International Journal of Research in IT and Management (IJRIM)
Available online at: http://euroasiapub.org
Vol. 8, Issue 9, September -2018
ISSN(o): 2231-4334 | ISSN(p): 2349-6517 | Impact Factor: 6.505

Research on hedging theory of modern portfolio management

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Abstract: The research for the theory of futures hedging ratio has matured at present, and take the foreign theories and models as the leading factor. Considering that the hedging models for risk minimization are easy to understand and calculate, they are widely used in research fields. Hedging ratios based on risk minimization are divided into two categories, static hedge ratio and dynamic hedge ratio. The static hedge ratio is derived from the assumption that market conditions remain unchanged, and the value is fixed, and it can be obtained by the least square model (OLS), vector auto-regressive model (B-VAR), error correction model (ECM). While the dynamic hedging ratio is obtained by generalized auto-regressive conditional heteroscedastic model (GARCH).
Keywords: OLS model, B-VAR model, ECM model, GARCH model, the optimal hedge ratio

1.Least squares model

The OLS model is proposed by Johnson (1960)[1], after that, Ederington (1979) [2] et al. applied it to the empirical analysis of hedging. Johnson (1960) using econometric model to make linear regression between spot price and futures price, and get the optimal hedge ratio estimate by the least squares (OLS)model. Generally, the price of assets is used as the variable of the model, and the optimal hedge ratio can be obtained by a series of calculations, and the optimal hedging ratio is just the coefficient of slope in the model. Cause this method is simple, intuitive and easy to operate, it has been widely used. The OLS model formula is as follows:

$$S_t = \alpha + \beta F_t + \varepsilon \qquad (1\text{-}1)$$

In the formula(1-1), S_t stands for the spot price; F_t stands for the futures price; α stands for the intercept item; β stands for the equation regression coefficient; ε stands for the random interference.

In order to eliminate the heteroscedasticity of the model equation, the logarithmic form of spot price and futures price are used to replace the spot price and futures price. Then the formula (1-1) becomes the formula(1-2):

$$\Delta \ln S_t = \alpha + \beta \Delta \ln F_t + \varepsilon_t \qquad (1\text{-}2)$$

In the formula(1-2), $\Delta \ln S_t$ stands for the logarithmic return of spot price; $\Delta \ln F_t$ stands for the logarithmic return of futures price; α stands for the intercept item; β stands for the equation regression coefficient; ε_t stands for the independent and identically distributed random variables, the mean value is 0, and the variance is constant.

Avoid price risk, after hedging, the profit or loss is expressed as:

$$P = \Delta \ln S_t + h(\Delta \ln F_t - c) \qquad (1\text{-}3)$$

In the formula(1-3), c stands for the cost of holding a unit futures contract, such as taxes and fees.

If the risk is expressed by variance, the risk of hedging is:

$$Var(P) = \delta_S^2 + h^2\delta_F^2 + 2h\delta_{SF} \qquad (1\text{-}4)$$

In the formula(1-4), δ_S stands for the standard deviation of return on spot assets; δ_F stands for the standard deviation of return on futures assets; δ_{SF} stands for the covariance of $\Delta \ln S_t$ and $\Delta \ln F_t$; ρ stands for the correlation coefficient of $\Delta \ln S_t$ and $\Delta \ln F_t$; h stands for the optimal hedge ratio.
For formula (1-4), the derivative of h ,and is equal to 0, then get that:

$$\frac{\partial Var(P)}{\partial h} = 2h\delta_F^2 + 2\delta_{SF} = 0 \qquad (1\text{-}5)$$

$$h = -\frac{\delta_{SF}}{\delta_F^2} = -\frac{\rho\delta_S\delta_F}{\delta_F^2} = -\frac{\rho\delta_S}{\delta_F} \qquad (1\text{-}6)$$

Therefore, h can be expressed as:

$$h = -\frac{Cov(\Delta \ln S_t, \Delta \ln F_t)}{Var(\Delta \ln F_t)} \qquad (1\text{-}7)$$

