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The influence of modulation on the estimation of Critical permeability in 2D Bioporous convection

Dr. Roopa M C
Associate Professor,
Department of Mathematics,
Maharani's Arts, Commerce & Management College for Women,
Bangalore, Karnataka, India.

Abstract

Bioconvection is a new area of fluid mechanics and it describes the phenomenon of spontaneous pattern formation in suspensions of micro-organisms like Bacillus subtilis and algae. The term "bioconvection" refered to macroscopic convection induced in water by the collective motion of a large number of self propelled motile micro-organisms. The direction of swimming micro-organisms is dictated by the different tactic nature of the micro-organisms. They may be gyrotactic, phototactic, chemotactic, etc.. The present study deals with the linear stability analysis of a suspension of gyrotactic micro-organisms in a horizontal sparsely packed porous fluid layer of finite depth subject to adverse temperature gradient. This problem is relevant to certain species of thermophilic micro-organisms that live in hot environment. Finally it is concluded that the permeability as well as the modulation parameters have strong opposing influences on the onset of bio-convection.Thus, by controlling these parameters it is possible to enhance or suppress bio-convection.

Keywords-component; Bio-porous convection, eccentricity,critical permeability microorganism, critical Rayleigh number and critical wave number

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Introduction

Bioconvection is an exciting, complex, yet experimentally tractable, approach for studying the mutual interdependence of physics and biology, where the overall phenomenology greatly exceeds its primitive components. Bioconvection is the name given to pattern formation in suspensions of microorganisms, such as bacteria and algae, due to up-swimming of the micro-organisms (Pedley & Kessler 1992). Bioconvection has been observed in several bacterial species, including aerobic, anaerobic, and magnetotactic organisms, as well as in algal and protozoan cultures (Kessler and Hill (1997)). All have in common the sudden appearance of a pattern when viewed from above. In all cases the microorganisms are denser than water and on average they swim upwards (although the reasons for up-swimming may be different for different species). The algae (e.g. Chlamydomonas) are approximately 5% denser than water, whereas the bacteria are nearly 10% denser than water. Microorganisms respond to certain stimuli by tending to swim in particular directions. These responses are called taxes, examples being gravitaxis, phototaxis, chemotaxis and gyrotaxis. Gyrotaxis is swimming directed by the balance between the torque due to gravity acting on a bottom-heavy cell and the torque due to viscous forces arising from local shear flows. This chapter is concerned with the 2D stability analysis of bioconvection in a suspension of motile gyrotactic microoganisms in an electrically conducting fluid saturated porous medium subject to in modulated and unmodulated environments. The method employed is perturbation technique and the results are obtained by using the computational tools viz. **Maple and Mathematica**. The effect of modulation parameter on critical permeability is studied and the results are studied through graphs.

Nomenclature : C_a : The acceleration coefficient. D : Tthe diffusivity of the microorganisms (this assumes that all random motions of the microorganisms can be approximated by a diffusive process). g : The gravitational acceleration. n : The number density of motile microorganisms. n_o : The number density of microorganisms in the basic state. p : The excess pressure(above hydrostatic) $\hat{p}$: The unit vector indicating the direction of swimming of microorganisms . t : The time; u, v and w are the x-,y and z-velocity components respectively. $\vec{v}$: The velocity vector: (u,v,w) . $W_c\hat{p}$: The vector of average swimming velocity of microorganisms relative to the fluid (W_c is assumed to be constant). x y and z : The Cartesian coordinates (y is vertical coordinate). θ : The average volume of microorganisms. $\Delta\rho$: The density difference $\rho_{cell} - \rho_0$. μ : The dynamic viscosity. assumed to be approximately the same as that of water. ρ_o : The density of water, $\vec{g} = g_o G*$ where $G*$: modulation parameter $G* = 1+(g(t)/g_o)$ *where* $\vec{g} = g(t) + g_o$ is the acceleration due to gravity

This section deals with a detailed study of the estimation of critical permeability in a 2D-BPC under the influence of modulation. The solution is obtained by using a perturbation technique and a numerical procedure wherein the s/w tool like Maple is used for the purpose.

