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Effects of high Brinkmann number on the laminar flow and heat transfer of a visco-elastic fluid between two coaxial circular cylinders in the presence of heat sources

P. Paikray[1]

Department of Physics, KPAN College Bankoi Khordha (Odisha) India

S. Dash[2]

Department of Physics, Regional Institute of Education, Bhubaneswar, India

S. Biswal[3]

Former Professor of Physics, SERC Bhubaneswar (Odisha) India

Abstract:

This paper deals with the study of the effects of high Brinkmann number on the momentum and heat transfer by laminar visco-elastic flow between two coaxial circular cylinders in the presence of internal heat generating sources. Constitutive equations of continuity, momentum and energy have been developed taking into account the Maxwell and Noll non-Newtonian fluid model. The problem is analytically formulated and solved by the Runge-Kutta numerical method of integration. The effects of high Brinkmann number of the velocity, temperature and pressure field are shown by the graphs. Numerical values of the Skin friction and rate of heat transfer are entered in the tables. Lastly truncation error involved in the estimation by computations is calculated. It is observed that the velocity at any point of the flow field falls with the rise of the Brinkmann number (Br). But the velocity rises at any point of the flow field with rise of the non-Newtonian parameter(R_c).As a result of which, the effect of Br is duely reversed to that of R_c .

Keywords: Visco-elastic flow, heat transfer, Brinkmann number, Coaxial circular cylinders, heat sources.

Introduction:

The study of heat transfer by laminar flow of Newtonian and non-Newtonian fluids has gained considerable importance in the field of technology as well as engineering sciences. Schlichting[1] has discussed several cases of exact solutions of heat transfer in viscous fluids. Jain[2] has studied the problem of heat transfer by laminar flow of elastico-viscous liquids through parallel walls when both the walls are at uniform temperatures and Mishra[3] has extended the problem to the case when the walls are at linearly varying temperature. Both of them obtained an approximate solution of the differential equation governing the velocity field by considering the elasticity of the liquid to be small. Earlier to his work of 1965. Mishra[4] had discussed a problem of heat transfer by laminar elastico-viscuos flow through parallel porous walls. Again, Mishra[5] studied the problem of heat transfer by laminar flow of an elastico-viscous liquid in a circular cylinder with linearly varying wall temperature. Mishra and Roy[6] have studied the problem of heat transfer by the rotator flow of an elastico-viscous liquid between two coaxial circular cylinders. Further, the two workers[7] have discussed the problem of heat transfer by laminar motion of an elastico-viscous liquid between two coaxial circular cylinders due to longitudinal motion of inner cylinder. Raju and Devanathan[8] have studied the problem of heat transfer to non-Newtonian fluids and laminar flow through concentric annuli with or without suction. Dash and Behera[9] have studied the laminar flow development and transfer of a visco-elastic liquid in a converging channel.

Besides above, a number of researchers have devoted themselves to the study of heat transfer problems relating to the flow of non-Newtonian fluids in several geometric shapes and for different physical situations. Heat transfer in the flow of second order fluids over an enclosed rotating disc has been studied by Sharma and Bhatia[10]. Bhatnagar and Bhatnagar[11] have analysed the problem of heat transfer due to a sphere steadily rotating in an infinitely extending non-Newtonian fluid. Again, in 1970, Bhatnagar[12] alone has discussed a problem on heat transfer in a visco –elastic fluid flowing around a steadily rotating and thermally insulated sphere. Sharma and Bhatnagar[13] have studied the problem of low Reynolds number heat transfer from a sphere in a laminar flow of non –Newtonian fluids. Heat transfer characteristics of some non-Newtonian fluids has also been discussed by Rao and Kloor[14]. Kapur and Tyagi[15] have discussed the problem of heat transfer in some flows of a certain class of non-Newtonian fluids. Krishnan and Pandya[16] have studied the problem of heat transfer to non-Newtonian fluids in jacketed agitated vessels. Combined laminar free and forced convection heat transfer to non-Newtonian fluids has been analysed by Kubair and Pei[17]. Pandian and Raja Rao[18] have discussed the problem of heat transfer to non-Newtonian liquids in agitated vessels. Rajsekharan, Kubair and Kuloor[19] have studied the problem of heat transfer to non-Newtonian fluids in coiled pipes in laminar flow. Mahalingam, Tilton and Coulson[20] have studied the problem of heat transfer in laminar flow of non-Newtonian fluids. Rath and Bastia[21] have studied a problem of steady flow and heat transfer in a visco-elastic fluid between two coaxial rotating disks. Acharya and Padhy[22] have studied a problem of heat transfer by laminar visco- elastic flow between two coaxial circular cylinders having different temperatures.

