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The Effect of Viscous Dissipation on MHD nonsimilar Boundary Layer Flow with an Exponentially decreasing Velocity Distribution

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Abstract

The objective of this paper is to investigate the effect of viscous dissipation on non-similar boundary layer MHD flow with an exponentially decreasing velocity distribution. The governing nonlinear partial differential equations in the velocity and temperature fields are written in non-dimensional form using suitable similarity transformations. The resulting final set of coupled nonlinear partial differential equations is solved using an implicit finite difference scheme in combination with a quasilinearization technique. It is found that the heat transfer coefficient is strongly affected by the viscous dissipation parameter. Also, it is observed that due to heat produced by viscous dissipation, the direction of heat transfer gets reversed.

Keywords: *Viscous dissipation*, MHD, Velocity, Skin friction, Heat transfer

INTRODUCTION

A wide range of boundary layer flow problems have come into prominence that demands a detailed analysis of the flow with the boundary layer taking non similarity into account in the recent years. A review of the non-similarity solution methods along with the citations of some relevant publications is given in an earlier study by Dewey and Gross [1].

In the presence of an adverse pressure gradient, the thickness of the boundary layer increases (since momentum is consumed both by the wall shear and the pressure gradient), the direction of flow changes and a region of recirculating flow is created. Hence the boundary layer breaks away from the surface. The point at which this happens is known as separation point. In steady flows separation is defined by

the point where the surface shear stress vanishes. There are large number of studies of the phenomenon of separation such as Brown and Stewartson [2], Williams [3] and Davis and Walker [4].Venkatachala and Nath [5] investigated non similar incompressible boundary layers with vectored mass transfer. The development and separation of laminar boundary layer with an exponentially increasing pressure gradient was studied by Curl [6]. A numerical solution for the laminar boundary layer with an exponentially decreasing velocity distribution, with and without mass transfer parameter is presented by Chiam [7]. Jayakumar.et.al [8] studied the effect of magnetic field on laminar boundary layer flow with an exponentially decreasing velocity distribution. Roy [9] examined the non similar solution of steady laminar boundary layer flow into exponentially decreasing free stream flow with multiple slot suction and injection. Recently, nonsimilar flow and heat transfer flow with an exponentially decreasing velocity distribution along with an applied magnetic field was investigated by Jayakumar and Eswara [10]. The main aim of the present paper is to study the effect of viscous dissipation on MHD laminar boundary layer flow and heat transfer with an exponentially decreasing velocity distribution.

MATHEMATICAL FORMULATION

Consider a semi-infinite porous plate at a uniform temperature. The plate is fixed in a vertical position with leading edge horizontal. The physical co-ordinates (x,y) are chosen such that x is measured from the leading edge in the stream wise direction and y is measured normal to the surface of the plate. The co-ordinate system and flow configuration are shown in Fig.1(a). Consider laminar boundary layer flow and heat transfer in the presence of magnetic field with an exponentially decreasing velocity distribution. Uniform suction is imposed at the surface. The applied magnetic field is assumed to be uniform and is in the direction normal to the surface. All fluid properties are assumed to be constant and the magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected. No electric field is assumed to exist and the Hall Effect is negligible. The effect of viscous dissipation is included in the analysis. The equations governing this boundary layer forced convection flow situation are based on the balance laws of mass, linear momentum and energy. These can be written as[11]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=u_e\frac{\partial u_e}{\partial x}+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}(u-u_e) \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma B_0^2}{\rho c_p}u^2 \qquad (3)$$

The physics of the problem suggests the following boundary conditions

$$\begin{aligned} &u=v=0, && T=T_w \quad \text{at} && y=0\\ &u\to u_e(x) && T=T_\infty \quad \text{as} && y\to\infty \end{aligned} \qquad (4)$$

Defining the stream function in the usual way such that $u=\frac{\partial \psi}{\partial y}$ and $v=-\frac{\partial \psi}{\partial x}$, substituting the following dimensionless variables

$$\eta=\left(\frac{u_e}{2\upsilon x}\right)^{1/2}y;\quad \xi=\frac{x}{c};\quad \psi=(2u_e\upsilon x)^{1/2}f(\xi,\eta);\quad T=T_\infty+(T_w-T_\infty)G(\xi,\eta) \qquad (5)$$

into Eqns.**(**1**)**-**(**3**)**. We see that the continuity Eq. **(**1**)** is identically satisfied and Eqns. **(**2**)** and **(**3**)** results in the following non similar equations

$$f'''+\left(\frac{m+1}{2}\right)ff''+m\left(1-f'^{2}\right)-M\left(f'-1\right)=\xi\left(f'f'_{\xi}-f''f_{\xi}\right) \quad (6)$$

