ON ROUGH BI-IDEALS OF NEAR-RINGS

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Abstract

This paper is an extension work of the paper by N.Thillaigovindan and V.S.Subha[13]. In this paper we introduce the notion of rough quasi-ideals and rough bi-ideals in near-rings, which is a generalization of the concepts of quasi-ideals and bi-ideals in near-rings. We study some properties of such ideals and characterize near-ring in terms of quasi -ideals and bi-ideals.

Keywords: Near-rings, Rough bi-ideals, Rough quasi-ideals, Lower approximation, Upper approximation.

1 INTRODUCTION

The notion near-ring is introduced by Pilz[9]. Tamilchelvan and Ganesan[11] developed bi-ideals in near-rings. Pawlak introduced the notion of rough set in his papers[6-8]. Rough set theory, a new mathematical approach to deal with in exact, uncertain or vague. Knowledge has recently received wide attention on the research areas in both of the real-life applications and theory itself. It has found practical applications in many areas such as knowledge discovery machine learning, data analysis, approximate classification, conflict analysis and so on. The algebraic approach of rough set was studied by some others such as[1,2,3,4,5,10,12]. Thillaigovindan and Subha[13] introduced rough ideals in near-rings.

In this paper we introduce the notion of rough quasi-ideals and rough bi-ideals in near-rings, which is a generalization of the concepts of quasi-ideals and bi-ideals in near-rings. We study some properties of such ideals and characterize near-ring in terms of quasi -ideals and bi-ideals.

2 PRELIMINARIES

Let U be a universal set. for an equivalence relation θ on U, the set of elements of U that are related to $x \in U$, is called the equivalence class of x and is denoted by $[x]_\theta$. Let U/θ denote the family of all equivalence classes induced on U by θ. U/θ is a partition or classification of U such that each element of U is contained in exactly on element of U/θ.

Definition 2.1.[8] A pair (U, θ) where $U \neq \emptyset$ and θ is an equivalence relation on U, is called an approximation space.

Definition 2.2.[8] For an approximation space (U, θ) by a rough approximation in (U, θ) we mean a mapping $\rho: \wp(U) \to \wp(U) \times \wp(U)$ defined as $\rho(X) = (\underline{\rho}(X), \overline{\rho}(X))$ for $X \subseteq U$, where

$\underline{\rho}(X) = \{x \in X | [x]_\rho \subseteq X\}$ and $\overline{\rho}(X) = \{x \in X | [x]_\rho \cap X \neq \emptyset\}$, $\underline{\rho}(X)$ is called lower rough approximation of X in (U, θ), where $\overline{\rho}(X)$ is called upper approximation of X in (U, θ).

Definition 2.3.[8] Given an approximation space (U, θ) a pair $(A, B) \in \wp(U) \times \wp(U)$ is caled *rough set* in (U, θ) if and only if $(A, B) \in \rho(X)$ for some $X \subseteq U$.

Definition 2.4.[8] A subset X of U is called definable if $\underline{\rho}(X) = \overline{\rho}(X)$. If $X \subseteq U$ is given by predicate P and $x \in U$, then

i) $x \in \underline{\rho}(X)$ means that x *certainly has property* P.

ii) $x \in \overline{\rho}(X)$ means that x *possibly has property* P.

iii) $x \in U \backslash \overline{\rho}(X)$ means that x *definitely does not have property* P.

3. ROUGH IDEALS IN NEAR-RINGS

Throughout this chapter N denotes a near-ring unless otherwise mentioned. A *near-ring* is a non-empty set N together with two binary operations + and · such that:

i. $(N, +)$ is a group (not necessarily abelian)
ii. $(N, \cdot)$ is a semigroup
iii. $(n_1 + n_2) \cdot n_3 = n_1 \cdot n_3 + n_2 \cdot n_3$, for all $n_1, n_2, n_3 \in N$.

In near-ring, $0 \cdot x = 0$ and $(-x) \cdot y = -x \cdot y$, but in general $x \cdot 0 \neq 0$ for some $x \in N$.

