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Stability Analysis of Oscillatory Motions in Magnetothermohaline Convection Problem with Temperature Dependent Viscosity

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Abstract

In this paper, the problem of thermohaline convection under the influence of temperature dependent viscosity and uniform magnetic field acting opposite to the gravity is studied mathematically using linear stability theory for general nature of boundary conditions. A sufficient condition for the stability of oscillatory motions for this general problem is derived, using some integral estimates obtained from the non-dimensional linearized perturbations equations governing the problem and hence the bounds for the complex growth rate for arbitrary neutral or unstable oscillatory modes (when it exist) are prescribed, which include the effects of both temperature dependent viscosity and the magnetic Prandtl number and are uniformly valid for all cases of boundary conditions.

Keywords: Thermohaline convection; Linear stability; Growth rate; Magnetic field; Temperature dependent viscosity, Oscillatory motions.

1. Introduction

Thermohaline convection has matured into a subject possessing fundamental departures from its classical counterpart, namely, thermal convection and is of direct relevance to the fields of limnology, oceanography, geophysics, astrophysics, chemical engineering, etc. The various applications of the problem have aroused the interest of many research workers and this led to numerous research papers in various journals in the recent past. Two fundamental configurations have been studied in the context of the thermohaline instability problem, one by Stern [1], wherein the temperature gradient is stabilizing and the concentration gradient is destabilizing and another by Veronis [2], wherein the temperature gradient is destabilizing and the concentration gradient is stabilizing. Gupta *et. al.* [3] studied the stability of thermohaline configurations of Veronis and Stern types in horizontal layer geometry and prescribed the upper limits for the growth rate of the oscillatory motions of neutral or growing amplitude in such a way that it also results in the sufficient conditions for the stability for an initially top-heavy or an initially bottom-heavy configuration. For a broad view of the subject one may be referred to Turner [4], Brandt and Fernando[5] and Schmitt[6].

When an electrically conducting fluid is subjected to the influence of a magnetic field, the electrical conductivity of the fluid and the prevalence of magnetic fields contribute to two kinds of effects: first, by the motion of electrically conducting fluid across the magnetic lines of force, electric currents are generated and the associated magnetic field contribute to change in the existing fields and second, the fluid elements carrying currents transverse magnetic lines of force contributes to additional forces (Lorentz Force) acting on the fluid elements. Chandrasekhar [7] has investigated the effect of magnetic field on the stability of thermal convection problem in detail. Banerjee et. al. [8] studied a more general problem, namely, magnetohydrodynamic thermohaline convection problem and derived a characterization theorem. N. Rudraiah [9, 10] has made a study of double diffusive magnetoconvection and have shown that magnetic field destabilizes the double diffusive system under certain conditions.

It is well known fact, that the viscosity of a fluid is one of the properties which are most sensitive to temperature (cf. Straughan [11]). In the majority of the cases, viscosity becomes the only property which may have considerable effect on the heat transfer and the temperature variation and the dependence of other thermo-physical properties to temperature is often negligible. Torrance & Turcotte [12] observed that the viscosity of the liquids decreases with increasing temperature, while the reverse trend is observed in gases. Therefore, the type of fluid and the range of operating temperatures are very important and crucial parameters in the study of fluid dynamics, and in particular in the fields of oceanography, astrophysics etc. In most of the studies pertaining to single or double-diffusive convection, authors have considered the viscosity of the fluid to be constant. Because, when the viscosity of the fluid is varying with temperature in convective instability problems, then the eigenvalue problems results in the forms of differential equations with variable coefficients, which introduces additional mathematical complexities. Some authors, including Trompert and Hansen [13], Palm [14], Stengel et al. [15] and Dhiman and Kumar [16]) have considered the effect of variable viscosity on the thermal convection problems. Gupta and Kaushal [17] have also analyzed the stability of double-diffusive convection problems of Veronis and Stern types, by taking into account the variations in viscosity. Dhiman and Kumar [18] studied the effect of temperature dependent viscosity on the onset of instability in thermohaline convection problems of Veronis and Stern type configurations, using linear stability theory and derived some general qualitative and quantitative results and discussed the effect of temperature dependent viscosity on the onset of stationary convection for each combination of rigid and dynamically free boundary conditions.

Motivated by the above discussions and the importance of the effect of viscosity variation on the onset of convection and also keeping in view the effect of magnetic field in convective processes, in the present paper, the problem of thermohaline convection under the influence of temperature dependent viscosity and uniform magnetic field acting opposite to the gravity is studied mathematically using linear stability theory for general nature of boundary conditions. A sufficient condition for the stability of oscillatory motions for this general problem is derived, using some integral estimates obtained from the non-dimensional linearized perturbations equations governing the problem and hence the bounds for the complex growth rate for arbitrary neutral or unstable oscillatory modes (when it exist) are prescribed, which include the effects of both temperature dependent viscosity and the magnetic Prandtl number and are uniformly valid for all cases of boundary conditions.