Further, Dash and Biswal[23] have investigated the problem of laminar visco elastic flow and heat transfer between two coaxial circular cylinders in the presence of internal heat generating sources. Our aim here in this paper to study the effects of high Brinkmann number on the laminar flow and heat transfer of a visco-elastic fluid between two coaxial circular cylinders in the presence of heat sources. The elastico-viscous fluid model given in the equation below with the definition of $\tilde{P}_{ij}$ has been considered to formulate the problem.

$$P_{ij} + \lambda \tilde{P}_{ij} = 2\eta_0 e_{ij} + 4\mu_c e_{i\alpha} e_{\alpha j} , \qquad (1.1)$$

And the tensor $\tilde{P}_{ij}$ has been defined by the equation,

$$\frac{\delta b_{ik}}{\delta t} = \frac{\partial b_{ik}}{\partial t} + V^m b_{ik,m} + V^m_{,i} b_{mk} + V^m_{,k} b_{im} \qquad (1.2)$$

Where $\frac{\delta}{\delta t}$ denotes the convective derivative and b_{ik} is a second order tensor.

[2]

μ_c is the co-efficient of cross-viscosity, λ_1 is the relaxation time, which is given by $\lambda_1 = \frac{\eta_0}{G}$,

η_0 is the co-efficient of viscosity and G is modulus of elasticity.

The non-linear equations of motion and energy have been solved by the Runge-Kutta numerical method of integration.

Formulation of the problem:

The flow under consideration takes place between two coaxial, infinitely long, horizontal circular cylinders maintained at different temperature. The inner cylinder of radius r_1 is assumed to be at a lower temperature T_1 than the temperature T_0 of the outer cylinder of radius r_0. We select cylindrical polar co-ordinates with z-axis along the axis of the tubes.

It is supposed that the flow takes place as the inner cylinder moves longitudinally. The outer cylinder is held fixed. The velocity component v_r, v_θ and v_z compatible with the equation of continuity are,

$$v_r = 0, \quad v_\theta = 0 \quad \text{and} \quad v_z = w(r) \tag{2.1}$$

From equation (2.1) and (1.1) , the various components of stress are,

$$P_{rr} = \mu_c \left(\frac{dw}{dr} \right)^2 \tag{2.2}$$

$$P_{rz} = \eta_0 \frac{dw}{dr} + \lambda_1 \mu_c \left(\frac{dw}{dr} \right)^3 \tag{2.3}$$

And

$$P_{zz} = \left(\mu_c + 2\lambda_1 \eta_0 \right) \left(\frac{dw}{dr} \right)^2 + 2\lambda_1 \mu_c \left(\frac{dw}{dr} \right)^4 \tag{2.4}$$

Since the motion is as due to shear only , we have

$$\frac{\partial p}{\partial z} = 0 \tag{2.5}$$

Using equation(2.5), the momentum equations are given by,

$$\frac{dp}{dr} = \mu_c \left\{ 2 \frac{dw}{dr} \frac{d^2 w}{dr^2} + \frac{1}{r} \left(\frac{dw}{dr} \right)^2 \right\} \tag{2.6}$$

$$0=\eta_0\left(\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}\right)+\lambda_1\mu_c\left[3\left(\frac{dw}{dr}\right)^2\frac{d^2w}{dr^2}+\frac{1}{r}\left(\frac{dw}{dr}\right)^3\right] \tag{2.7}$$

[3]

The equation (2.7) under the following boundary conditions yields the velocity distribution while the equation (2.6) determines the pressure field.

$r=r_1$: $w=w_1$

$r=r_0$: $w=0$ (2.8)

Where w_1 is the velocity of the inner cylinder in z-direction.

The equation of energy in the present problem is given by,

$$\alpha\left(\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}\right)+\phi+S^+\left(T-T_0\right)=0 \tag{2.9}$$

Where $\alpha=\frac{K}{\rho C_p}$ is the thermal diffusivity, ϕ is the viscous dissipation function and S^+ is the dimensional source parameter.

The dissipation function ϕ is prescribed as

$$\phi=2e_{rz}p_{rz}=\frac{dw}{dr}\left\{\eta_0\frac{dw}{dr}+\lambda_1\mu_c\left(\frac{dw}{dr}\right)^3\right\} \tag{2.10}$$

From equation(2.9) and (2.10) the equation of energy is

$$\alpha\left(\frac{d^2T}{dr^2}+\frac{1}{r}\frac{dT}{dr}\right)+\eta_0\left(\frac{dw}{dr}\right)^2+\lambda_1\mu_c\left(\frac{dw}{dr}\right)^4+S^+\left(T-T_0\right)=0 \tag{2.11}$$

With the boundary conditions

$r = r_1$: $T = T_1$

$r = r_0$: $T = T_0$ (2.12)