Usually we take the opposite number of h .Therefore, the slope of the regression equation is the optimal hedge ratio:

$$h = \frac{Cov(\Delta \ln S_t, \Delta \ln F_t)}{Var(\Delta \ln F_t)} = \beta \qquad (1\text{-}8)$$

2.B-VAR model

With the development of time series econometrics, there are many shortcomings of the ordinary least squares model has been shown. For example, it needs some assumptions to calculate the optimal hedging ratio by the ordinary least squares method, that is, it needs to assume that the changes of futures price and spot price are stable, and the error terms which is independent and identical to the mean value 0 and the variance is a fixed constant do not have sequence correlation and are unique.
Herbst (1989), Myers[3] & Thompson (1989) found that the residuals in the model have a sequence correlation, which will affect the optimal hedge ratio calculated by OLS model, and some other variables should be added to the model at the same time. Therefore, they proposed to calculate the optimal hedge ratio, we can abandon the ordinary least squares model and adopt a more accurate double-vector auto-regressive model (B-VAR) to calculate the hedging ratio. The form of vector auto-regression model is:

$$\Delta \ln S_t = \alpha_S + \sum_{i=1}^{k} \beta_{S_i} * \Delta \ln S_{t-i} + \sum_{i=1}^{k} \gamma_{S_i} * \Delta \ln F_{t-i} + \varepsilon_{S_t} \qquad (2\text{-}1)$$

$$\Delta \ln F_t = \alpha_F + \sum_{i=1}^{k} \beta_{F_i} * \Delta \ln S_{t-i} + \sum_{i=1}^{k} \gamma_{F_i} * \Delta \ln F_{t-i} + \varepsilon_{F_t} \qquad (2\text{-}2)$$

In the formula(2-1)and(2-2), α_S and α_F stands for the intercept item; β_{S_i} , β_{F_i} , γ_{S_i} , γ_{F_i} stands for the regression coefficient; $\Delta \ln S_{t-i}$ stands for the lagged gains in spot market; ε_{S_t} , ε_{F_t} stands for the white noise sequence.
The optimal hedge ratio is:

$$h = \frac{Cov(\varepsilon_{S_t}, \varepsilon_{F_t})}{Var(\varepsilon_{F_t})} \qquad (2\text{-}3)$$

3.Error correction model

B-VAR solves all kinds of problems in the hypothesis of OLS model, but it also has its own shortcomings. In real life, there is often a non-stationary problem in financial time series. In order to solve this problem, economists usually have to change the data to a certain extent, so as to obtain a stationary time series. Although this method is feasible and effective in statistical and quantitative calculation, it often changes the meaning of the data itself and makes the research results incomplete. In order to solve this problem, Granger (1981) proposed the co-integration theory, and six years later Granger and Engle further extended co-integration theory.

Granger (1981) & Engle (1987)[4] holds that if two time series are non-stationary in themselves, but their linear regression is stationary, then it is certain that there is a co-integration relationship between them. This co-integration relationship exists a formula for error correction. Ghosh (1993)[5] calculates the optimal hedging ratio by using vector error correction model (ECM) on the basis of co-integration theory. The greatest improvement of vector error correction model (ECM) is that the co-integration relationship between futures price and spot price is taken into account, so the optimal hedging ratio calculated by this model is better than that calculated by other models. The form of error correction model (ECM) considering co-integration is as follows:

$$\Delta \ln S_t = \lambda_S * Z_{t-1} + \varepsilon_{S_t}, \Delta \ln F_t = \lambda_F * Z_{t-1} + \varepsilon_{F_t} \quad (3\text{-}1)$$

In the formula(3-1), Z_{t-1} stands for the error correction term; λ_S , λ_F stands for the regression coefficient. Then get that:

$$\Delta \ln S_t = \alpha_S + \sum_{i=1}^{k} \beta_{S_i} * \Delta \ln S_{t-i} + \sum_{i=1}^{k} \gamma_{S_i} * \Delta \ln F_{t-i} + \lambda_S * Z_{t-1} + \varepsilon_{S_t} \quad (3\text{-}2)$$

$$\Delta \ln F_t = \alpha_F + \sum_{i=1}^{k} \beta_{F_i} * \Delta \ln S_{t-i} + \sum_{i=1}^{k} \gamma_{F_i} * \Delta \ln F_{t-i} + \lambda_F * Z_{t-1} + \varepsilon_{F_t} \quad (3\text{-}3)$$

