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RETAILER'S OPTIMAL INVENTORY POLICY FOR DETERIORATING ITEMS WITH TRADE CREDIT, INFLATION AND EFFECT OF LEARNING

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ABSTRACT

In this paper an inventory model of decaying items with trade credit period and time dependent demand has been developed with the effect of learning. The model is developed under inflationary environment. Sometimes it is observed that by repeated course of action the retailer can reduce some associated cost known as learning effect. The proposed model is structured under three different cases of trade credit limit. The objective of this paper is to minimize the total average cost of the system. Numerical examples for all the cases are discussed. Further a sensitivity analysis is also executed to check the stability of the model with respect to different system parameters.

INTRODUCTION

There are so many products such as steel, hardware and gasoline etc. which have a very low or negligible rate of deterioration. For these items we can say that the rate of deterioration is constant. But for the food items, vegetables and fruits the deterioration rate increases with time. To maintain and control the inventory of such products and to satisfy the customers need is a very important task. In last few decades many researchers show their interest in the inventory of deteriorating items. Ghare and Schrader (1963) were the first who execute their opinion in the field of deteriorating inventory. This model was developed with constant rate of deterioration and got considerable attention. Further Covert and Philip (1973) developed an inventory model for time variable rate of deterioration. Chakrabarty *et. al.*, (1998) extended this model with linear rate of demand and shortages. Lo *et. al.*, (2007) proposed an integrated production-inventory model with imperfect production processes and Weibull deterioration under inflationary environment. Tayal *et. al.*, (2014) introduced an inventory model for multi items having selling price and expiration date dependent demand. For the products which have a certain expiration date, the demand rate decreases as the product is nearer to its expiration date. Further Tayal et al. (2014) presented a two echelon supply chain model for deteriorating items with effective investment in preservation technology by which the effect of deterioration can be reduced. Singh *et. al.*, (2014) presented a production inventory problem for deteriorating products with limited storage space. Khurana *et. al.*, (2015) developed a supply chain production inventory model for deteriorating product with stock dependent demand under inflationary environment and partial backlogging. Tayal *et. al.*, (2015) introduced an EPQ model for non- instantaneous deteriorating item with time dependent holding cost and exponential demand rate. Singh *et. al.*, (2016) investigated an inventory model for deteriorating items having seasonal and stock-dependent demand with allowable shortages. Rastogi *et. al.*, (2017) came forward with two warehouse inventory policy with price dependent demand and deterioration under partial backlogging.

Traditionally in the development of inventory models it is assumed that the retailer has to pay at the time of arrival of stock, but generally the supplier allows a certain time period for the payment. If the retailer clears all the outstanding in this time period than no interest is charged but after this time limit an interest will be charged on unpaid amount. Goyal (1985) introduced an economic order quantity under conditions of permissible delay in payments. Hwang and Shinn (1997) developed retailer's pricing and lot sizing policy for exponentially deteriorating products under the condition of permissible delay in payments. Teng (2002) presented an inventory model on the economic order quantity under conditions of permissible delay in payments. Kumar *et. al.,* (2009) introduced an inventory model for decaying items with quadratic demand rate, trade credits and inflation. Tayal *et. al.,* (2016) investigated an integrated production inventory model for perishable products with trade credit period and investment in preservation technology. Singh *et. al.,* (2016) came forward with an economic order quantity model for deteriorating products having stock dependent demand with trade credit period and preservation technology. Rastogi *et. al.,* (2017) presented an EOQ model with variable holding cost and partial backlogging under credit limit policy and cash discount.

Most of the above discussed models are developed without consideration of rate of inflation, which plays a very important role in any inventory management. Inventory models without considering the inflation rate misleads the results. Buzacott (1975) was the first to introduce the rate of inflation in the development of an inventory model. Misra (1979) presented a note on optical inventory management under inflation. Yang *et. al.,* (2001) developed a deterministic

inventory lot-size models under inflation with shortages and deterioration for fluctuating demand. Singh and Sharma (2013) introduced an integrated model with variable production and demand rate under inflation. Yadav *et. al.,* (2015) presented a retailer's optimal policy under inflation in fuzzy environment with trade credit.

Here in this paper we have tried to combine all above mentioned factors together to meet the market conditions. First the model was developed mathematically and then illustrated numerically. The convexity nature of the objective function is revealed with a graph.

ASSUMPTIONS

The following assumptions are made in the development of this model.