We now introduce the non-dimensional parameters:

$1=h-r_1$, $\sigma=\frac{1}{r_1}$, $\lambda=\frac{r-r_i}{r_0-r_1}$, $r=r_1\left(1+\sigma\lambda\right)$,

$$\bar{w} = \frac{w}{w_1},\ R_c = \frac{\lambda_1 \mu_0 w_1^2}{1^2 \eta_0},\ \upsilon_0 = \frac{\mu_c}{p_1^2},\ P = \frac{p}{\rho w_1^2},$$

$$\theta = \frac{T - T_i}{T_0 - T_1},\ B_r = \frac{\eta_0 W_i^2}{\alpha\left(T_0 - T_1\right)},\ P_r = \frac{\upsilon}{\alpha}, \qquad (2.13)$$

$$R = \frac{W_1 r_1}{\upsilon},\ S = 4\frac{S^+ \upsilon}{W_1^2},$$

Where,

R_c , is the elastic parameter,

υ_c , is the cross-viscous parameter,

B_r, is the Brinkmann number,

P_r, is the Prandtl number,

R, is the Reynolds number,

S, is the source – strength,

And λ, is the normalized space variable.

With the aid of the quantities defined in equation(2,13), the equations(2.6) – (2.8) and (2.11),(2.12) are transformed as

$$\frac{dp}{d\lambda} = \upsilon_c\left[2\frac{d\bar{w}}{d\lambda}\frac{d^2\bar{w}}{d\lambda^2} + \frac{\sigma}{1+\sigma\lambda}\left(\frac{d\bar{w}}{d\lambda}\right)^2\right] \qquad (2.14)$$

$$\frac{d^2\bar{w}}{d\lambda^2}\left\{1 + 3R_c\left(\frac{d\bar{w}}{d\lambda}\right)^2\right\} + \frac{\sigma}{1+\sigma\lambda}\frac{d\bar{w}}{d\lambda}\left\{1 + R_c\left(\frac{d\bar{w}}{d\lambda}\right)^2\right\} = 0 \qquad (2.15)$$

$$\frac{d^2\theta}{d\lambda^2} + \frac{\sigma}{1+\sigma\lambda}\frac{d\theta}{d\lambda} + B_r\left(\frac{d\bar{w}}{d\lambda}\right)^2 + R_c B_r\left(\frac{d\bar{w}}{d\lambda}\right)^4 + \frac{P_r R^2 S\theta}{4} = 0 \qquad (2,16)$$

Together with the boundary conditions

λ=0 : $\bar{w}$=1 , θ=0

λ=1 : $\bar{w}$=0 , θ=1 (2.17)

Solution of the equations:

We solved equation (2.15) and (2.16) by Runge-Kutta method of numerical integration. We proceed with the step by step integration as illustrated in Ralton and Wilf[24]. We only derive below the set of the simultaneous equations each of order one and the set of primary equations used in the method. These equations are obtained by setting

$Y_1=\lambda$, $y_2=\bar{w}$, $y_3=\bar{w}'$, $y_4=\theta$ and $y_5=\theta'$ (3.1)

Where a prime(') denotes differentiation w.r.to λ.

Then the desired first order equations called to be 'reduced equation' from (2.15) and (2.16) are

$$y_1'=0 \tag{3.2}$$

$$y_2'=y_3 \tag{3.3}$$

$$y_3'=-\frac{\sigma y_3\left(1+R_c y_3^2\right)}{\left(1+\sigma y_1\right)\left(1+3R_c y_3^2\right)} \tag{3.4}$$

$$y_4'=y_5, \tag{3.5}$$

$$y_5'=-\frac{\left[\sigma y_5 B_r y_3^2\left(1+\sigma y_1\right)\left(1+R_c y_3^2\right)+\frac{P_r R^2 S y_4}{4}\left(1+\sigma y_1\right)\right]}{\left(1+\sigma y_1\right)} \tag{3.6}$$

Where,

$$y_1(0)=0,\ y_2(0)=1,\ y_4(0)=0$$

And $$y_1(1)=1,\ y_2(1)=0,\ y_4(1)=1 \tag{3.7}$$

The rough starting values supplied to $y_3(0)$ and $y_5(0)$ which are not known are corrected By means of the corrective procedure as outlined in Fox[25]. This method is used in correcting the rough starting values supplied for $y_3(0)$ and $y_5(0)$ to the desired degree of accuracy 'e'.