In near-ring, $0 \cdot x = 0$ and $(-x) \cdot y = -x \cdot y$, but in general $x \cdot 0 \neq 0$ for some $x \in N$. More precisely the above near-ring is right near-ring. $N_0 = \{n \in N \,/\, n \cdot 0 = 0\}$ is called the zero-symmetric part of N and $N_c = \{n \in N \,/\, n \cdot 0 = n\} = \{n \in N \,/\, n \cdot n' = n \text{ for all } n' \in N\}$ is called the constant part of N. N is called zero-symmetric if $N = N_0$ and N is called constant if $N = N_c$.

Definition 3.1.[13] Let $(N, +, \cdot)$ be a near-ring. A subset I of N is called an *ideal* of N if

1) $(I, +)$ is anormal subgroup of $(N, +)$
2) $IN \subseteq I$
3) $n_1 \cdot (n_2 + i) - n_1 \cdot n_2 \in I \in N$.

For all $i \in I$ and $n_1, n_2 \in N$.

If I satisfies (1) and (2) then it is called a right ideal of N. If I satisfies (1) and (3), then it is called a left ideal of N.

Let I be an ideal of N and X be a non-empty subset of N. Then the sets $\underline{\rho}_{\mathrm{I}}(X) = \{x \in N / \, x + \mathrm{I} \subseteq X\}$ and $\overline{\rho}_{\mathrm{I}}(X) = \{x \in N / \, (x + \mathrm{I}) \cap \mathrm{X} \neq \emptyset\}$ are called respectively the *lower* and *upper* approximations of the set X with respect to the ideal I.

For any ideal I of N and $a, b \in N$, we say a is *congruent* to b mod I, written as $a \equiv b(mod\ I)$ if $a - b \in I$.

It is easy to see that relation $a \equiv b(mod\ I)$ is an *equivalence relation*. Therefore, when $U = N$ and θ is the above equivalence relation, we use the pair (N, I) instead of the approximation space (U, θ).

Also, in this case we use the symbols $\underline{\rho}_{\mathrm{I}}(X)$ and $\overline{\rho}_{\mathrm{I}}(X)$ instead of $\underline{\rho}(\mathrm{X})$ and $\overline{\rho}(\mathrm{X})$. If X is a subset of N, then X^C will be denoted by $N - X$.

Lemma 3.2. [13] For every approximation space (N, I) and every subsets $M, P \subseteq N$, the following hold:

1) $\underline{\rho}_{\mathrm{I}}(N - P) = \mathrm{N} - \overline{\rho}_{\mathrm{I}}(P)$
2) $\overline{\rho}_{\mathrm{I}}(N - P) = \mathrm{N} - \underline{\rho}_{\mathrm{I}}(P)$
3) $\overline{\rho}_{\mathrm{I}}(M) = (\underline{\rho}_{\mathrm{I}}(M^C))^{\mathrm{C}}$
4) $\underline{\rho}_{\mathrm{I}}(\mathrm{M}) = \left(\overline{\rho}_{\mathrm{I}}(M^C)\right)^{\mathrm{C}}$.

Theorem 3.3. [13] For every approximation space (N, I) and ever subsets $\mathrm{M}, \mathrm{P} \subseteq N$, then the following hold:

1) $\underline{\rho}_{\mathrm{I}}(\mathrm{M}) \subseteq M \subseteq \overline{\rho}_{\mathrm{I}}(\mathrm{M})$,

2) $\underline{\rho}_I(\emptyset) = \emptyset = \overline{\rho}_I(\emptyset)$
3) $\underline{\rho}_I(N) \subseteq N \subseteq \overline{\rho}_I(N)$
4) $\overline{\rho}_I(M \cup P) = \overline{\rho}_I(M) \cup \overline{\rho}_I(P)$
5) $\underline{\rho}_I(M \cap P) = \underline{\rho}_I(M) \cap \underline{\rho}_I(P)$
6) If $M \subseteq P$, then $\underline{\rho}_I(M) \subseteq \overline{\rho}_I(P)$ and $\overline{\rho}_I(M) \subseteq \overline{\rho}_I(P)$
7) $\overline{\rho}_I(M \cap P) \subseteq \overline{\rho}_I(M) \cap \overline{\rho}_I(P)$
8) $\underline{\rho}_I(M \cup P) \supseteq \underline{\rho}_I(M) \cap \underline{\rho}_I(P)$
9) If J is an ideal of N such that $I \subseteq J$, then $\underline{\rho}_I(A) \supseteq \underline{\rho}_J(A)$ and $\overline{\rho}_I(A) \subseteq \overline{\rho}_J(A)$
10) $\underline{\rho}_I(\underline{\rho}_I(M)) = \underline{\rho}_I(M)$
11) $\overline{\rho}_I\left(\overline{\rho}_I(M)\right) = \overline{\rho}_I(M)$
12) $\overline{\rho}_I(\underline{\rho}_I(M)) = \underline{\rho}_I(M)$
13) $\underline{\rho}_I(\overline{\rho}_I(M)) = \overline{\rho}_I(M)$
14) $\underline{\rho}_I(x + I) = \overline{\rho}_I(x + I)$ for all $x \in N$.