2. The Physical Configuration and the Basic Equations

A horizontal layer of viscous incompressible fluid of infinite horizontal extension and finite vertical depth is statically confined between two horizontal boundaries $z = 0$ and $z = d$, which are respectively maintained at uniform temperatures $T_0$and $T_1(< T_0)$ and uniform solute concentrations S_0 and $S_1(< S_0)$ and is kept under the influence of uniform vertical magnetic field. Let the viscosity of the fluid depend upon temperature. Let the origin be taken on the lower boundary $z = 0$ with z-axis perpendicular to it. It is further assumed that cross diffusion effects are neglected.

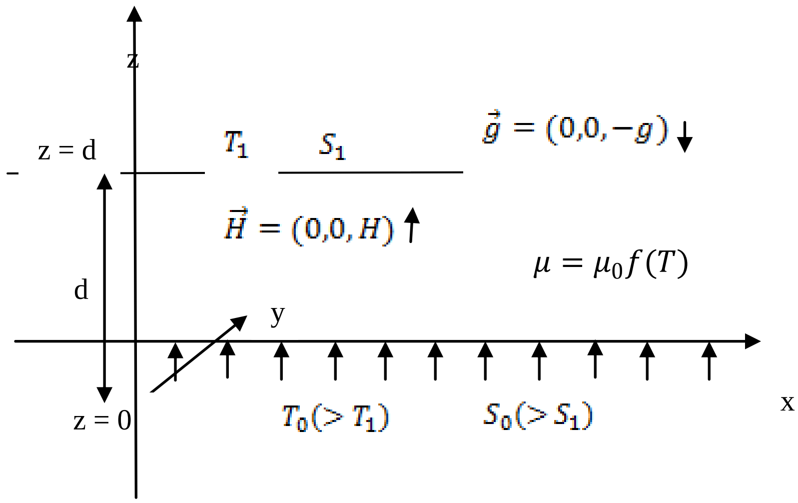

Fig: Geometrical configuration of the problem

Following the usual steps of linear stability theory and proceeding analogously as in the analysis of Gupta et. al. [3] and Dhiman and Kumar [18], one can easily see that the non-dimensional linearized perturbation equations and the boundary conditions, which govern the initiation of the convection in thermohaline instability problems, when the viscosity is an arbitrary function of temperature, are given by

$$f\left(D^2-a^2\right)^2 w-\frac{p}{\sigma}\left(D^2-a^2\right)w+2(Df)D\left(D^2-a^2\right)w+D^2 f\left(D^2+a^2\right)w$$
$$=R_T a^2\theta-R_S a^2\phi-QD\left(D^2-a^2\right)h_z \quad (1)$$

$$\left(D^2-a^2-p\right)\theta=-w \quad (2)$$

$$\left(D^2-a^2-\frac{p}{\tau}\right)\phi=-\frac{w}{\tau} \quad (3)$$

$$\left(D^2-a^2-\frac{p\sigma_1}{\sigma}\right)h_z=-Dw \quad (4)$$

together with following cases of the boundary conditions

Case I. $w=0=\theta=\phi=D^2w=h_z$ at $z=0$ and $z=1$, (5)

(Both dynamically free boundaries)

Case II. $w=0=\theta=\phi=Dw=h_z$ at $z=0$ and $z=1$, (6)

(Both rigid boundaries)

Case III. $w = 0 = \theta = \phi = D^2 w = h_z$ at $z = 0$ and

$w = 0 = \theta = \phi = Dw = h_z$ at $z = 1$, (Lower free and upper rigid)

or

$w = 0 = \theta = \phi = D^2 w = h_z$ at $z = 1$ and

$w = 0 = \theta = \phi = Dw = h_z$ at $z = 0$, (7)

(Lower rigid and upper free)

$Dh_z = \mp k h_z$ (on the upper and lower boundary respectively if the region outside the liquid are insulating). (8)

In the foregoing equations, z is the real independent variable, $D = d/dz$ is the differentiation with respect to z, a^2 is the square of the wave number, σ is the thermal Prandtl number, σ_1 is the magnetic Prandtl number, τ is the Lewis number, R is the thermal Rayleigh number, R_s is the concentration Rayleigh number, Q is the Chandrasekhar number, $p(= p_r + ip_i)$ is the complex growth rate and w, θ, ϕ and h_z are the perturbations in the vertical velocity, temperature, concentration and magnetic field respectively and are complex valued functions of the vertical coordinate z only.

Also, in deriving the above equations, the viscosity of the fluid $\mu = \mu_0 f(T)$ is taken to be temperature dependent; μ_0 is the viscosity at the lower boundary and $f(T)$ is an arbitrary function (which is twice continuously differentiable) of vertical coordinate z. Also, we have followed the assumptions of Stengel, regarding the small ratio of the viscosities at the top to the bottom boundaries.

System of equations (1) – (4) together with either of the boundary conditions (5) – (8) governing the thermohaline convection problem with temperature dependent viscosity in the presence of magnetic field constitutes an eigen value problem for R_T for given values of the other parameters, namely, p, Q, R_s, σ and a^2. Further, a given state of system is stable, neutral or unstable according as p_r is negative, zero or positive. Further if $p_r = 0$ implies $p_i = 0$ for every wave number a, then the *principle of the exchange of stabilities* (PES) is valid, which means that instability sets in as stationary convection, otherwise we shall have *overstability* at least when instability sets in as certain modes.