Let 'e' be one of the values of $y_3(0)$ and $y_5(0)$. Differentiating the primary equations(3.1) with regard to 'e' and denoting

$$\frac{dy_i}{de}=P_i, \qquad (i=1,2,\ldots\ldots\ldots,5)$$

The following equations are obtained.

$$p_1' = 0, \tag{3.8}$$

$$p_2' = p_3 \tag{3.9}$$

$$p_3' = \frac{\sigma p_1\left(1+R_c y_3^2\right)}{\left\{\left(1+\sigma y_1\right)^2\left(1+3R_c y_3^2\right)\right\}} + \frac{\sigma R y_3 P_3\left(1+R_c y_3^2\right)}{\left(1+\sigma y_1\right)\left(1+3R_c y_3^2\right)} - \frac{\sigma P_3}{\left(1+\sigma y_1\right)} \tag{3.10}$$

$$p_4' = p_5 \tag{3.11}$$

$$p_5' = \frac{\sigma^2 p_1 y_5}{\left(1+\sigma y_1\right)^2} - \frac{\sigma p_5}{\left(1+\sigma y_1\right)} - 4B_r R_c y_3^3 p_3 - 2B_r y_3 p_3 - \frac{P_r R^2 S p_4}{4} \tag{3.12}$$

The initial conditions for the equations(3.) _ () for purpose of integration are

$$p_i = \delta_{3i}$$

$$p_i = \delta_{5i} \quad , \text{i=1,2,3,4,5.} \tag{3.13}$$

After solutions of the functions y_i, i=1,2,......5,we next calculate the pressure gradient $\frac{dp}{d\lambda}$, and the Skin friction τ_1 and τ_0 at the walls. From equations(2.14) and (2.15) , $\frac{dp}{d\lambda}$ is given by

$$\frac{dp}{d\lambda} = \upsilon_c \sigma y_3^2 \frac{\left(R_c y_3^2 - 1\right)}{\left(1+\sigma y_1\right)\left(1+3R_c y_3^2\right)} \tag{3.14}$$

Using the quantities defined in (2.13) in equation (2.3), the Skin frictions at the inner and outer walls are given by

$$\tau_1 = \left[\frac{p_{rz^1}}{\eta_0 w_1}\right]_{\lambda=0} = y_3\left(0\right) = R_c y_3^3\left(0\right) \tag{3.15}$$

$$\tau_0 = \left[\frac{p_{rz^1}}{\eta_0 w_1}\right]_{\lambda=1} = y_3\left(1\right) + R_c y_3^3\left(1\right) \tag{3.16}$$

Moreover , the rates of heat transfer from the walls are $\left(\frac{d\theta}{d\lambda}\right)_{\lambda=0}$ and $\left(\frac{d\theta}{d\lambda}\right)_{\lambda=1}$. These quantities are equal to

$$q_1 = \left(\frac{d\theta}{d\lambda}\right)_{\lambda=0} = \left(\theta'\right)_{\lambda=0} = y_5\left(0\right) \quad (3.17)$$

$$q_0 = \left(\frac{d\theta}{d\lambda}\right)_{\lambda=1} = \left(\theta'\right)_{\lambda=1} = y_5\left(1\right) \quad (3.18)$$

The integration by Runge-Kutta method of the equations (3.2) to (3.6) subject to the conditions(3.7) and the equations (3.8) to (3.12) subject to the conditions (3.13) for correcting the rough values of $y_3\left(0\right)$ and $y_5\left(0\right)$ are performed in $0 \le \lambda \le 1$ with step size h=0.1. for the purpose of determining the error of truncation, the above equations are also integrated with step size h=0.2 for some set of values of the parameters.

Result and Discussion:

The flow phenomenon is characterized by the important parameter R_c, B_r and S which are respectively known to be the elastic parameter, Brinkmann number and the source strength. The effects of these parameters on the velocity field, temperature field, Skin friction and rate of heat transfer have been studied by means of diagrams and tables.