1. This model is developed for deteriorating items.

2. The deterioration is considered at a weibullrate.

3. The demand rate is a linear function of time.

4. The inflation rate is considered in the development of thismodel.

5. The effect of learning is applied on holding cost and ordering cost.

6. The deteriorated items are completely discarded.

7. The model is developed for trade credit policy.

8. The shortages are not allowed here.

9. Replenishment rate is infinite.

NOTATIONS

These are the used notations throughout this model.

T replenishment cycle

α, β deterioration parameter, $\alpha>0$ and $\beta>1$

a, b demand parameters, a,b>0

Q initial inventory level

c purchasing cost per unit

r rate ofinflation

M allowed trade credit period

h_0 constant part of holding cost parameter

h_1 variable part of holding cost parameter

n number ofcycles

α_1, α_2 positive constants

d deterioration cost per unit

O_0 constant part of holding cost parameter

O_1 variable part of holding cost parameter

p selling price perunit

I_c rate of interest charged

I_e rate of interestearned

U unpaid amount at the end of trade creditperiod

MATHEMATICAL MODELLING

Figure (1) shows the level of inventory at different time. Initially ‘Q’ is the amount of inventory which decreases due to the combined effect of demand and deterioration during [0, T] and at last becomes zero. Then inventory is replenished at t=T and the process goeson.

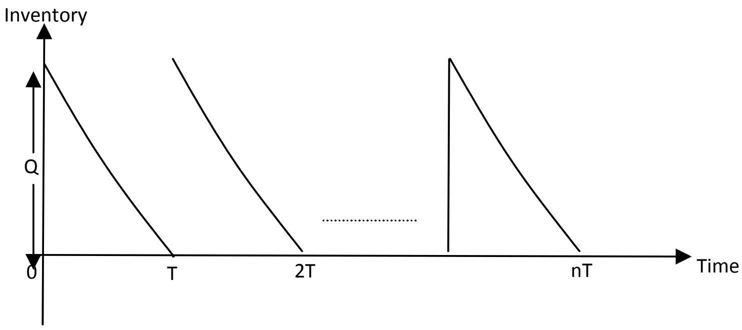

Fig. 1: Inventory time graph for the System

The differential equation is given as follows:

$$\frac{dI(t)}{dt} = -\alpha\beta t^{\beta-1} I(t) - (a+bt) \qquad 0 \le t \le T \qquad (1)$$

With boundary condition:

$I(T) = 0$

The solution of this equation is given by:

$$I(t) = \{a(T-t) + \frac{b}{2}(T^2 - t^2) + \frac{a\alpha}{\beta+1}(T^{\beta+1} - t^{\beta+1}) + \frac{b\alpha}{\beta+2}(T^{\beta+2} - t^{\beta+2})\}e^{-\alpha t^{\beta}} \qquad (2)$$

Using equation (2) the inventory level I (0) is given by:

$$I(0) = Q = aT + \frac{b}{2}T^2 + \frac{a\alpha}{\beta+1}T^{\beta+1} + \frac{b\alpha}{\beta+2}T^{\beta+2} \qquad (3)$$

Purchasing Cost

Since Q shows the inventory level and c is standing for purchasing cost per unit, hence the purchasing cost will be:

$$P.C. = cQe^{-rM}$$

$$P.C. = c(1-rM)(aT+\frac{b}{2}T^2+\frac{a\alpha}{\beta+1}T^{\beta+1}+\frac{b\alpha}{\beta+2}T^{\beta+2}) \quad (4)$$

Holding Cost

Holding cost is a function that depends on the time and it occurs till the time when inventory holds in the system. The holding cost can be calculated asfollows:

$$H.C. = (h_0+\frac{h_1}{n^{\alpha_2}})\int_0^T I(t)e^{-rt}dt$$

$$H.C. = (h_0+\frac{h_1}{n^{\alpha_1}})(\frac{aT^2}{2}+\frac{b}{3}T^3+\frac{2a\alpha T^{\beta+2}}{\beta+2}-\frac{a\alpha T^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{b\alpha T^{\beta+3}}{\beta+2}-\frac{b\alpha T^{\beta+3}}{(\beta+2)(\beta+3)}-\frac{aT^3r}{6}-\frac{brT^4}{8}$$

$$\frac{a\alpha T^{\beta+2}}{\ldots} \quad \frac{b\alpha T^{\beta+3}}{\ldots 1)} \quad \frac{b\alpha T^{\beta+3}}{2(\ldots\beta+3)} \quad (5)$$