2 Mathematical formulation and analysis:. The major assumptions utilized in this chapter are ; **(i)** the porous matrix does not absorb microorganisms **(ii)** the suspension is dilute.**(iii)** the medium is an isotropic electrically conducting fluid saturated porous medium of uniform porosity (**iv**) no macroscopic motion of the fluid occurs and **(v)** all the microoganisms are swimming vertically upwards. The governing equations for a two dimensional unsteady flow in a porous medium are obtained by volume averaging the modified equations of Pedley et al., [1988] model, utilizing the volume averaging procedure described in Whitaker [1999]. This procedure resulted in the replacement of the Laplacian viscous terms with the Darcian terms that described viscous resistance in a porous medium (Nield and Bejan [1999]). The governing equations are:

$$C_a\rho_o\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial y}-\frac{\mu v}{K}-n\theta\Delta\rho gG^* \quad ..(1) \qquad C_a\rho_o\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial x}-\frac{\mu u}{K} \quad ..(2) \qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad ..(3)$$

$$\frac{\partial n}{\partial t}=-div(n\vec{v}+nw_c\hat{p}-D\Delta_n) \quad ..(4)$$

Pressure Elimination: Eliminating the pressure from eqns(1) and (2), the following eqns are obtained, $c_a\rho_o\frac{\partial}{\partial t}(\frac{\partial v}{\partial x})=-\frac{\partial^2 p}{\partial y\partial x}-\frac{\mu}{K}\frac{\partial v}{\partial x}-\theta\Delta\rho gG^*\frac{\partial n}{\partial x}$ ..(5)

$$c_a\rho_o\frac{\partial}{\partial t}(\frac{\partial u}{\partial y})=-\frac{\partial^2 p}{\partial x\partial y}-\frac{\mu}{K}\frac{\partial u}{\partial y} \quad ..(6) \qquad c_a\rho_o\frac{\partial}{\partial t}(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x})=-\frac{\mu}{K}(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x})+\theta\Delta\rho gG^*\frac{\partial n}{\partial x} \quad ..(7)$$

3. Stability analysis: For the discussion of the stability analysis, the following basic state is considered. Here a uniform suspension of microorganisms with the number density n_o in an infinite region occupied by an isotropic fluid saturated porous medium of uniform porosity is considered with the assumptions that **(i)** no macroscopic motion of the fluid occurs.**(ii)** all microorganisms are swimming vertically upwards and **(iii)** all captured microorganisms are fixed and attached to the porous matrix. However because of the infinite size of the domain, the upswimming does not change the cell concentration or the fluid density. We now introduce the following**: (i)** the perturbations of the cell concentration **(ii)** fluid velocity components and **(iii)** the unit vector $\hat{p}$ that indicates the direction of bacterial swimming. $n(t,x,y) = n_o + \varepsilon n^1(t,x,y)$ ..(8) $u(t,x,y) = \varepsilon u^1(t,x,y)$ .. (9)

$v(t,x,y) = \varepsilon v^1(t,x,y)$ ..(10) $\hat{p}(t,x,y) = \hat{k} + \varepsilon \hat{p}^1(t,x,y)$ …(11)

where $\hat{k}$ is the vector in the vertically upwards y-direction, a prime denotes a perturbation quantity and ε is the small perturbation amplitude. Substituting equations (8)-(11) in to the governing equations (7), (3) and (4) and linearizing gives the following equations for perturbation quantities (i.e. the terms containing ε are considered)

$$c_a \rho_o \frac{\partial}{\partial t}\left(\frac{\partial u^1}{\partial y} - \frac{\partial v^1}{\partial x}\right) = -\frac{\mu}{K}\left(\frac{\partial u^1}{\partial y} - \frac{\partial v^1}{\partial x}\right) + \theta \Delta \rho G^* \frac{\partial n^1}{\partial x} \quad ..(12)$$

$$\frac{\partial u^1}{\partial x} + \frac{\partial v^1}{\partial y} = 0 \quad ..(13) \qquad \frac{\partial n^1}{\partial t} = -div[n_o(\vec{v}^1 + w_c \hat{p}^1) + n^1 w_c \hat{k} \; - D\nabla n^1) \quad ..(14)$$