Velocity distribution: -

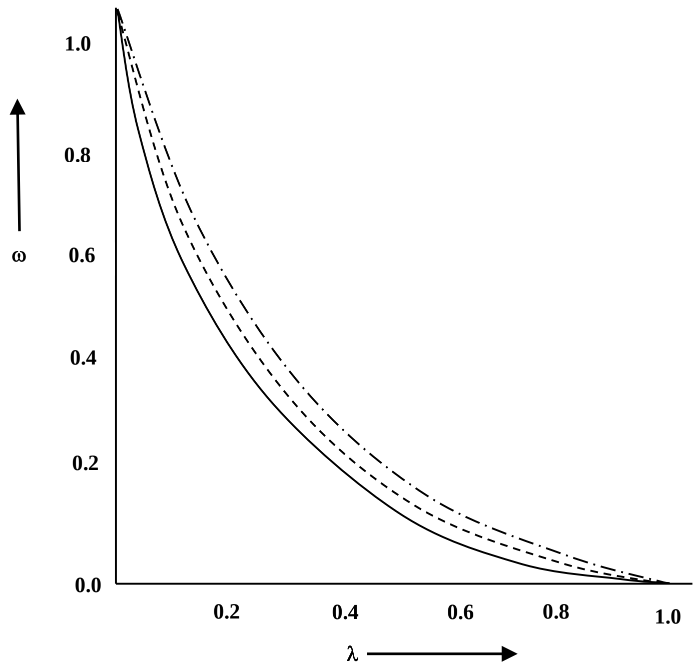

FIGURE: 1

EFFECT OF R_C ON VELOCITY FIELD

B_r = 1.0 , P_R=1.0, R=5.0 , S=0.1 ,σ =4.0

R_C=0.0 , - - - - - - R_C=0.05, - - - - - - R_C=0.1

Figure-1 depicts the effects of the elastic element on the fluid flow. It is observed from the velocity profiles that the velocity rises at every point of the flow field with the increase of R_c . However, the nature of the velocity curves remains the same for both Newtonian(R_c=0) and non-Newtonian(R_c>0) fluids. It is worth mentioning here that the flow pattern remains invariant with heat source in the present problem and without heat source in the work of Padhy[22].

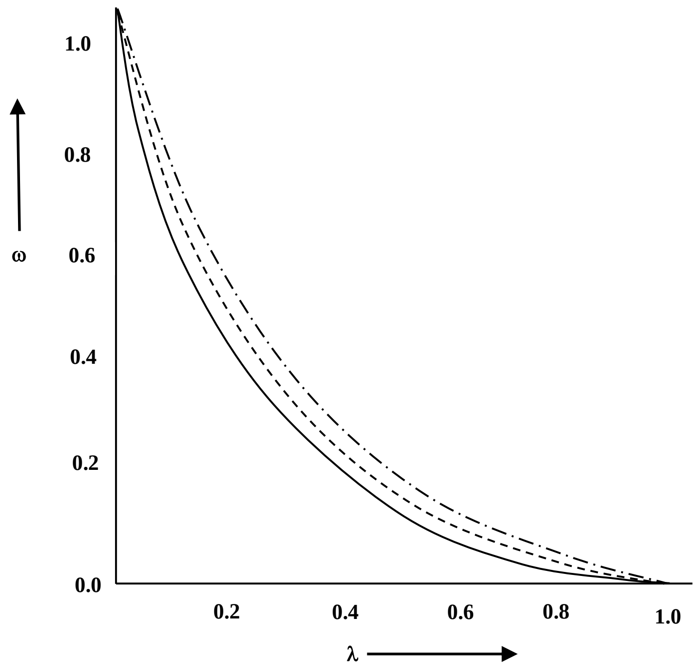

FIGURE: 2

EFFECT OF LOW VALUE OF B_r ON VELOCITY FIELD

R_C = 0.05 , P_R=1.0, R=5.0 , S=0.1 ,σ =4.0

B_r=0.0 , - - - - - - B_r=1.0, - - - - - - B_r=2.0

Figure-2 is drawn to exhibit the effect of low Brinkmann number on the flow characteristics of the flow characteristics of the visco-elastic fluid. It is noted that the velocity at any point of the flow field decreases as Brinkmann number B_r increases. Consequently, the effect of B_r is duely reversed to that of R_c.

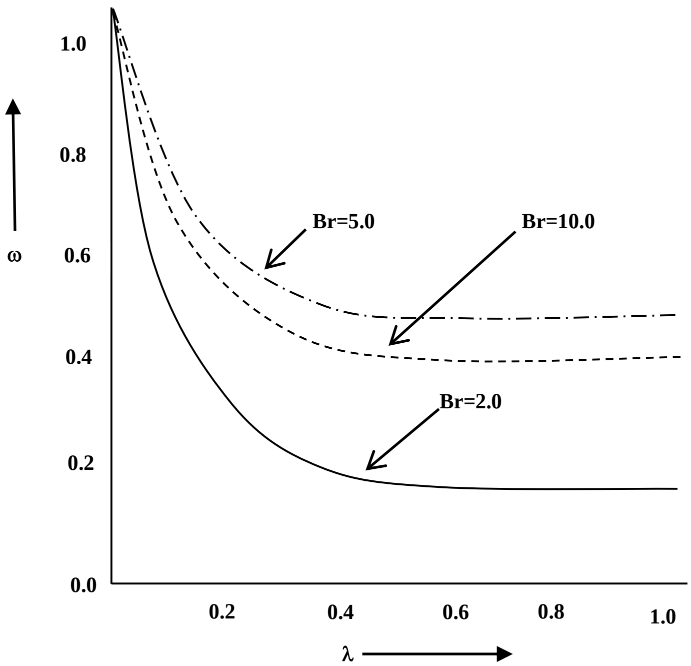

FIGURE: 3

EFFECT OF HIGH VALUE OF B_r ON VELOCITY FIELD

R_C = 0.05 , P_R=1.0, R=5.0 , S=0.1 ,σ =4.0

B_r=10.0 , - - - - - - B_r=5.0, - - - - - - B_r=2.0

Figure-3 shows the effect of high B_r on the velocity field. It is observed that the visco-elastic flow of the fluid is accelerated with the rise of Brinkmann number (B_r) from B_r=2.0 to 5.0. But for higher value of Brinkmann number (B_r=10.0), the velocity of the fluid decreases. Thus, the behaviour of the fluid for high value of Brinkmann number is ambiguous, which rests on the present theoretical study pertaining to numerical computation.

