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Option pricing and sensitivity analysis

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Abstract Option is a kind of financial derivative, which has been developed rapidly for many years. How to price option is the most important issue of option trading. We should not only know the direction of the influence of various factors on the option price, but also the extent of the factors. In order to solve this problem it is necessary to analyze the sensitivity of option price, the parameter of analysis is also called the Greek value. In this paper, we study 5 parameters (Delta, Gamma, Rho, Theta, Vega) and their detailed derivation for the Black-Scholes model. These financial parameters are used to describe the risk characteristics of options and portfolios from different angles. Finally, two numerical methods of American option pricing, the binary tree method and the finite difference method, are introduced, and how to calculate the sensitivity parameters are explained.

Keywords：Black-Scholes model; sensitivity analysis; Greek value; Numerical solution

1. Black-Scholes model

In 1973, Scholes and his colleague Black developed a complex formula for option pricing. Blake-Scholes Option Pricing Model, which was founded and developed by them, lays a foundation for the rational pricing of various derivatives in the emerging derivative financial markets, including stocks, bonds, currencies and commodities, which are priced at variable market prices.

At the same time, Merton also found the same formula and many other useful conclusions about options. The two papers were published almost simultaneously in different publications. Therefore, the Black-Scholes pricing model can also be called the Black-Scholes-Merton pricing model. Merton expanded the connotation of the original model and made it applied to many other forms of financial transactions as well.

There are 7 important assumptions in the B-S model:

(1)Stock price behavior obeys logarithmic normal distribution model;

(2)In the validity period of the option, the risk free interest rate and the financial asset return variable are constant;

(3)There is no friction in the market, that is, there is no tax and transaction costs;

(4)The option is European option, that is, it can not be implemented before the expiration of the option;

(5)There is no risk arbitrage opportunity.

(6)Securities trading is continuous;

(7)Investors can borrow at risk-free rates.

The specific formula of the Black-Scholes model is as follows:

$$c = SN(d_1) - Xe^{-rt}N(d_2) \qquad (1\text{-}1)$$

$$d_1 = \frac{\ln(S / Xe^{-rt})}{\sigma\sqrt{t}} + \frac{\delta\sqrt{t}}{2}, \quad d_2 = d_1 - \sigma\sqrt{t} \qquad (1\text{-}2)$$

In Formula(1-1)and(1-2), S stands for the common stock price, X stands for the option execution price, R is the risk-free interest rate, t is the remaining expiration time of the option, δ indicates the volatility of the underlying asset price of the continuous compound interest, i.e. the standard deviation. N (x) denotes the cumulative probability distribution function of a variable that obeys the standard normal distribution, i. e. the probability that the variable is less than x.

2. Greek value

2.1 Delta(Δ)

Delta is the most important sensitive index of option price. It indicates the degree to which changes in the present value s of future cash flows affect the option price. For example, if the present value s of future cash flows rises by 1 cents and the option premium rises by 0.3 cents, the Delta of the option is 0.3. The Delta value of the option is the partial derivative of the generalized Black-Scholes model for S.

$$
\begin{aligned}
Delta = \Delta = \frac{\partial C}{\partial S} &= e^{(b-\xi)T} N(d_1) + Se^{(b-\xi)T} \frac{\partial N(d_1)}{\partial S} - Xe^{\xi T} \frac{\partial N(d_2)}{\partial S} \\
&= e^{(b-\xi)T} N(d_1) + Se^{(b-\xi)T} \frac{e^{-\frac{1}{2}d_1^2}}{\sqrt{2\pi}} \frac{\frac{1}{S}}{\sigma\sqrt{T}} - Xe^{\xi T} \frac{e^{-\frac{1}{2}d_2^2}}{\sqrt{2\pi}} \frac{\frac{1}{S}}{\sigma\sqrt{T}} \qquad (2\text{-}1) \\
&= e^{(b-\xi)T} N(d_1) + \frac{e^{-\frac{1}{2}d_1^2 T}}{\sigma\sqrt{2\pi T}} (e^{bT} - \frac{X}{S} e^{-\frac{1}{2}d_2^2}) \\
&= e^{(b-\xi)T} N(d_1)
\end{aligned}
$$

The above formula shows that the present value of future cash flows rises by 1 cents, and the Delta value of the call option premium rises between(0,1). This shows that the change of option price is in the same direction as that of future cash flow.

Delta represents the probability of price risk in the target market. It also shows that how much profit or loss a particular option contract makes when the underlying option price changes. When the call option is at parity, when the option is at full profit, the call option is close to 1; when the option is at a loss, the contract will not be executed. At this time it tends to 0,and that called the neutral state of Delta. meanwhile, the price of the option is not affected by the volatility of the underlying assets, thus achieving hedging. Cause Delta's neutral hedge only happens in a moment. Delta varies with S、 T、 r andσ, therefore, we need to constantly adjust the hedging position so that the hedging portfolio can be repositioned in the Delta neutral state. This adjustment is called rebalancing, which is called dynamic hedge.