If A and B are nonempty subsets of N, then $AB = \{a.b \ / \ a \in A \ and \ b \in B\}$.
Hereafter we write ab instead of $a.b$.
A near-ring N is called zero-symmetric if $A * B = AB$.

Lemma 3.4. [13] Let I be an ideal of N and A, B nonempty subsets of N. If $A \subseteq B$ then

i) $\overline{\rho}_I(A) \subseteq \overline{\rho}_I(B)$. and
ii) $\underline{\rho}_I(A) \subseteq \overline{\rho}_I(B)$

Theorem 3.5. [13] Let I be an ideal of N and A, B nonempty subsets of N, then $\overline{\rho}_I(A).\overline{\rho}_I(B) = \overline{\rho}_I(AB)$.

Theorem 3.6. [13] Let I be an ideal of N and A, B nonempty subsets of N, then $\underline{\rho}_I(A).\underline{\rho}_I(B) \subseteq \underline{\rho}_I(AB)$.

Theorem 3.7. [13] Let I be an ideal of N and A, B nonempty subsets of N, then $\overline{\rho}_I(A) + \overline{\rho}_I(B) = \overline{\rho}_I(A + B)$.

Theorem 3.8. [13] Let I be an ideal of N and A, B nonempty subsets of N, then $\underline{\rho}_I(A) + \underline{\rho}_I(B) \subseteq \underline{\rho}_I(A + B)$.

Lemma 3.9. [13] Let I, J be two ideals of N and A a nonempty subset of N, then

(i) $\underline{\rho}_I(A) \cap \underline{\rho}_J(A) \subseteq \underline{\rho}_{I\cap J}(A)$
(ii) $\overline{\rho}_{I\cap J}(A) \subseteq \overline{\rho}_I(A) \cap \overline{\rho}_J(A)$.

Theorem 3.10.[13] If I and J are two ideals (resp. sub near-rings) of N, then $\overline{\rho}_I(J)$ is an ideal (resp. sub near-ring) of N.

Theorem 3.11. If I and J are two ideals (resp. sub near-rings) of N, then $\underline{\rho}_I(J)$ is an ideal (resp. sub near-ring) of N.

4. ROUGH NEAR-RINGS AND IDEALS

Definition 4.1.[13] Let I be an ideal of N and $\rho_I(A) = (\underline{\rho}_I(A), \overline{\rho}_I(A))$ a rough set in the approximation space (N, I). If $\underline{\rho}_I(A)$ and $\overline{\rho}_I(A)$ are ideals (resp. sub near-rings) of N, then we call $\rho_I(A)$ *rough ideal (resp. near-ring).*

Note that a rough sub near-ring is also called a rough near-ring. Clearly every rough ideal is a rough near-ring but the converse need not be true in general.

Lemma 4.2. [13]

i) *Let I, J be two ideals of N, then $\rho_I(I)$ and $\rho_I(J)$ are rough ideals*

ii) *Let I be an ideal and J a sub near-ring of N, then $\rho_I(J)$ is a rough near-ring.*

Remark 4.3. [13] If I is not an ideal and J is an ideal (resp. sub near-ring) of N, then $\rho_I(J)$ is not a rough ideal (resp. rough near-ring) which is shown in the following example.

Theorem 4.4.[13] *Let I, J be two ideals of N and K be a sub near-ring of N. Then*

(i) $\overline{\rho}_I(K).\overline{\rho}_J(K) \subseteq \overline{\rho}_{I+J}(K)$

(ii) $\underline{\rho}_I(K).\underline{\rho}_J(K) = \underline{\rho}_{(I+J)}(K)$.