$$\Pr^{-1}G''+\left(\frac{m+1}{2}\right)fG'+Ec\left(\frac{f''^{2}}{2}+Mf'^{2}\right)=\xi\left(f'G_{\xi}-G'f_{\xi}\right) \quad (7)$$

where m is a dimensionless pressure gradient parameter defined by

$$m=\frac{\xi}{u_e}\frac{du_e}{d\xi} \quad (8)$$

And

$$u=u_e f';\quad v=-\left(\frac{\upsilon u_e}{2x}\right)^{1/2}\{f+mf+2\xi f_{\xi}+\eta f'(m-1)\}$$

$$M=\frac{\sigma B_0^2 x}{\rho u_e};\qquad \Pr=\frac{\upsilon}{\alpha};\qquad Ec=\frac{u_e^2}{c_p\left(T_w-T_{\infty}\right)};\quad \mathrm{Re}_x=\frac{u_e x}{\nu} \quad (9)$$

Here u_e is the external flow velocity at the edge of the boundary layer and is given by

$$u_e=u_0\left(1-\varepsilon e^{\xi}\right)\quad 0<\varepsilon<1 \quad (10)$$

Where u_0 is constant.
The dimensionless form of the boundary conditions in the absence of suction become

$$f=0;\qquad f'=0\quad G=1\quad \text{at}\quad \eta=0$$
$$f'=1\quad G=0\quad \text{as}\quad \eta\to\infty \quad (11)$$

If the uniform suction with velocity v_0 is introduced, the condition in the equation is replaced by

$$f(\xi,0)=A\sqrt{\xi} \quad (12)$$

where the suction parameter

$$A=-v_0\sqrt{\frac{c}{2u_0\upsilon}}$$

is positive.
Important physical parameters for this flow and heat transfer are local skin friction and the localNusselt number. These are defined as follows:

$$Cf=\frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{\rho u_e^2}=\frac{f''}{\sqrt{2}}\left(\mathrm{Re}_x\right)^{-1/2} \quad (13)$$

$$Nu=\frac{-x\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w-T_{\infty}}=-\frac{G'}{\sqrt{2}}\left(\mathrm{Re}_x\right)^{1/2} \quad (14)$$

Here, u and v are velocity components in x and y direction; F is dimensionless velocity; T and G are dimensional and dimensionless temperatures, respectively; ξ,η, are transformed co-ordinates; ψ and f are the dimension and dimensionless stream functions respectively; Pr is the Prandtlnumber ; Ec is the Eckert number ; ν, α are respectively kinetic viscosity and thermal diffusivity ; w_0 and ∞ denote conditions at the edge of the boundary layer on the wall and in the free stream respectively and prime $(')$ denotes derivatives with respect to η.

It is remarked here that when M = 0.0 Eqn. (6) becomes

$$f'''+\left(\frac{m+1}{2}\right)ff''+m\left(1-f'^{2}\right)=\xi\left(f'f'_{\xi}-f''f_{\xi}\right) \qquad (15)$$

which is exactly same as those of T.S.Chiam[7]. Also, It is worth mentioning here that when Ec = 0.0 Eqns. (6) and (7) becomes

$$f'''+\left(\frac{m+1}{2}\right)ff''+m\left(1-f'^{2}\right)-M\left(f'-1\right)=\xi\left(f'f'_{\xi}-f''f_{\xi}\right) \qquad (16)$$

$$\Pr^{-1}G''+\left(\frac{m+1}{2}\right)fG'=\xi\left(f'G_{\xi}-G'f_{\xi}\right) \qquad (17)$$

which is exactly same as those of Jayakumar and Eswara[10] in the presence of magnetic field.

METHOD OF SOLUTION

The non-linear coupled partial differential equations (6) and (7) under the boundary conditions (11) have been solved numerically using an implicit finite difference scheme in combination with the quasi-linearization technique [12]. Quasi-linearization technique can be viewed as a generalization of the Newton–Raphson approximation technique in functional space. An iterative sequence of linear equations is carefully constructed to approximate the non-linear Eqns.(6) and (7) under the boundary conditions(11) achieving quadratic convergence and monotonicity of the sequence of solutions.