Deterioration Cost

$$D.C. = d\int_0^T \{I(0)-(a+bt)\}e^{-rt}dt$$

$$D.C. = d(aT^2+\frac{b}{2}T^3+\frac{a\alpha T^{\beta+2}}{\beta+1}+\frac{b\alpha T^{\beta+3}}{\beta+2}-aT-\frac{bT^2}{2}-\frac{arT^3}{2}+\frac{brT^4}{4}+\frac{arT^2}{2}+\frac{brT^3}{3}) \quad (6)$$

Ordering Cost

Ordering cost per order can be calculated as follows:

$$O.C. = o_0 \quad (7)$$

$+ \ 1\, n^{\alpha_1}$

Permissible Delay

On the basis of credit time limit, following two cases arise:

Case 1: When $M \square T$

The inventory time graph of this case is shown in fig 2:

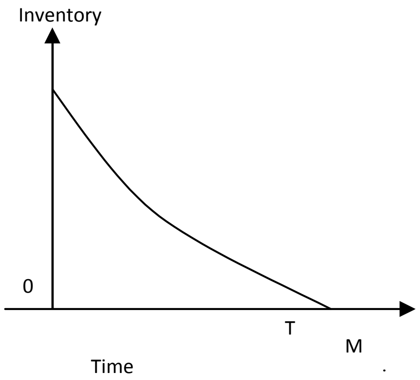

Fig. 2: Inventory time graph for the

In this case allowed trade credit limit is greater than the time when the retailer sold all the stock, so at the time of payment retailer has enough money to pay all his dues. So interest charged would bezero.

$\boldsymbol{I_C}$ □ **0**

Interest would be earned during [0, M]

$$I.E = pI \{ \int_0^T (a+bt)te^{-rt}dt + (M-T)\int_0^T (a+bt)e^{-rt}dt \}$$

$$I.E_1 = pI_e\left(\frac{-aT^2}{2} + \frac{arT^3}{6} - \frac{bT^3}{6} + \frac{brT^4}{12} + MaT - \frac{MarT^2}{2} + \frac{MbT^2}{2} - \frac{MbrT^3}{3}\right) \quad (8)$$

Case 2: When M □ T

The inventory time graph of this case is shown in fig 3:

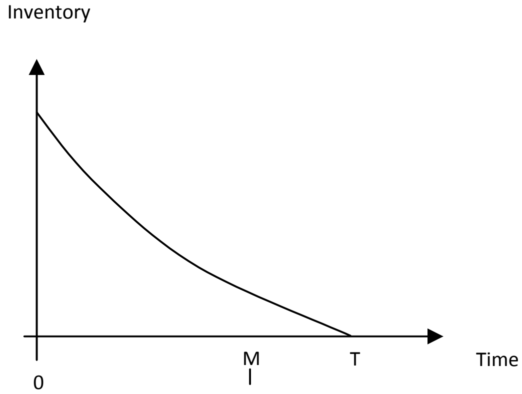

Fig. 3: Inventory time graph for the system

In this case allowed trade credit limit is less than the time when the retailer sold all the stock. On the basis of the availability of amount at the time of payment, two case4s arise:

Case 2.1: When M □ T and $pD[0, M]$ □ $I.E_{2.1}[0, M]$ □ cI (0)

In this case at the time of payment retailer has enough money to clear all his dues. So interest charged would be zero.

$I.C_{2.1} = 0$

Interest would be earned during [0, M]

$$I.E_{2.1} = pI_e \int_0^M (a + bt)te^{-rt}dt$$

$$I.E_{2.1} = pI_e \left(\frac{aM^2}{2} - \frac{arM^3}{3} + \frac{bM^3}{3} - \frac{brM^4}{4} \right) \quad (9)$$

Case2.2: When $M < T$ and $pD[0, M] + I.E_{2.2}[0, M] < cI(0)$

In this case retailer is not able to pay all his dues at the time of payment. So interest would be charged on unpaid amount.