Where $\vec{v}\,'$ is the vector composed of perturbations of the corresponding velocity components, For a two dimensional problem, the results of Pedly et al. [1988] can be presented as $\hat{p}^1 = (-B\xi, 0)$ .. ..(15)

where B is the time scale for the reorientation of microorganisms by the gravitational torque against viscous resistance. In Pedley and Kessler [1987] this parameter is called the “gyrotactic orientation parameter”, which can be expressed as: $B = \frac{\alpha_\perp \mu}{2h\rho_o g}$ ..(16)

where $\alpha_\perp$ is the dimensionless constant relating viscous torque to the relative angular velocity of the cell and h is the displacement of center of mass of the cell from the center of buoyancy. The components ξ of vector $\hat{p}^1$ in eqn.(15) is connected to perturbations of velocity component by the following eq: $\xi = (1+\alpha_o)\frac{\partial u^1}{\partial y} - (1-\alpha_o)\frac{\partial v^1}{\partial x}$ . ..(17) α_o is the cell eccentricity which is given by the following equation. $\alpha_o = \frac{a^2 - b^2}{a^2 + b^2}$ ….(18)

where a and b are the semi-major and semi-minor axes of the spheroidal cell respectively.The following equation for n^1 is obtained by substituting eqn.(15) in to (14) and then using eqn.(17)

Equation (26) can be solved for the growth rate parameter σ This results in two roots.

$$\sigma = \frac{1}{2}(-D\,C_a\,\rho_o\,K\,k^3 - k\,C_a\,\rho_o\,K\,i\,m\,W_c - k\,\mu + \text{sqrt}(D^2\,C_a^{\;2}\,\rho_o^{\;2}\,K^2\,k^6 + 2\,D\,C_a^{\;2}\,\rho_o^{\;2}\,K^2\,k^4\,i\,m\,W_c - 2\,D\,C_a\,\rho_o\,K\,k^4\,\mu + k^2\,C_a^{\;2}\,\rho_o^{\;2}\,K^2\,i^2\,m^2\,W_c^{\;2} - 2\,k^2\,C_a\,\rho_o\,K\,i\,m\,W_c\,\mu + k^2\,\mu^2 + 4\,C_a\,\rho_o\,K^2\,B\,n_o\,G\,g\,l^2\,\Delta\rho\,\theta\,W_c\,m^2 \ldots\text{etc}$$

and

$$\sigma = \frac{1}{2}(-D\,C_a\,\rho_o\,K\,k^3 - k\,C_a\,\rho_o\,K\,i\,m\,W_c - k\,\mu - \text{sqrt}(D^2\,C_a^{\;2}\,\rho_o^{\;2}\,K^2\,k^6 + 2\,D\,C_a^{\;2}\,\rho_o^{\;2}\,K^2\,k^4\,i\,m\,W_c - 2\,D\,C_a\,\rho_o\,K\,k^4\,\mu + k^2\,C_a^{\;2}\,\rho_o^{\;2}\,K^2\,i^2\,m^2\,W_c^{\;2} - 2\,k^2\,C_a\,\rho_o\,K\,i\,m\,W_c\,\mu + k^2\,\mu^2 + 4\,C_a\,\rho_o\,K^2\,B\,n_o\,G\,g\,l^2\,\Delta\rho\,\theta\,W_c\,m^2 \ldots\text{etc} \quad ..(27)$$

In the above equation and in the subsequent equations $G = G^*$. In order to prove that critical permeability exists it is absolutely necessary to prove that **(i)** the system is stable, when the permeability of the porous medium is close to zero and **(ii)** the system becomes unstable when the permeability is sufficiently large. Accordingly instability appeared only when the real part of σ is positive. It is observed that since the root with the positive sign in front of the forth term in eqn.(27) of first root has a greater real part, analysis is concentrated on this root it self. The Taylor series expansion of this root about the point K=0 is found . By neglecting the quadratic and higher order terms in this expansion, the following solution (valid only for the small values of permeability) is obtained;