Theorem 4.5. [13] *Let I, J be two ideals of N and K a sub near-ring of N. Then*

(i) $\underline{\rho}_{(I+J)}(K) = \underline{\rho}_I(K) + \underline{\rho}_J(K)$

(ii) $\overline{\rho}_{I+J}(K) = \overline{\rho}_I(K) + \overline{\rho}_J(K)$.

5. ROUGH QUASI-IDEALS AND BI-IDEALS IN NEAR-RINGS

In this section we introduce the notion of rough quasi-ideals and rough bi-ideals in near-rings and characterize near –ring in terms of rough quasi-ideals and rough bi-ideals.

Definition 5.1. A subgroup Q of $(N, +)$ is said to be *quasi-ideal* of N if $QN \cap NQ \cap N * Q \subseteq Q$.
A subgroup B of $(N, +)$ is said to be a *bi-ideal* of N if $BNB \cap (BN) * B \subseteq B$.

Let N be a *zero-symmetric* near-ring. A subgroup Q of $(N, +)$ is said to be a *quasi-ideal* of N if $QN \cap NQ \subseteq Q$.

A subgroup B of $(N, +)$ is said to be a *bi-ideal* of N if $BNB \subseteq B$.

Hereafter near-ring N is a zero-symmetric near-ring unless otherwise mentioned.

Definition 5.2. Let I be an ideal of N. A sub near-ring A of N is called

(i) an *upper rough quasi-ideal* of N if $\overline{\rho}_I(A)$ *is a quasi-ideal* of N, and

(ii) a *lower rough quasi-ideal* of N if $\underline{\rho}_I(A)$ is a quasi-ideal of N.

Definition 5.3.. Let I be an ideal of N. A sub near-ring A of N is called

(i) an *upper rough bi-ideal* of N if $\overline{\rho}_I(A)$ is a bi-ideal of N, and

(ii) a *lower rough bi-ideal* of N if $\underline{\rho}_I(A)$ is a bi-ideal of N.

Definition 5.4. Let I be an ideal of N. A sub near-ring A of N is called

(i) a *rough quasi-bi-ideal* of N if $\overline{\rho}_I(A)$ and $\underline{\rho}_I(A)$are quasi-ideals of N, and

(ii) a *rough bi-ideal* of N if $\overline{\rho}_I(A)$ and $\underline{\rho}_I(A)$ are bi-ideals of N.

Theorem 5.5. *Let I be an ideal of N. If Q is a quasi-ideal of N, then $\overline{\rho}_I(Q)$ is a quasi-ideal of N.*

Proof. Let Q is a quasi-ideal of N. Let $x \in \overline{\rho}_I(Q)N \cap N\overline{\rho}_I(Q)$. Then there exists $a, d \in \overline{\rho}_I(Q)$ and $b, c \in N$ such that $x = ab \in \overline{\rho}_I(Q)N$ and $x = cd \in N\overline{\rho}_I(Q)$. This means that $(a + I) \cap Q \neq \emptyset$ and $(d + I) \cap Q \neq \emptyset$. This implies that there exists y such that $y \in a + I$ and $y \in Q$. Now $y - a \in I$. Since I is a right ideal and Q is a quasi-ideal of N, $yb - ab \in I$ and $yb \in Q$. This means that $(ab + I) \cap Q \neq \emptyset$ and $x \in \overline{\rho}_I(Q)$. Now $\overline{\rho}_I(Q)N \cap N\overline{\rho}_I(Q) \subseteq \overline{\rho}_I(Q)N \subseteq \overline{\rho}_I(Q)$. Thus $\overline{\rho}_I(Q)$ is a quasi-ideal of N.