$$I.C_{2.2} = I_c U$$

$$U = cI(0) - (I.E_{2.2}[0, M] + pD[0, M])$$

$$U = \{c(1 - rM)(aT + \frac{b}{2}T^2 + \frac{a\alpha}{\beta+1}T^{\beta+1} + \frac{b\alpha}{\beta+2}T^{\beta+2})\} - \{pI_e(\frac{aM^2}{2} - \frac{arM^3}{3} + \frac{bM^3}{3} - \frac{brM^4}{4})$$
$$\frac{bM^2}{2} \quad \frac{arM^2}{2} \quad \frac{brM^3}{3} \tag{10}$$

Interest would be earned during [0, M]

$$I.E_{2.2} = pI_e \int_0^M (a + bt)te^{-rt}dt$$

$$I.E_{2.2} = pI_e(\frac{aM^2}{2} \quad \frac{arM^3}{3} \quad \frac{bM^3}{3} - \frac{brM^4}{4}) \tag{11}$$

Total Average Cost

The total average cost for the system can be derived by dividing the difference of total associated costs and interest earned with the time unit T.

$$T.A.C. = \frac{1}{T}(P.C. + H.C. + D.C. + O.C. + I.C. - I.E.) \tag{12}$$

Numerical Example

In this section we have demonstrated the model with the help of a numerical example. The optimal value of time for the inventory and total average cost has been calculated here.

Case 1: When $M \geq T$

$c = 20 rs/unit,\ r = 0.06,\ a = 350 units,\ b = 25 units,\ \beta = 1.2,\ h_0 = 2 rs/unit,\ h_1 = 3 rs/unit,\ n = 4$
$\alpha_1 = 0.02,\ \alpha_2 = 0.025,\ d = 21 rs/unit,\ o_0 = 700\ rs,\ o_1 = 150\ rs,\ I_e = 0.15,\ p = 30\ rs/unit$
$M = 0.5 year,\ \alpha = 0.001$

After solving this model corresponding to these values we get:

T = 0.3025 year, Q = 106.995units, T .A.C. = 4237.54 rs

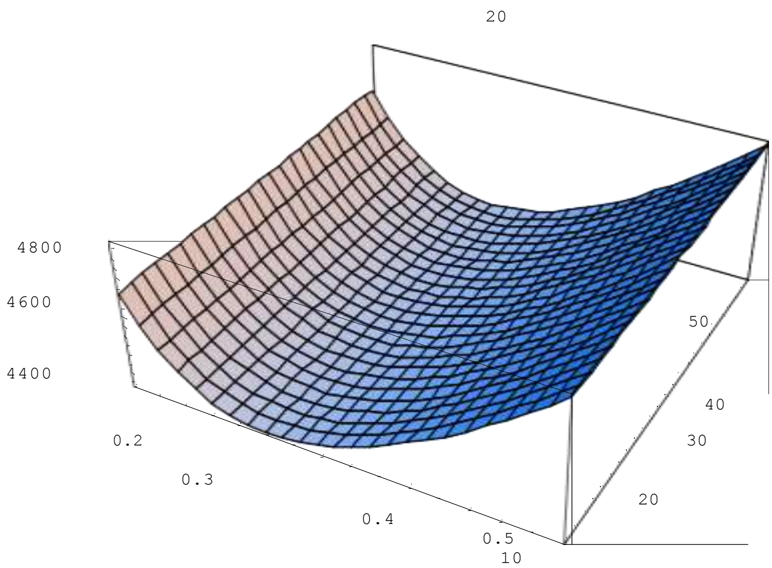

Fig. 4: Convexity of the Total Average Cost function

Case 2.1: When $M < T$ **and** $pD[0, M] + I.E_{2.1}[0, M] \geq cI(0)$

$c = 20 rs/unit,\ r = 0.06,\ a = 350 units,\ b = 25 units, \beta = 1.2,\ h_0 = 2 rs/unit,\ h_1 = 3 rs/unit,\ n = 4$
$\alpha_1 = 0.02,\ \alpha_2 = 0.025,\ d = 21 rs/unit,\ o_0 = 700\ rs,\ o_1 = 150\ rs,\ I_e = 0.15,\ p = 30\ rs/unit$
$M = 0.05 year,\ \alpha = 0.001$

After solving this model corresponding to these values we get:

$T = 0.3156\ year,\ Q = 111.618\ units,\ \ T.A.C. = 4967.51 rs$

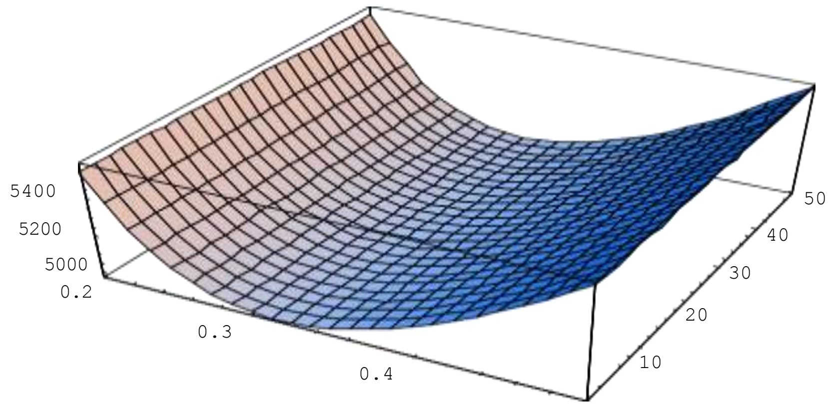

Fig. 5: Convexity of the Total Average Cost function

Case 2.2: When $M < T$ **and** $pD[0, M] + I.E_{2.2}[0, M] < cI(0)$

$c = 20 rs/unit,\ r = 0.06,\ a = 350 units,\ b = 25 units,\ \beta = 1.2,\ h_0 = 2 rs/unit,\ h_1 = 3 rs/unit,\ n = 4$

$\alpha_1 = 0.02,\ \alpha_2 = 0.025,\ d = 21 rs/unit,\ o_0 = 700\ rs,\ o_1 = 150\ rs,\ I_e = 0.15,\ p = 30\ rs/unit$

$M = 0.05 year,\ \alpha = 0.001,\ I_c = 0.2$

After solving this model corresponding to these values we get:

$T = 0.2920\ year,\ Q = 103.241 units,\ \ T.A.C. = 5291.48\ rs$

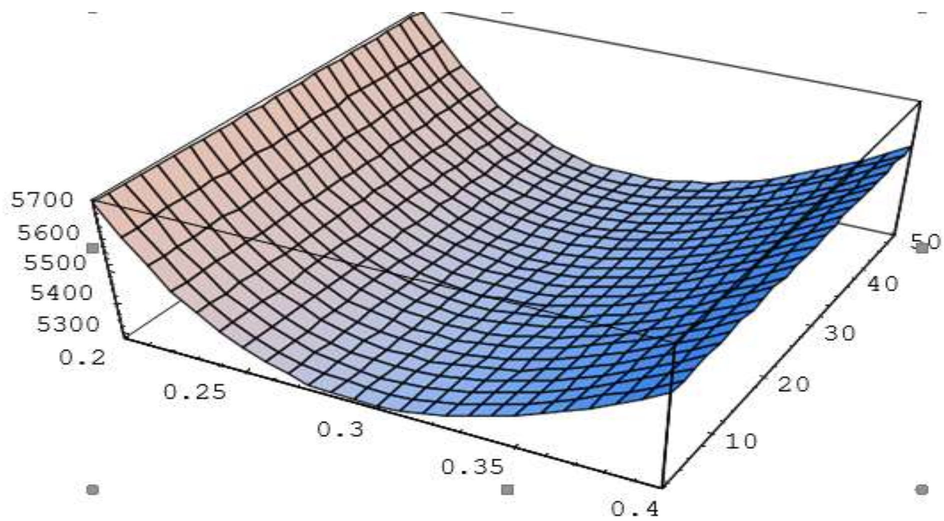

Fig. 6: Convexity of the Total Average Cost function

SENSITIVITY ANALYSIS

A sensitivity Analysis is carried out with respect to different associated system parameters taking one at a time and other variable unchanged.

Table 1: Variation in Total Average Cost with respect to demand parameter'a':

% variation in a	a	T	T.A.C.
-20	280	0.3372	3922.69
-15	297.5	0.3275	4007.46
-10	315	0.3185	4087.98
-5	332.5	0.3102	4164.57
0	350	0.3025	4237.54
5	367.5	0.2953	4307.13
10	385	0.2886	4373.58
15	402.5	0.2824	4437.1
20	420	0.2765	4497.86