$$\sigma = \frac{1}{2}\frac{-k\,\mu + \sqrt{k^2\,\mu^2}}{C_a\,\rho_o\,k}K^{-1} + \frac{1}{2}\frac{-D\,C_a\,\rho_o\,k^3 - k\,C_a\,\rho_o\,i\,m\,W_c + \dfrac{\frac{1}{2}\sqrt{k^2\,\mu^2}\,(-2\,k^2\,C_a\,\rho_o\,i\,m\,W_c\,\mu - 2\,D\,C_a\,\rho_o\,k^4\,\mu)}{k^2\,\mu^2}}{C_a\,\rho_o\,k}$$

$$+\frac{1}{2}\sqrt{k^2\,\mu^2}\left(\frac{1}{2}(k^2\,C_a^{\;2}\,\rho_o^{\;2}\,i^2\,m^2\,W_c^{\;2} + D^2\,C_a^{\;2}\,\rho_o^{\;2}\,k^6 + 2\,D\,C_a^{\;2}\,\rho_o^{\;2}\,k^4\,i\,m\,W_c\right.$$
$$+ 4\,C_a\,\rho_o\,B\,n_o\,G\,g\,l^2\,\Delta\rho\,\theta\,W_c\,m^2 + 4\,C_a\,\rho_o\,B\,n_o\,G\,g\,l^2\,\Delta\rho\,\theta\,W_c\,m^2\,\alpha_o$$

$$+4\,C_a\,\rho_o\,B\,n_o\,G\,g\,l^4\,\Delta\rho\,\theta\,W_c - 4\,C_a\,\rho_o\,B\,n_o\,G\,g\,l^4\,\Delta\rho\,\theta\,W_c\,\alpha_o)\,/\,(k^2\,\mu^2)$$

$$-\frac{1}{8}\,\frac{(-2\,k^2\,C_a\,\rho_o\,i\,m\,W_c\,\mu - 2\,D\,C_a\,\rho_o\,k^4\,\mu)^2}{k^4\,\mu^4}\Bigg)\Big/\,(C_a\,\rho_o\,k)\,K + O(K^2) \qquad ..(28)$$

From the above equation it is observed that for K=0, σ has a negative real part $-k^2D$, which is independent of the modulation parameter suggests that for sufficiently small values of permeability the system is stable. Therefore in order to prove that the system becomes unstable with the increase in K, it is absolutely necessary to show that the real part of σ will be positive. Suppose m=o (which corresponds to the case of no vertical disturbances). In this case the root of the eqn. (27) with the greater real part is found to be

$$\sigma = \frac{1}{2}\,\frac{-k\,\mu + \sqrt{k^2\,\mu^2}}{C_a\,\rho_o\,k\,K} + \frac{\frac{1}{2}\left(-D\,C_a\,\rho_o\,k^3 - \frac{\sqrt{k^2\,\mu^2}\,D\,C_a\,\rho_o\,k^2}{\mu}\right)}{C_a\,\rho_o\,k} + \frac{1}{2}\sqrt{k^2\,\mu^2}\Bigg($$

$$\frac{1}{2}\,\frac{D^2\,C_a^{\,2}\,\rho_o^{\,2}\,k^6 + 4\,C_a\,\rho_o\,B\,n_o\,G\,g\,l^4\,\Delta\rho\,\theta\,W_c - 4\,C_a\,\rho_o\,B\,n_o\,G\,g\,l^4\,\Delta\rho\,\theta\,W_c\,\alpha_o}{k^2\,\mu^2}$$

$$-\frac{1}{2}\,\frac{D^2\,C_a^{\,2}\,\rho_o^{\,2}\,k^4}{\mu^2}\Bigg)K\,/\,(C_a\,\rho_o\,k) \qquad \ldots(29)$$

The upper limit of the critical permeability is computed by solving the equation (29) for σ =0

$$K_{crit}^{upper} = -\frac{D\,\mu}{B\,n_o\,G\,g\,\Delta\rho\,\theta\,W_c\,(-1+\alpha_o)} \qquad ..(30)$$

The above equation clearly proves and predicts that the existence of critical permeability and further it is observed that the upper limit of the above solution coincides with that of unmodulated case (Kuznetsov(2002)) $G^* = 1$

The critical permeability is the minimum value of K for all allowable wavenumbers and critical permeability is less when compared to the unmodulated case .