The optimal hedge ratio is:

$$h = \frac{Cov(\varepsilon_{S_t}, \varepsilon_{F_t})}{Var(\varepsilon_{F_t})} \quad (3\text{-}4)$$

4.Generalized auto-regressive conditional heteroscedastic model

These models follow a hypothesis that the variance of the residual sequence in the equation is a fixed value. In financial research, the residual represents the volatility which is the risk of securities. Therefore, this hypothesis is equivalent to the hypothesis that in hedging, the risk of spot and futures is a fixed value, and will not change with time.

However, in real life, the risk of commodity futures and spot is not fixed, economists also found that time series have the characteristics of volatility clustering, which shows that the variance of residual series is not invariable, but has the characteristics of changing with time. In order to reflect the changing characteristics of financial time series, Engle proposed an auto-regressive conditional heteroscedastic (ARCH) model in 1982. Bollerslev[6] extended the ARCH model and proposed the generalized auto-regressive conditional heteroscedastic (GARCH) model in 1986. The introduction of GARCH model provides a more accurate method for calculating the optimal hedging ratio. Cause the optimal hedging ratio calculated by GARCH model takes into account the characteristics of futures and spot risks at any time, so the optimal hedge ratio calculated by this model is called dynamic hedging ratio. More and more researchers are also using the dynamic hedge ratio.

Park, Switzer (1995) & Lien[7] (1996) used the GARCH model to calculate the optimal hedge ratio in the spot market of the futures period, and compared the results with the previous static hedging ratio, obtained that the optimal hedge ratio obtained by using this dynamic model is better than those of static hedge ratio. The form of the GARCH model is as follows:

$$\Delta \ln S_t = \alpha_S + \beta_S \Delta \ln F_t + \varepsilon_{S_t} \quad (4\text{-}1)$$

$$\Delta \ln F_t = \alpha_F + \beta_F \Delta \ln S_t + \varepsilon_{F_t} \quad (4\text{-}2)$$

$$\varepsilon_{S_t} \sim N(0, \sigma^2_{S,t}), \varepsilon_{F_t} \sim N(0, \sigma^2_{F,t}) \quad (4\text{-}3)$$

$$\sigma^2_{S,t} = \omega_S + \theta_S * \varepsilon^2_{S,t-1} + \delta_S * \sigma^2_{S,t-1}, \sigma^2_{F,t} = \omega_F + \theta_F * \varepsilon^2_{F,t-1} + \delta_F * \sigma^2_{F,t-1} \quad (4\text{-}4)$$

In the formula(4-1)to(4-4), $\sigma^2_{S,t}$ stands for the conditional variance of spot return; $\sigma^2_{F,t}$ stands for the conditional variance of futures return; ω_S , ω_F stands for the constant term; θ_S , θ_F , δ_S , δ_F stands for the parameter coefficient; $\varepsilon^2_{S,t-1}$, $\varepsilon^2_{F,t-1}$ stands for the lag 1 periods residual square.

The optimal hedge ratio is:

$$h = \frac{Cov(\varepsilon_{S_t}, \varepsilon_{F_t})}{Var(\varepsilon_{F_t})} \quad (4\text{-}5)$$

5.Conclusion

This paper studies the definition and principle of hedge theory, introduces the derivation process of linear regression model and linear mean-variance model in modern portfolio management, and obtains the optimal hedge ratio. Some basic models such as B-VAR model and ECM model considering co-integration relation are studied, and the formula of the model and the process of deriving the optimal hedge ratio are given. In the dynamic model, the formulas of GARCH model and the derivation process of the optimal hedge ratio are given. On the basis of understanding other scholars, the specific theory and equation of hedge model are introduced in detail, and the coefficients in the model are further discussed and explained.

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