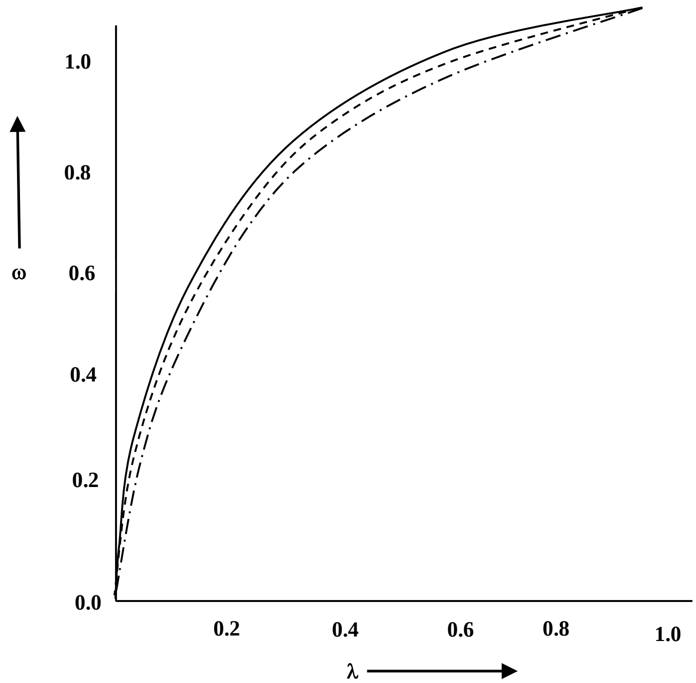

FIGURE: 4

EFFECT OF R_C ON TEMPERATURE FIELD

$B_r = 1.0$, $P_r=1.0$, R=5.0 , S=0.1 ,σ =4.0

$R_c=0.0$, - - - - - - $R_c=0.05$, - - - - - - $R_c=0.1$

Figure-4 presents the temperature distribution for various values of elastic parameter R_c. As the elasticity of the fluid becomes more, the temperature of the flow field becomes more. But the findings of Padhy[22] without heat source is opposite. Therefore, it is concluded that the heat source dominates over the elastic property of the fluid in modifying the temperature distribution.

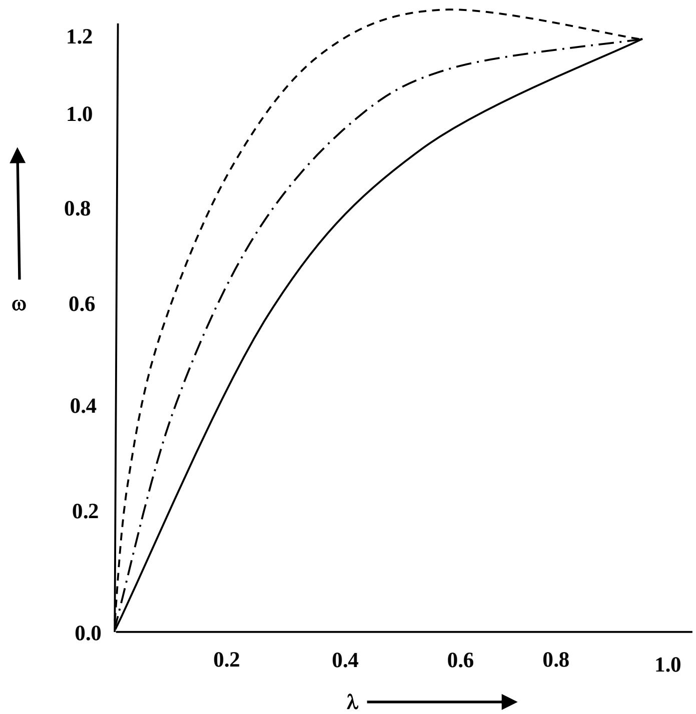

FIGURE: 5

EFFECT OF B_r ON TEMPERATURE FIELD

P_r = 1.0 , R_c=0.05, R=5.0 , S=0.1 ,σ =4.0

B_r=0.0 , - - - - - - B_r=1.0, - - - - - - B_r=2.0

Figure-5 shows the effect of B_r on the temperature distribution, with the increase of B_r, temperature rises at any point of the flow field.