Applying quasilinearization technique [12], we replace the nonlinear partial differential equations (6) and (7) by an iterative sequence of linear equations as follows:

$$F''^{(k+1)}+X_1^{(k)}F'^{(k+1)}+X_2^{(k)}F^{(k+1)}+X_3^{(k)}F_{\xi}^{(k+1)}=U_1^{(k)} \qquad (18)$$

$$G''^{(k+1)}+Y_1^{(k)}G'^{(k+1)}+Y_2^{(k)}F'^{(k+1)}+Y_3^{(k)}F^{(k+1)}+Y_4^{(k)}G_{\xi}^{(k+1)}+=U_2^{(k)} \qquad (19)$$

Where the coefficient functions with iterative index k are known and functions with iterative index $k+1$ are to be determined. The boundary conditions become

$$F^{(k+1)}(\xi,0)=0,\quad G^{(k+1)}(\xi,0)=1 \quad and$$

$$F^{(k+1)}(\xi,\infty)=1,\quad G^{(k+1)}(\xi,\infty)=0 \qquad (20)$$

The coefficients in (16) and (17) are given by

$$X_1^{(k)}=\left(\frac{m+1}{2}\right)f+\xi f_{\xi}$$

$$X_2^{(k)}=-2mF-M-\xi F_{\xi}$$

$$X_3^{(k)}=-\xi F$$

$$U_1^{(k)}=-m-mF^2-M-\xi FF_{\xi}$$

$$Y_1^{(k)}=\left(\frac{m+1}{2}\right)\Pr f+\xi f_{\xi}\Pr$$

$$Y_2^{(k)} = \mathrm{Ec\,Pr\,F'}$$

$$Y_3^{(k)} = 2MEc\,\mathrm{Pr}\,F - \xi G_\xi\,\mathrm{Pr}$$

$$Y_4^{(k)} = -\xi\,\mathrm{F\,Pr}$$

$$U_2^{(k)} = \frac{1}{2}\mathrm{Ec\,Pr\,F'^2} + \mathrm{Ec\,Pr\,M\,F^2} - \xi\,\mathrm{G_\xi\,Pr\,F}$$

The resulting linear partial differential equations (18) and (19) along with (20) were expressed in difference form, considering central difference in η-direction and backward difference in ξ-direction. The equations were then reduced to a system of linear algebraic equations with a block tridiagonal structure which is solved using Varga's algorithm [13]. In order to obtain grid independent numerical results, the grid sizes $\Delta\eta$ and $\Delta\xi$ have been optimized. The value of η_∞ (i.e., the edge of the boundary layer) has been taken as 8.0 throughout the computation. A convergence criterion based on the relative difference between the current and the previous iteration has been used. The solution is assumed to have converged and the iterative process is terminated when $\mathrm{Max}\left[\left|(F')_w^{(k+1)} - (F')_w^{(k)}\right|,\ \left|(G')_w^{(k+1)} - (G')_w^{(k)}\right|\right] < 10^{-4}$

RESULTS AND DISCUSSION

To asses the accuracy of the method which we have used the skin friction parameter $f_{\eta\eta}(\xi,0)$ have been obtained by solving the Eqn.(15) in the absence of magnetic field. Also, the velocity and temperature profiles are computed by solving the Eqns. (16) and (17) without viscous dissipation. Our results are found to be in excellent agreement with those of Chiam [7] and Jayakumar and Eswara[10]. The comparison is shown in Figure 1(b) and Figure 2(a) and 2(b) respectively.