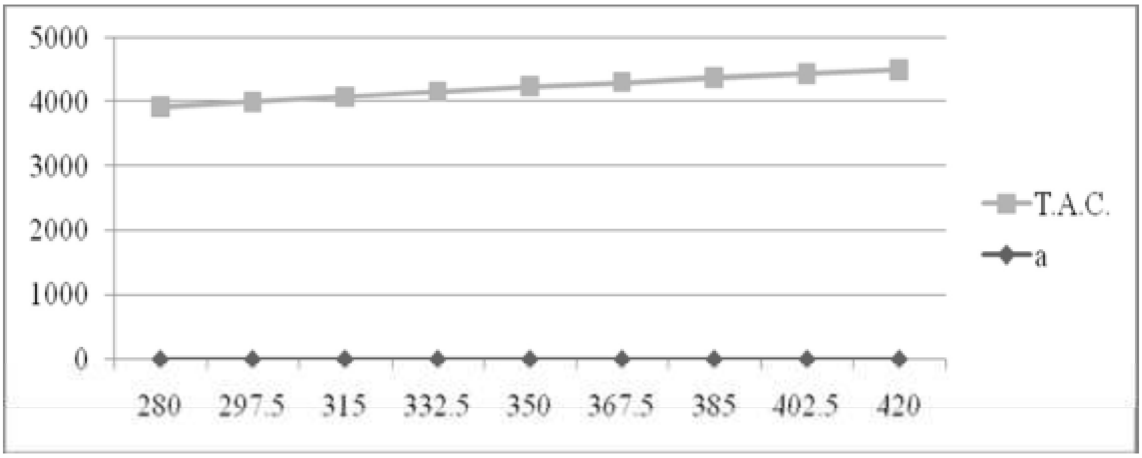

Fig. 7: Variation in T.A.C. with respect to a

Table 2: Variation in Total Average Cost with respect to demand parameter'b':

% variation in b	**b**	**T**	**T.A.C.**
-20	20	0.30298	4234.3
-15	21.25	0.30285	4235.11
-10	22.5	0.30273	4235.92
-5	23.75	0.3026	4236.73
0	25	0.3025	4237.54
5	26.25	0.30235	4238.35
10	27.5	0.3022	4239.15
15	28.75	0.30209	4239.96
20	30	0.30197	4240.76

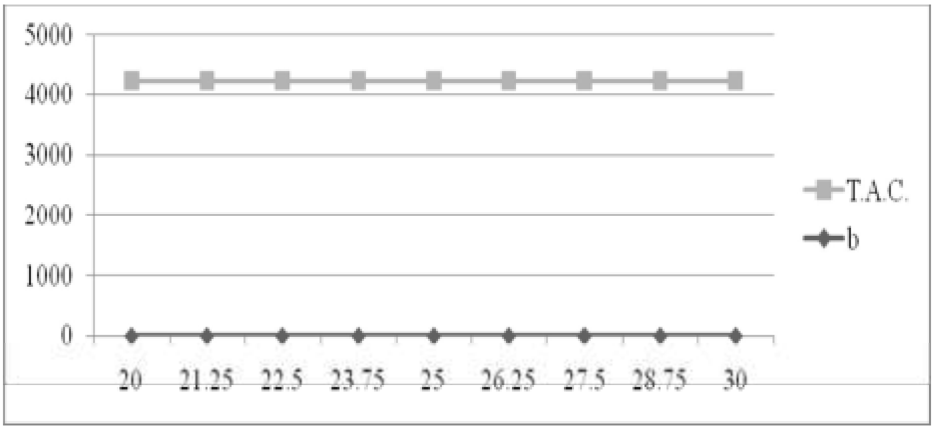

Fig. 8: Variation in T.A.C. with respect to b

Table 3: Variation in Total Average Cost with respect to holding cost parameter 'h_0'

% variation in h_0	**h_0**	**T**	**T.A.C.**
-20	1.6	0.3036	4216.15
-15	1.7	0.3033	4221.5
-10	1.8	0.3031	4226.85
-5	1.9	0.3028	4232.2
0	2	0.3025	4237.54
5	2.1	0.3022	4242.87
10	2.2	0.3019	4248.2
15	2.3	0.3016	4253.53
20	2.4	0.3013	4258.85