4 Estimation of the value of the critical permeability

For this purpose the linearized, eqn. (28) is considered and Re(σ)=0 results in the following equation:

$$\frac{1}{2}\frac{-k\,\mu+\sqrt{k^2\,\mu^2}}{C_a\,\rho_o\,k\,K}+\frac{\frac{1}{2}\left(-D\,C_a\,\rho_o\,k^3-\frac{\sqrt{k^2\,\mu^2}\,D\,C_a\,\rho_o\,k^2}{\mu}\right)}{C_a\,\rho_o\,k}+\frac{1}{2}\sqrt{k^2\,\mu^2}\left(\frac{1}{2}(D^2\,C_a^{\;2}\,\rho_o^{\;2}\,k^6\right.$$
$$+4\,C_a\,\rho_o\,B\,n_o\,g\,G\,l^2\,\Delta\rho\,\theta\,W_c\,m^2+4\,C_a\,\rho_o\,B\,n_o\,g\,G\,l^2\,\Delta\rho\,\theta\,W_c\,m^2\,\alpha_o$$
$$+4\,C_a\,\rho_o\,B\,n_o\,g\,G\,l^4\,\Delta\rho\,\theta\,W_c-4\,C_a\,\rho_o\,B\,n_o\,g\,G\,l^4\,\Delta\rho\,\theta\,W_c\,\alpha_o)\,/\,(k^2\,\mu^2) \quad ..(31)$$
$$\left.-\frac{1}{2}\frac{D^2\,C_a^{\;2}\,\rho_o^{\;2}\,k^4}{\mu^2}\right)K\,/\,(C_a\,\rho_o\,k)=0$$

In the above equation K= $\tilde{K}$, G= G^*

where $\tilde{K}$ denotes the value of K for which real part of σ =0. Solving the equation for $\tilde{K}$ results in:

$$\tilde{K}=\frac{D\,k^4\,\mu}{B\,n_o\,G\,g\,l^2\,\Delta\rho\,\theta\,W_c\,(m^2+m^2\,\alpha_o+l^2-l^2\,\alpha_o)}$$

Substituting $k^2=l^2+m^2$

$$\frac{D\,(m^2+l^2)^2\,\mu}{B\,n_o\,G\,g\,l^2\,\Delta\rho\,\theta\,W_c\,(m^2+m^2\,\alpha_o+l^2-l^2\,\alpha_o)} \quad ..(32)$$

It is important to note that $\tilde{K}_{min}$ is the critical permeability for all the allowable wave numbers. Accordingly the extremum conditions yields the following equation:

$$\frac{\partial\tilde{K}}{\partial l}=\frac{\partial\tilde{K}}{\partial m}= \quad 0= \quad 4\frac{D\,(m^2+l^2)\,\mu}{B\,n_o\,G\,g\,l\,\Delta\rho\,\theta\,W_c\,(m^2+m^2\,\alpha_o+l^2-l^2\,\alpha_o)}$$
$$-\frac{2\,D\,(m^2+l^2)^2\,\mu}{B\,n_o\,G\,g\,l^3\,\Delta\rho\,\theta\,W_c\,(m^2+m^2\,\alpha_o+l^2-l^2\,\alpha_o)}-\frac{D\,(m^2+l^2)^2\,\mu\,(2\,l-2\,l\,\alpha_o)}{B\,n_o\,G\,g\,l^2\,\Delta\rho\,\theta\,W_c\,(m^2+m^2\,\alpha_o+l^2-l^2\,\alpha_o)^2}=0$$

on simplification we get,

$$-2\frac{D\,(m^2+l^2)\,\mu\,m^2\,(l^2-3\,l^2\,\alpha_o+m^2+m^2\,\alpha_o)}{B\,n_o\,G\,g\,l^3\,\Delta\rho\,\theta\,W_c\,(m^2+m^2\,\alpha_o+l^2-l^2\,\alpha_o)^2}=0$$