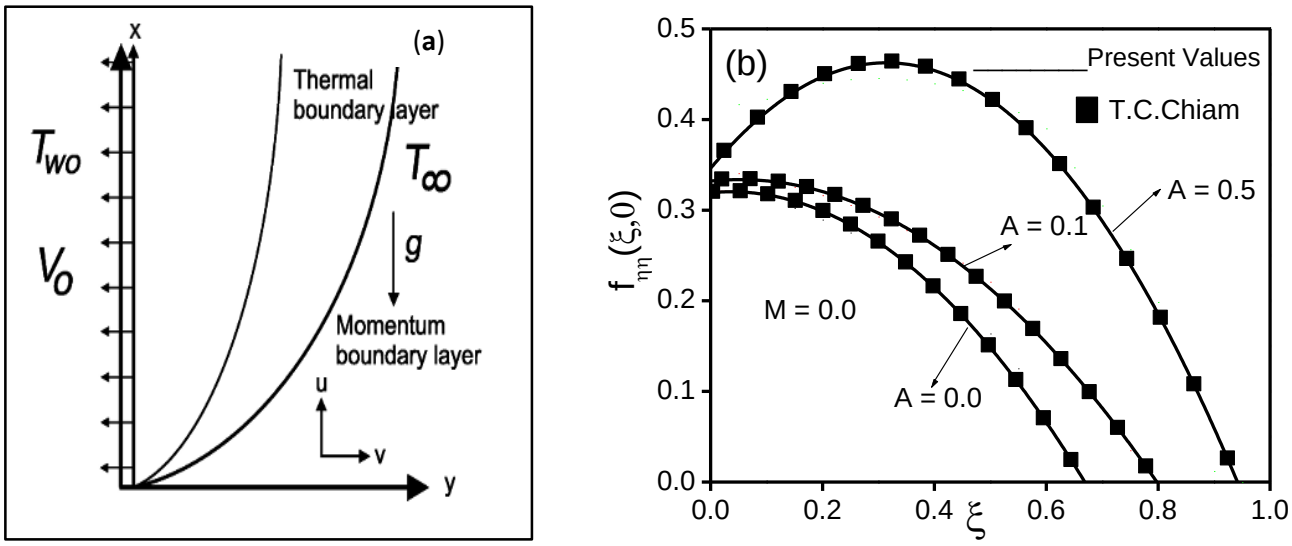

Fig.1. (a) Flow Model (b) Comparison of skin friction parameter with Chiam [7]

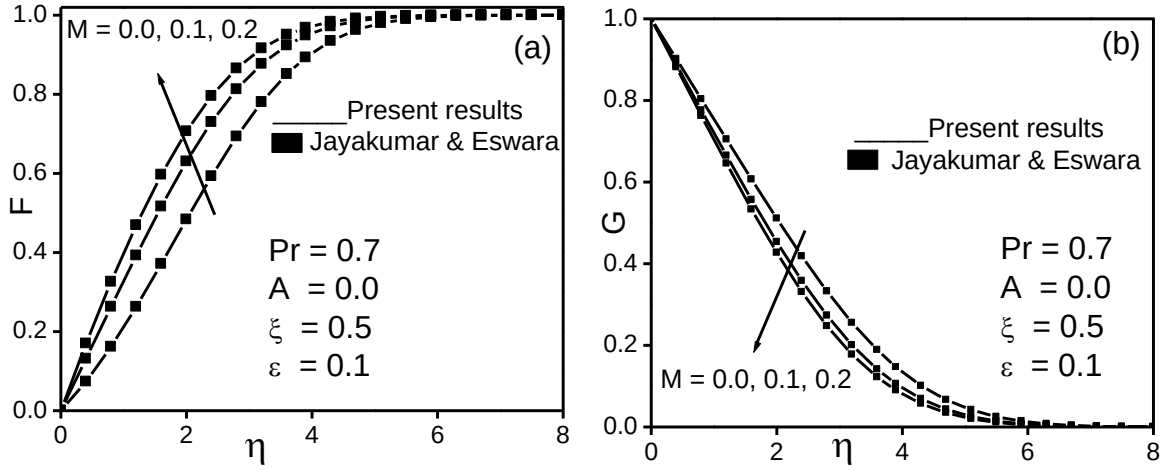

Fig. 2. Comparison of (a) velocity (b) temperature profiles with Jayakumar and Eswara [10].

The role of viscous dissipation (Ec) on dimensionless heat transfer coefficient [$\mathrm{Nu}(\mathrm{Re}_L)^{-1/2}$]in the absence and presence of suction parameter (A) with magnetic field (M = 0.2) for air (Pr = 0.7) are shown for different stream wise locations ξ in Fig.3. It is observed that when Ec increases (Ec=0.5) the heat transfer coefficient ($\mathrm{Nu}(\mathrm{Re}_L)^{-1/2}$) decreases, whereas, Ec decreases (Ec= - 0.5) the $\mathrm{Nu}(\mathrm{Re}_L)^{-1/2}$ increases in both the cases. Whereas, the heat transfer coefficient increases for few values of ξ and then decrease to attain a steady value showing the effect of suction parameter clearly. In fact, the percentage of decrease of heat transfer coefficient when Ec=0.5 is 9.72% and the percentage of increase in $\mathrm{Nu}(\mathrm{Re}_L)^{-1/2}$ when Ec= - 0.5, is 9.7% in the absence of suction parameter. When A = 0.5 the corresponding decrees in $\mathrm{Nu}(\mathrm{Re}_L)^{-1/2}$ 13.98% and the increase in heat transfer coefficient is 14% at the stream wise location ξ = 0.5.