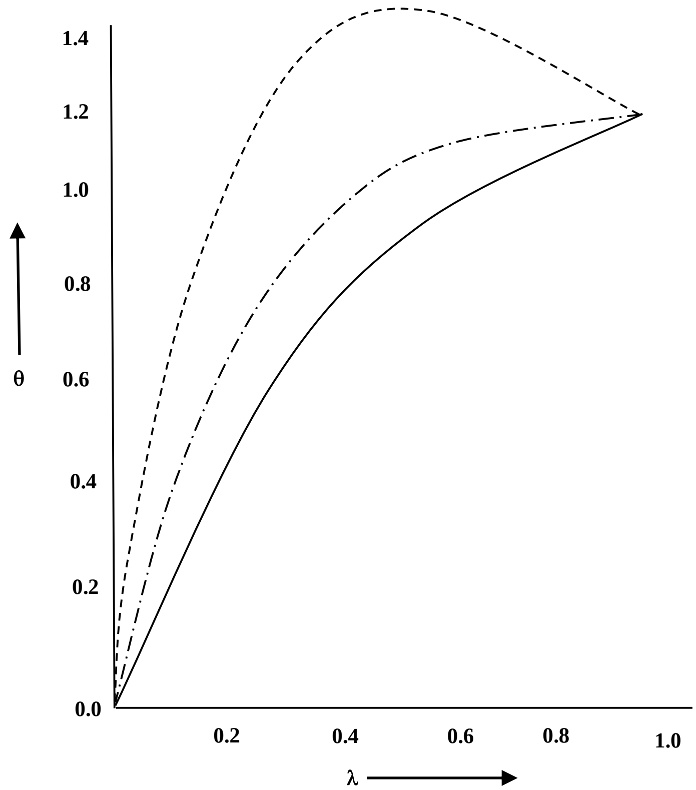

FIGURE: 6

EFFECT OF S ON TEMPERATURE FIELD

R_C = 0.05 , P_r=1.0, R=5.0 , B_r=1.0 ,σ =4.0

S = 0.0 , - - - - - - S = 0.1, - - - - - - S = 0.5

Figure-6 has been shown the effect of the source parameter S. Here, an increasing behavior of the temperature profiles is marked with the increase of the source strength. Moreover, the rise in temperature is significant with the increase of S from 0.1 to 0.5.This can be quantitatively presented as 40.86%.

Pressure field: -

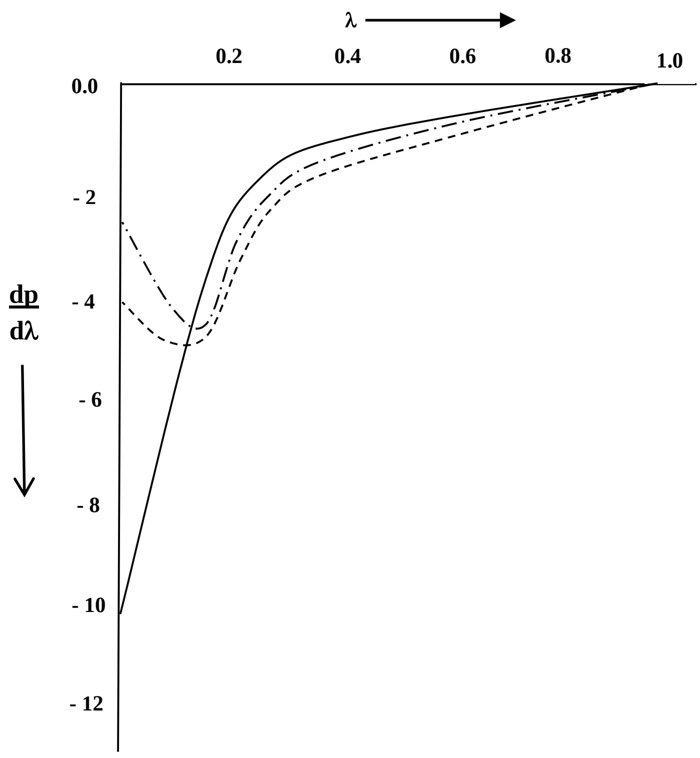

FIGURE: 7

EFFECT IF R_C ON PRESSURE GRADIENT

$V_C = 0.5$,$\sigma = 4.0$

R_C=0.0 , - - - - - - R_C=0.05, - - - - - - R_C=0.1

Figure-7 shows the effect of R_c on $\frac{dp}{d\lambda}$, the pressure gradient . The cross viscous parameter υ_c has no effect either on the velocity or on the temperature field, but it modifies the pressure field. From equation (3.14), it is noticed that the magnitude of $\frac{dp}{d\lambda}$ is magnified in the scale of υ_c. It is further observed from the curves of figure-6 that $\frac{dp}{d\lambda}$ increases with the increase in the value of R_c near the vicinity of the surface λ=0. But an opposite effect is marked in the region $0.1 < \lambda \leq 1$. Thus, a comparative study of the present work with that of the previous work[22] without heat sources reveals that the source parameter effects the pressure gradient in the region away from the surface λ=0 .

