^2 + 2ab)$$
$$v = \frac{1}{2}(a^2 - 3b^2 - 6ab) \qquad (17)$$

Since our interest is on finding integer solutions, we have choose a and b suitably so that u and v are integers. Let us take $a = 2A$ & $b = 2B$ we have

$$u = 2A^2 - 6B^2 + 4AB$$
$$v = 2A^2 - 6B^2 - 12AB \qquad (18)$$
$$z = 4A^2 + 12B^2$$

Thus the radii of bottom and top circles of the frustum are given by,

$$R = 4A^2 - 12B^2 - 8AB$$
$$r = 16AB \qquad (19)$$

The heights of the frustum are given by

Case 1	Case 2	Case 3	Case 4	Case 5
$h = 4A^2 + 12B^2 - 9$	$h = 4A^2 + 12B^2 - 45$	$h = 4A^2 + 12B^2 - 55$	$h = 4A^2 + 12B^2 - 99$	$h = 4A^2 + 12B^2 - 297$

Set 4:

Instead of (16), write $1=\frac{(1+4i\sqrt{3})(1-4i\sqrt{3})}{49}$ (20)

Substituting (11) & (20) in (15), and using factorization method,

$$u=\frac{1}{7}(4a^2-12b^2+2ab)$$
$$v=\frac{1}{7}(a^2-3b^2-24ab) \qquad (21)$$

Since our interest is on finding integer solutions, we have choose a and b suitably so that u and v are integers. Let us take $a=7A$ & $b=7B$ we have

$$u=28A^2-84B^2+14AB$$
$$v=7A^2-21B^2-168AB \qquad (22)$$
$$z=49A^2+147B^2$$

Thus the radii of bottom and top circles of the frustum are given by,

$$R=35A^2-105B^2-154AB$$
$$r=21A^2-63B^2+182AB \qquad (23)$$

The heights of the frustum are given by

Case 1	Case 2	Case 3	Case 4	Case 5
$49A^2+147B^2-9$	$49A^2+147B^2-45$	$49A^2+147B^2-55$	$49A^2+147B^2-99$	$49A^2+147B^2-297$

Note:

For the Jarasandha numbers 81, 2025, 9801, 88209 we also get the relations,

1. $\frac{3V}{\pi h}-CS_h+Star_h-10\,Obl_h+3T_{12,h}=81$ (24)

2. $\frac{3V}{\pi h}-CS_h+Star_h-50\,Obl_h+11T_{12,h}=2025$ (25)

3. $\frac{3V}{\pi h}-CS_h+Star_h-110\,Obl_h+23T_{12,h}=9801$ (26)

4. $\frac{3V}{\pi h}-CS_h+Star_h-330\,Obl_h+67T_{12,h}=88209$ (27)

which also reduces to $3u^2 + v^2 = z^2$,

where $z = h - 9$ for (24)

$z = h - 45$ for (25)

$z = h - 99$ for (26)

$z = h - 297$ for (27)

Conclusion:

In this paper, we have discussed the connection between frustum of the cone with some special numbers and jarasandha numbers. To conclude, one may consider connections between some special numbers and jarasandha numbers with other geometrical figures.

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