Theorem 5.6. *Let I be an ideal of N. If Q is a quasi-ideal of N and $\underline{\rho}_I(Q)$ is nonempty, then $\underline{\rho}_I(Q)$ is a quasi-ideal of N.*

Proof. Since Q is a subgroup of $(N,+)$, $\underline{\rho_I}(Q)$ is a subgroup of N. Let $x \in \underline{\rho_I}(Q)N \cap N\underline{\rho_I}(Q)$. Then $x \in \underline{\rho_I}(Q)N$ and $x \in N\underline{\rho_I}(Q)$. This implies that $x = an_1 \in \underline{\rho_I}(Q)N$ and $= n_2 b \in N\underline{\rho_I}(Q)$. Now $a + I \subseteq Q, an_1 + I \subseteq Qn_1$. Thus $x + I \subseteq QN$. Again $x = n_2 b \in N\underline{\rho_I}(Q)$ implies $b \in \underline{\rho_I}(Q)$. This means that $B + I \subseteq Q$. Since $0 \in I, n_2 b + I \subseteq NQ$. Combining these results
$(x + I) \subseteq QN \cap NQ \subseteq Q$. Thus $x \in \underline{\rho_I}(Q)$. Hence $\underline{\rho_I}(Q)N \cap N\underline{\rho_I}(Q) \subseteq \underline{\rho_I}(Q)$ which implies that $\underline{\rho_I}(Q)$ is quasi-ideal of N.

Theorem 5.7. *Let I be an ideal of N. If Q is a quasi ideal of N and $\underline{\rho_I}(Q)$ is nonempty, then $\rho_I(Q) = (\underline{\rho_I}(Q), \overline{\rho}_I(Q))$ is a quasi-ideal of N.*

Proof. It follows from Theorem 5.5 and Theorem 5.6.

Theorem 5.8.. *Let I be an ideal of N. If B is a bi-ideal of N, then $\overline{\rho}_I(B)$is a bi-ideal of N.*

Proof. Let B be a bi-ideal of N. Since by Theorem 3.3(3), $\overline{\rho}_I(N) = N$,

$$\overline{\rho}_I(B)N\overline{\rho}_I(B) = \overline{\rho}_I(BNB), \text{ by Theorem 3.8}$$
$$\subseteq \overline{\rho}_I(B) .$$

Thus $\overline{\rho}_I(B)$ is a bi-ideal of N.

Theorem 5.9. *Let I be an ideal of N. If B is a bi-ideal of N and $\underline{\rho_I}(B)$ is nonempty, then $\underline{\rho_I}(B)$ is a bi-ideal of N.*

Proof. Let B be a bi-ideal of N. Since by Theorem 3.3(3), $\underline{\rho_I}(N) = N$ and by Theorem 3.8,

$$\underline{\rho_I}(B)N\underline{\rho_I}(B) \subseteq \underline{\rho_I}(BNB)$$
$$\subseteq \underline{\rho_I}(B).$$

Thus $\underline{\rho_I}(B)$ is a bi-ideal of N.

Theorem 5.10. *Let I be an ideal of . if B is a bi-ideal of N. If B is a bi-ideal of N and $\underline{\rho_I}(B)$ is nonempty, $\underline{\rho_I}(B)$ is a rough bi-ideal of N.*

Proof. It is straight forward by Theorem 5.8 and theorem 5.9.

Lemma 5.11. *Let I be any ideal of N. If B is a quasi-ideal of N and $\underline{\rho_I}(B)$ is nonempty, then*

(i) *$\overline{\rho}_I(B)$ is a bi-ideal of N*

(ii) *$\underline{\rho_I}(B)$ is a bi-ideal of N.*

Proof. Let B be a quasi-ideal of N.

(i) By Theorem 5.5, $\overline{\rho}_I(B)$ is a quasi-ideal of N. Now

$$\overline{\rho}_I(B)N\overline{\rho}_I(B) = \overline{\rho}_I(BNB)$$
$$\subseteq \overline{\rho}_I(BNN)$$
$$\subseteq \overline{\rho}_I(BN).$$

Also

$$\overline{\rho}_I(B)N\overline{\rho}_I(B) = \overline{\rho}_I(BNB)$$
$$\subseteq \overline{\rho}_I(NNB)$$
$$\subseteq \overline{\rho}_I(NB).$$

Thus

$$\overline{\rho}_I(B)N\overline{\rho}_I(B) = \overline{\rho}_I(BN) \cap \overline{\rho}_I(NB)$$
$$\subseteq \overline{\rho}_I(B)N \cap N\overline{\rho}_I(B)$$

$$\subseteq \overline{\rho}_I(B).$$

Hence $\overline{\rho}_I(B)$ is a bi-ideal of N.