This implies

$$l^2-3\,l^2\,\alpha_o+m^2+m^2\,\alpha_o=0 \quad ..(33)$$

Solving the above eqn. for m (the vertical wave number) and considering only the positive root:

m= $\frac{\sqrt{(1+\alpha_o)(-1+3\alpha_o)}\, l}{1+\alpha_o}$

$$= l\sqrt{\frac{3\alpha_o - 1}{\alpha_o + 1}} \quad ..(34)$$

Substituting m value in eqn. (32) we have

$$K_{crit} = \frac{D\left(\frac{(-1+3\alpha_o)l^2}{1+\alpha_o}+l^2\right)^2 \mu}{B n_o G g l^2 \Delta\rho\theta W_c\left(\frac{(-1+3\alpha_o)l^2}{1+\alpha_o}+\frac{(-1+3\alpha_o)l^2\alpha_o}{1+\alpha_o}+l^2-l^2\alpha_o\right)} \quad \text{ie } K_{crit} = 8\frac{\mu\alpha_o D}{W_c\theta\rho\Delta g G n_o B(1+\alpha_o)^2} \quad ..(35)$$

..

,

Substituting m=0 in eqn.(32) we have following critical permeability for the case of $\alpha_o \le 1/3$

,

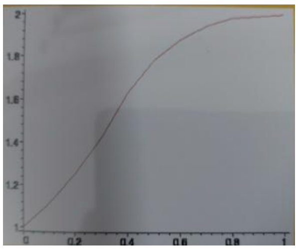
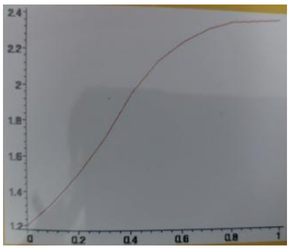
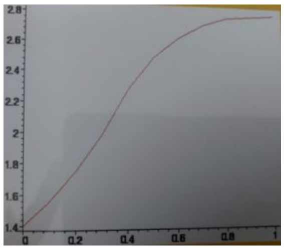
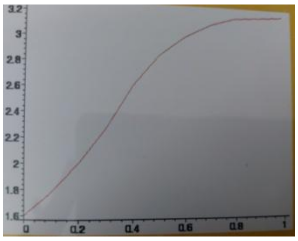
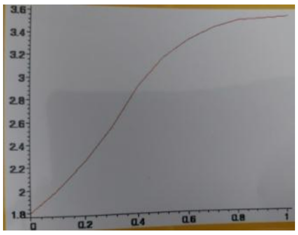
$$K_{crit} = \frac{D l^2 \mu}{B n_o G g \Delta\rho\theta W_c (l^2 - l^2\alpha_o)} \quad \text{on simplifying} \quad K_{crit} = -\frac{D\mu}{B n_o G g \Delta\rho\theta W_c(-1+\alpha_o)}$$

..(36)

It is observed that at $\alpha_o = 1/3$ eqns. (35) and (36) match

$$K_{crit} = \begin{matrix} \frac{1}{(1-\alpha_o)}\frac{D\mu}{Bn_oGgW_c\Delta\rho\theta} & for\ 0 \le \alpha_o \le 1/3 \\ \frac{8\alpha_o}{(1-\alpha_o)}\frac{D\mu}{Bn_oGgW_c\Delta\rho\theta} & for\ i/3 \le \alpha_o \le 1 \end{matrix} \quad ..(37)$$

5 Results: This case investigates the 2D- stability of bio- porousconvection in modulated as well as in unmodulated environments. As discussed earlier, gyrotactic microorganisms swim generally upward and the exact direction of swimming of these microorganisms is determined by the balance of two torques, The analysis is carried out under the main assumption that the porous matrix does not absorb the microorganisms and that the suspension is dilute. The criterion for the existence of critical permeability is determined. One of the important predictions is : the cell elongation and eccentricity of microorganisms have a dominating influence on the system considered. In figures 1 to 5, the graphs of Dar_{crit} vs α_o are presented for $G^* = 1.0$ to 1.8 . The critical permeability value increases continuously with α_o and remains constant ($\square 2$) for values of $\alpha_o \ge 0.7$ and $G^* = 1$. (unmodulated case) which is exactly the same as that of Kuznetsov (2002). However, as G^* increases from 1.0 to 1.8, it is observed that, the behavior of the profile is exactly the same as that of unmodulated case, but the critical permeability value remains constant for $\alpha_o \geq 0.6$ and the constant value from 2.0 to 3.6.

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