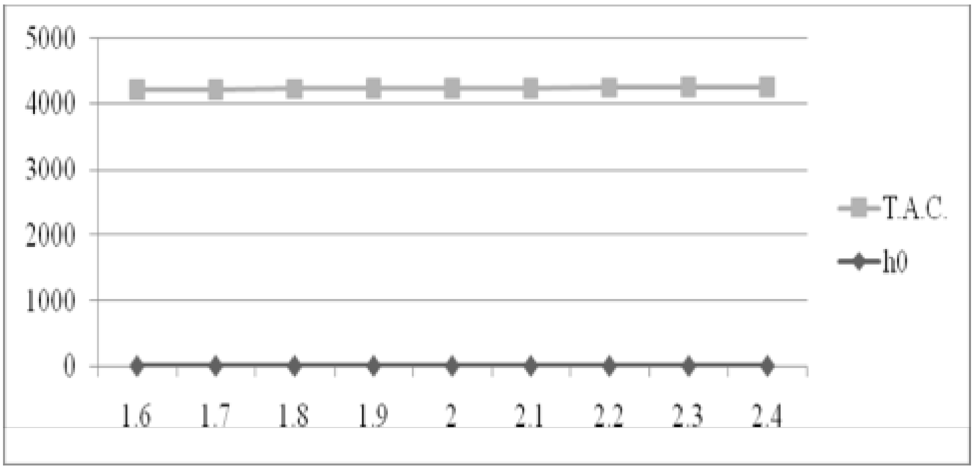

Fig. 9: Variation in T.A.C. with respect to h_0

Table 4: Variation in Total Average Cost with respect to holding cost parameter 'h_1':

% variation in h_1	**h_1**	**T**	**T.A.C.**
-20	2.4	0.3042	4206.52
-15	2.55	0.3037	4214.29
-10	2.7	0.3033	4222.05
-5	2.85	0.3029	4229.8
0	3	0.3025	4237.54
5	3.15	0.3021	4245.27
10	3.3	0.3016	4252.98
15	3.45	0.3012	4260.69
20	3.6	0.3008	4268.38

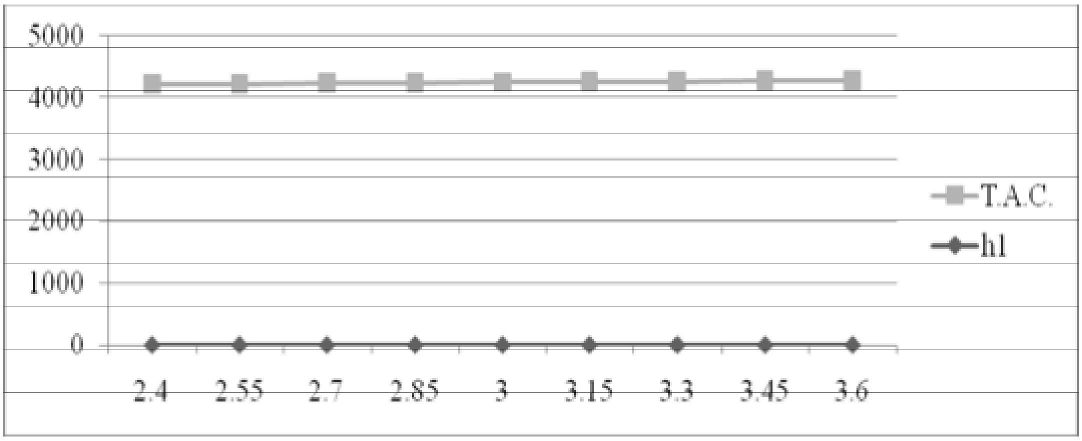

Fig. 10: Variation in T.A.C. with respect to h_1

Table 5: Variation in Total Average Cost with respect to Interest earned 'I_e':

% variation in e	**I_e**	**T**	**T.A.C.**
-20	0.12	0.3051	4347.4
-15	0.1275	0.3044	4319.97
-10	0.135	0.3038	4292.52
-5	0.1425	0.3031	4265.04
0	0.15	0.3025	4237.54
5	0.1575	0.3018	4210.01
10	0.165	0.3012	4182.46
15	0.1725	0.3006	4154.88
20	0.18	0.2999	4127.28