Skin- friction: -

Table-1, Values of τ_1 and τ_0 for B_r=1.0, σ =4.0, S=0.1, P_r=1.0, R=5.0

R_c	τ_1	τ_0
0.00	-2.2596964	-0.5237277
0.05	-2.4290450	-0.5629643
0.10	-2.5779325	-0.5974694

The values of the skin- friction τ_1 and τ_0 for different values of R_c are tabulated in table-1. It is noticed that both τ_1 and τ_0 decrease with the increase in the value of R_c. But , in the absence of heat source, opposite effect has been observed by Padhy[22].

Rate of heat transfer: -

Table-2,values of the rates of heat transfer from the wall for σ =4.0, S=0.1, P_r=1.0, R=5.0

B_r	$\left(\frac{d\theta}{d\lambda}\right)_{\lambda=0}$	$\left(\frac{d\theta}{d\lambda}\right)_{\lambda=1}$	R_c
0.0	2.7580631	0.2692154	0.00
	2.7755913	0.2661377	0.05
	2.7982312	0.2632541	0.10
0.1	2.8998189	0.2382400	0.00
	2.9083547	0.2356562	0.05
	2.9128751	0.2331478	0.10
1.0	4.0196075	-0.0126718	0.00
	4.0799589	-0.0415536	0.05
	4.1363172	-0.0659394	0.10

Table-2 is prepared to show the effect of R_c and B_r on the rate of the heat transfer from the surfaces of the inner and outer cylinder.With the increase in the value of R_c, the rate of heat transfer q_1 increases and q_0 decreases. Similar effect is observed for B_r. This is well in agreement with the results obtained by Mishra and Roy[7] for an elastico-viscous liquid without heat sources.

Estimation of truncation error: -

Table-3, the truncation error involved in computation, R_c=0.05, σ=4.0, B_r=1.0, S=0.1, P_r=1.0, R=5.0

λ	$\bar{w}(.1)$	$\bar{w}(.2)$	$T_n\bar{w}(.1)$	$\theta(.1)$	$\theta(.2)$	$T_n(\theta)(.1)$
0.2	0.6511904	0.6511899	0.03×10^{-6}	0.5286804	0.5286761	0.28×10^{-6}
0.4	0.4217781	0.4217769	0.08×10^{-6}	0.7837378	0.7837394	-0.1×10^{-6}
0.6	0.2505165	0.2505062	0.68×10^{-6}	0.4749736	0.4749893	-1.0×10^{-6}
0.8	0.1137994	0.1137844	1.0×10^{-6}	0.1920053	0.1920356	-1.3×10^{-6}

Table-3 gives the estimate of truncation error involved in the computations. For determining this, we use the following formula based on Richardson extrapolation {Cf. Hildebrand[26]} .

$$T_n\left(y_{n+1}^{(h)}\right) = y_{n+1} - y_{n+1}^{(h)} \frac{y_{n+1}^{(h)} - y_{n+1}^{(2h)}}{2^r - 1} \tag{4.1}$$

Where y_{n+1} is true ordinate at x_{n+1}. $y_{n+1}^{(h)}$ and $y_{n+1}^{(2h)}$ are the values obtained with spacing h and 2h respectively and which are obtained at two steps starting at x_{n-1} where r is the order of the Runge-Kutta formula. In the present case r being 4, with h=0.1 and h=0.2 the numerical integrations are performed for the set of values of the parameters R_c=0.05, B_r=1.0, σ =4.0 and S=0.1.We notice that the truncation errors involved in the calculations of $\bar{w}$, the velocity and θ, the temperature at λ=0.2,0.4,0.6 and 0.8 are of the orders $O(10^{-6})$.

Conclusion:

Following significant conclusions are drawn from the present theoretical investigation of the heat transfer by laminar visco-elastic flow between co-axial circular cylinders in the presence of heat sources.

i. The velocity rises at any point of the fluid with the increase of the non-Newtonian parameter R_c.
ii. The flow is decelerated with the rise of Brinkmann number.
iii. The effect of Brinkmann number on the velocity of the fluid is duely reversed to that of elastic parameter R_c.
iv. With rise of B_r , the temperature rises at any point of the fluid.

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