(ii) By Theorem 5.6, $\underline{\rho}_I(B)$ is a quasi-ideal of N.

Consider

$$\underline{\rho}_I(B)N\underline{\rho}_I(B) \subseteq \underline{\rho}_I(BNB)$$
$$\subseteq \underline{\rho}_I(BNN)$$
$$\subseteq \underline{\rho}_I(BN).$$

Again

$$\underline{\rho}_I(B)N\underline{\rho}_I(B) \subseteq \underline{\rho}_I(BNB)$$
$$\subseteq \underline{\rho}_I(NNB)$$
$$\subseteq \underline{\rho}_I(NB).$$

$$\underline{\rho}_I(B)N\underline{\rho}_I(B) \subseteq \underline{\rho}_I(BN) \cap \underline{\rho}_I(NB)$$
$$\subseteq \underline{\rho}_I(BN \cap NB) \text{ by Theorem 3.3(5)}$$
$$\subseteq \underline{\rho}_I(B).$$

Lemma 5.12. *Let I be any ideal of N. If B is a quasi-ideal of N, then $\rho_I(B)$ is a bi-ideal of N.*

Theorem 5.13.. *Let I be any ideal of N. If A is a left ideal (right ideal, sub near-ring, two-sided ideal) of N, then $\overline{\rho}_I(A)$ is a quasi-ideal of N.*

Proof. Let A be any ideal of N. Since N is a zero-symmetric near-ring, $NA \subseteq A$.
Consider $\overline{\rho}_I(A)N \cap N\overline{\rho}_I(A) \subseteq N\overline{\rho}_I(A) = \overline{\rho}_I(NA) \subseteq \overline{\rho}_I(A)$. Thus $\overline{\rho}_I(A)N \cap N\overline{\rho}_I(A) \subseteq \overline{\rho}_I(A)$. Hence $\overline{\rho}_I(A)$ is a quasi-ideal of N.

Theorem 5.14.. *Let I be an ideal of N. If A is a left ideal (right ideal, sub near-ring, two-sided ideal) of N, then $\overline{\rho}_I(A)$ is a bi-ideal of N.*

Proof. Let A be any ideal of N. By Theorem 5.5, $\overline{\rho}_I(A)$ is a quasi-ideal of N. By Theorem 5.8, $\overline{\rho}_I(A)$ is a bi-ideal of N.

Theorem 5.15.. *Let I be any ideal of N. If A is a left ideal (right ideal, sub near-ring, two-sided ideal) of N, then $\underline{\rho}_I(A)$ is nonempty, then$\underline{\rho}_I(A)$ a quasi-ideal of N.*

Proof. Let A be any ideal of N. Since N is a zero-symmetric near-ring, $NA \subseteq A$.
Consider $\underline{\rho}_I(A)N \cap N\underline{\rho}_I(A) \subseteq N\underline{\rho}_I(A) = \underline{\rho}_I(NA) \subseteq \underline{\rho}_I(A)$. Thus $\underline{\rho}_I(A)$is a quasi-ideal of N.

Theorem 5.16.. *Let I be an ideal of N. If A is a left ideal (right ideal, sub near-ring, two-sided ideal) of N, then $\underline{\rho}_I(A)$ is nonempty, then$\underline{\rho}_I(A)$ a bi-ideal of N.*

Proof. Let A be any ideal of N. By Theorem 5.6, $\underline{\rho}_I(A)$is a quasi-ideal of N. By Theorem 5.9, $\underline{\rho}_I(A)$is a bi-ideal of N.

Theorem 5.17. Let I be an ideal of N. If A is a left ideal (right ideal, sub near-ring, two-sided ideal) of N, then $\underline{\rho}_I(A)$ is nonempty, then $\rho_I(B)$ a rough bi-ideal of N.

Proof. The result follows from Theorem 5.14 and 5.16.

5.CONCLUSION

The rough set theory is regarded as a generalization of the classical set theory. A key notion in rough set is an equivalence relation. An equivalence is sometime difficult to be obtained in reward problems due to vagueness and incompleteness of human knowledge. In this paper the concept of rough bi-ideals and rough quasi-ideals were introduced in near-rings. This work is extended to Γ-near- rings and modules.

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