2.2 Gamma(Γ)

Delta can measure the impact of underlying securities price changes on royalty. This estimate is valid when the underlying security price changes little. However, when the underlying securities price changes greatly, the use of Delta alone will produce a large estimation error, which requires the introduction of another Greek letter Gamma. Gamma is the sensitivity of options to asset cash flow changes, it measures the effect of the underlying securities price change on Delta, that is, the underlying securities price changes by a unit, and the option Delta changes accordingly.

Theoretically, Gamma is defined as the two order partial derivative of the option value to the underlying securities price. According to the Black-Scholes model, The specific formula of Gamma is as follows:

$$\Gamma = \frac{N'(d_1)}{S\sigma\sqrt{T}} \qquad (2\text{-}2)$$

Gamma measures the sensitivity of Delta to underlying asset prices. When Gamma is small, Delta changes slowly, and trading adjustments to ensure Delta neutrality need not be too frequent. But when the absolute value of Gamma is very large, Delta is very sensitive to changes in underlying assets, in order to ensure Delta neutrality, it needs frequent adjustments.

Gamma the difference between the market price and the execution price of the main measurement options. The closer the market price is to the execution price and the closer to maturity, the higher the Gamma value. The longer the holding time, the lower the Gamma value. When the Gamma value is positive, the change of the Delta value is proportional to the change of the underlying market price, and buyers can profit from the rate of change of the underlying asset price.

2.3 Rho(P)

Rho is the sensitivity of option to interest rate, which indicates that interest rate changes the value of a unit period. The size of Rho depends not only on the relationship between the price of the subject matter and the agreed price, but also on the duration of the right. The longer the right period, the greater the absolute value of Rho; the shorter the right time, the smaller the absolute value of Rho. At the expiration date of the option, the Rho of any option will be 0.

Theoretically, Rho is defined as the first order partial derivative of the option value to interest rate.

2.4 Theta(Θ)

Theta measures the impact of time-to-maturity changes on royalties, i.e. changes in the amount of royalties that a unit should produce in the past. That is, when other conditions remain unchanged, the ratio of option prices to time varies.

In financial options trading, especially in the horizontal spread trading of financial options, The size of the Theta reflects how much value the option buyer loses over time, and how much value the option seller adds over time. Therefore, Theta is a useful sensitivity index.

2.5 Vega(υ)

Vega measures the impact of changes in the volatility of the underlying securities on royalties, that is, changes in the volatility of a unit of royalties should produce changes.

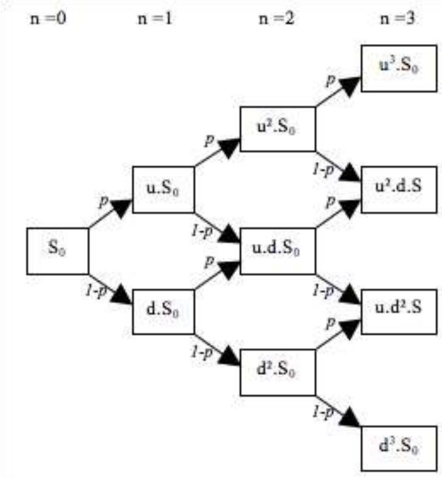
If the absolute value of Vega is large, the option value is sensitive to small changes in volatility; otherwise, if the absolute value of Vega is small, the change in volatility will no longer affect the value of the option. For options, the Vega value of the option is always positive, whether it is a call option or a put option, whether it is a futures option or a spot option. Parity options Vega is always larger, premium or loss, when volatility is higher, Vega is also larger; but when volatility is lower, Vega is smaller.

3.Numerical solution of option pricing

The numerical method of option pricing includes two fork tree method and finite difference method，the following two methods will be introduced and how to use the above methods to calculate option sensitivity parameters.

3.1 Binary tree method

Binomial option pricing model and Black-Hughes option pricing model are two complementary methods. The binomial option pricing model is relatively simple and more suitable for explaining the basic concept of option pricing. The binomial option pricing model is based on a basic assumption that there are two possible directions for the price movement of securities within a given time interval: up or down. Although this assumption is very simple, the binomial option pricing model is suitable for dealing with more complex options because it can be subdivided into smaller time units for a given period of time. The specific methods are as follows:



$$p = \frac{e^{rt/n} - d}{u - d}$$

$$u = e^{\sigma \sqrt{t/n}}$$

$$d = e^{-\sigma \sqrt{t/n}}$$

First: create two price trees, the creation of the two price tree is pushed step by step from the valuation day to the expiration date of the option; Then: find the value of options on each final node; after that: finding the value of the earlier node's upper right.

3.2 Finite difference method

Finite difference method refers to the method of using Taylor series expansion to write the derivative of a variable into a variable in the form of difference at different time or space points. By solving the differential equation of derivative securities, the finite difference method can be used to evaluate derivative securities. The specific steps are as follows:

First,regional grid generation of the derivative securities (Regional Division). Then,Establishing difference schemes.

4. conclusion

In view of the generalized Black-Scholes model, the 5 parameters of options are deeply analyzed and deduced. These parameters allow us to comprehensively observe the measurement methods of option value sensitivity when these variables change, and use these financial parameters to describe the risk characteristics of options and portfolios with options. Finally, the numerical solution of option pricing is studied.

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