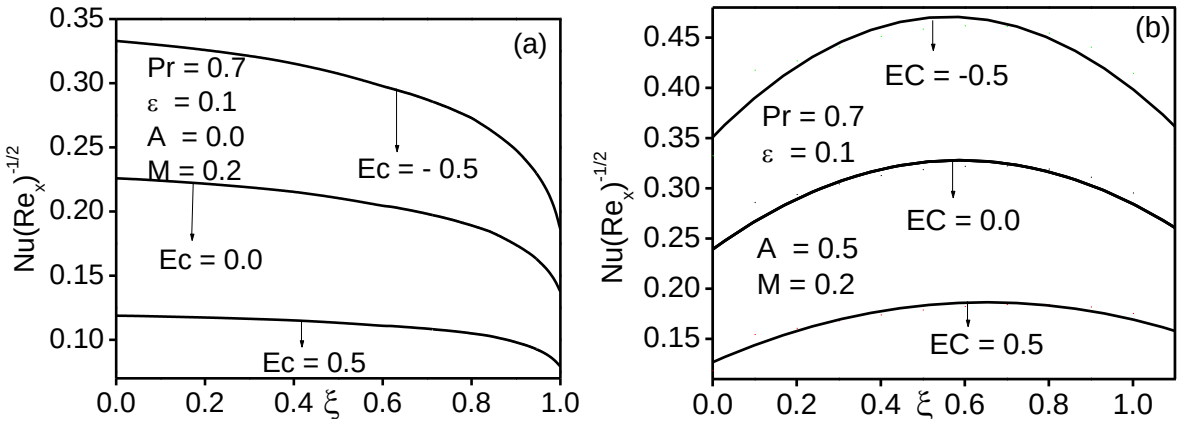

Fig.3.The effect of viscous dissipation on heat transfer coefficient (a) in the absence of suction parameter (A = 0.0) (b) in the presence of suction parameter (A = 0.5)

Fig.4 displays the effect of viscous dissipation on temperature profile (G) with and without suction parameter (A) for M = 0.2. It is clear from the numerical results that the thermal boundary layer is significantly affected by the viscous dissipation parameter (Ec) in both the cases. Further, we observe that there is an undershoot in the temperature profile [Fig.4(a) and 4(b)] which indicates the reverse direction of heat transfer i.e., instead of wall to fluid, heat transfer take place from fluid to wall.

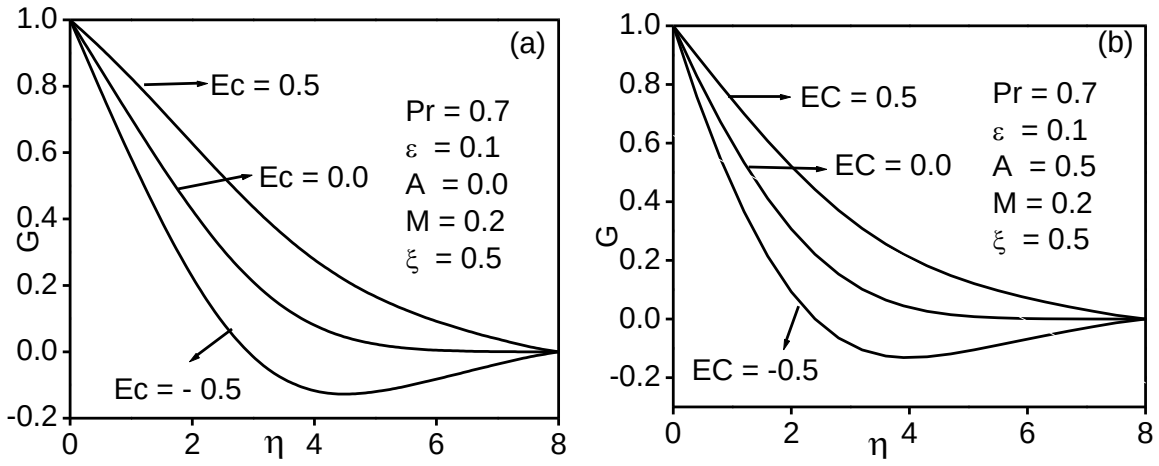

Fig.4. Viscous dissipation effect on temperature profile (a) in the absence of suction parameter (A = 0.0) (b) in the presence of suction parameter (A = 0.5)

CONCLUSIONS

Non similar solution of steady, laminar, incompressible MHD boundary layer flow with an exponentially decreasing velocity distribution with viscous dissipation has been obtained starting from the origin of stream wise coordinate to the exact point of separation. The transformed parabolic partial differential equations were solved by an implicit finite difference scheme along with quasilinearization technique. From the present investigation we observe the following in absence and presence of suction parameter:

1. The heat transfer coefficient increases as viscous dissipation parameter (EC) decreases, while, heat transfer coefficient decreases when Ec increases.
2. The viscous dissipation changes the direction of heat transfer, which takes place from fluid to wall, instead of initial, wall to fluid.

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