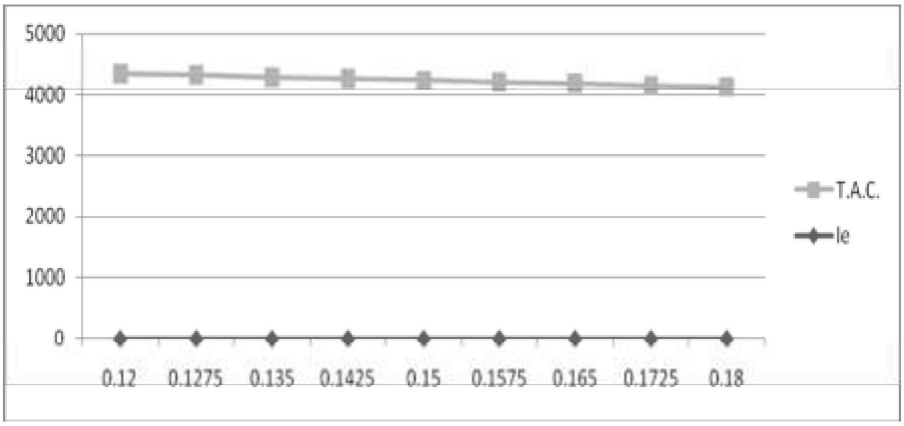

Fig. 11: Variation in T.A.C. with respect to Ie

n	T	T.A.C.
1	0.3029	4256.55
2	0.30269	4246.97
3	0.30256	4241.44
4	0.3025	4237.54
5	0.3024	4234.53
6	0.30235	4232.08
7	0.3023	4230.02
8	0.30226	4228.24
9	0.3022	4226.67
10	0.30219	4225.27

Table 6: Variation in Total Average Cost with respect to ‘n’:

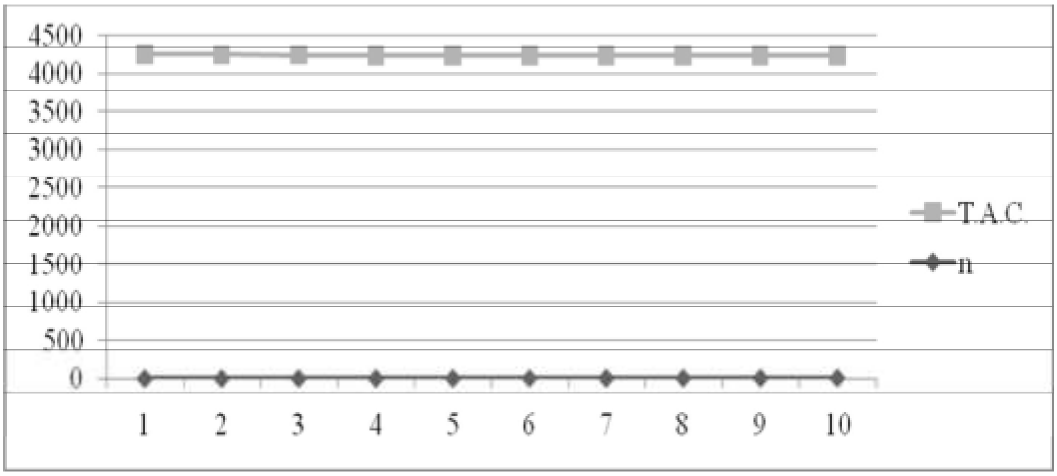

Fig 12: Variation in T.A.C. with respect to n

OBSERVATIONS

A sensitivity analysis is performed here with respect to some system parameters considering one by one.

1. Table (1) and table (2) list the variation in demand parameter ‘a’ and ‘b’. It is observed from these tables that as the demand rate increases, the value of purchasing cost increases and so the total average cost of the system also increases.
2. Table (2) and table (3) show the holding cost parameters at different points. With the increment in ‘h_0’ and ‘h_1’ the total average cost of the system goes onincreasing.

3. Table (5) contributes the variation in rate of interest earned at different points. It is concluded from this table that an increment in ‘I_e’ shows the reverse effect on total averagecost.
4. Table (5) shows the increment in number of replenishment cycles. It is seen from this table that as the value of ‘n’ increases, then due to the effect of learning the holding cost and ordering cost will decrease. As a result the total average cost of the system willdecrease.

CONCLUSION

In this paper an inventory model for two parameter weibull deterioration with the effect of learning has been proposed under inflationary environment. The model is developed with time varying demand and trade credit policy. It has been shown with the help of numerical example that due to the effect of learning the total system cost can be reduced. Three different cases for trade credit period are also discussed mathematically and numerically and it is found that a allowed trade credit period is very beneficial for the businessman. The total value of purchasing cost can be reduced with the help of permissible delay period. The convexity of the model has also been shown. It is found that the model is suitable for real life situations and has a unique solution.

For further research scope this model can be extended in numerous ways. The demand pattern can be taken as a stock dependent function and quantity discount can also be applied with trade credit period.

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