Furthermore, the eigen value problem represented by above equations describes

a) Veronis type thermohaline convection (VTHC), when $R_T > 0$ and $R_S > 0$ and

b) Stern's type thermohaline convection (STHC), when $R_T < 0$ and $R_S < 0$.

3. Mathematical Analysis

In this section, we shall mathematically analyze the stability of the considered problem.

a) **Stability of the oscillatory modes**

We shall investigate the character of the oscillatory modes.

Multiply equation (1) by w^* (the complex conjugate of w) and integrating the resulting equation by parts a suitable number of times and using the relevant boundary conditions (5)-(8) and equations (2)-(4), we have following equation

$$\int_0^1 f\left\{\left|D^2 w\right|^2 + 2a^2|Dw|^2 + a^4|w|^2\right\}dz + \frac{p}{\sigma}\int_0^1\left(|Dw|^2 + a^2|w|^2\right)dz + a^2\int_0^1\left(D^2 f\right)|w|^2 dz$$
$$= R_T a^2\int_0^1\left(|D\theta|^2 + a^2|\theta|^2\right)dz - R_S a^2\tau\int_0^1\left(|D\phi|^2 + a^2|\phi|^2\right)dz - Q\int_0^1\left(\left|D^2 h_z\right|^2 + 2a^2|Dh_z|^2 + a^4|h_z|^2\right)dz$$
$$+ a^2 p^*\left[R_T\int_0^1|\theta|^2 dz - R_S\int_0^1|\phi|^2 dz - \frac{Q\sigma_1}{\sigma}\int_0^1\left(|Dh_z|^2 + a^2|h_z|^2\right)dz\right]$$

Now equating the real and imaginary parts of both sides of above equation, we have

$$\int_0^1 f\left\{\left|D^2 w\right|^2 + 2a^2|Dw|^2 + a^4|w|^2\right\}dz + \frac{p_r}{\sigma}\int_0^1\left(|Dw|^2 + a^2|w|^2\right)dz + a^2\int_0^1\left(D^2 f\right)|w|^2 dz$$
$$- R_T a^2\int_0^1\left(|D\theta|^2 + a^2|\theta|^2\right)dz + R_S a^2\tau\int_0^1\left(|D\phi|^2 + a^2|\phi|^2\right)dz + Q\int_0^1\left(\left|D^2 h_z\right|^2 + 2a^2|Dh_z|^2 + a^4|h_z|^2\right)dz$$
$$= a^2 p_r\left[R_T\int_0^1|\theta|^2 dz - R_S\int_0^1|\phi|^2 dz - \frac{Q\sigma_1}{\sigma}\int_0^1\left(|Dh_z|^2 + a^2|h_z|^2\right)dz\right] \qquad (9)$$

and

$$p_i\left[\frac{1}{\sigma}\int_0^1\left(|Dw|^2+a^2|w|^2\right)dz+R_T a^2\int_0^1|\theta|^2\,dz-R_S a^2\int_0^1|\phi|^2\,dz-\frac{Q\sigma_1}{\sigma}a^2\int_0^1\left(|Dh_z|^2+a^2|h_z|^2\right)dz\right]=0$$

(10)

It is clear from equation (10) that p_i may or may not be zero, which in view of above discussions means that PES is not valid for the present problem in general and thus, instability can also arise through oscillations.

let us suppose that $p_i \neq 0$, i.e. PES is not valid. This fact means that the instability is through oscillations. So cancelling p_i though out from equation (10), we have

$$\frac{1}{\sigma}\int_0^1\left(|Dw|^2+a^2|w|^2\right)dz+R_T a^2\int_0^1|\theta|^2\,dz-R_S a^2\int_0^1|\phi|^2\,dz-\frac{Q\sigma_1}{\sigma}a^2\int_0^1\left(|Dh_z|^2+a^2|h_z|^2\right)dz=0$$

(11)

which yields, that;

$$R_S a^2\int_0^1|\phi|^2\,dz>\frac{1}{\sigma}\int_0^1\left(|Dw|^2+a^2|w|^2\right)dz-\frac{Q\sigma_1 a^2}{\sigma}\int_0^1\left(|Dh_z|^2+a^2|h_z|^2\right)dz \tag{12}$$

Since $w, \theta, \phi\ and\ h_z$ satisfy the boundary conditions:

$w(0) = 0 = w; \theta(0) = 0 = \theta(1); \phi(0) = 0 = \phi(1); h_z(0) = 0 = h_z(1);$

therefore, upon using Rayleigh-Ritz inequality [19], we have

$$\int_0^1|Dw|^2dz \geq \pi^2 \int_0^1|w|^2dz, \tag{13}$$

$$\int_0^1|D\theta|^2dz \geq \pi^2 \int_0^1|\theta|^2dz, \tag{14}$$

$$\int_0^1|D\phi|^2dz \geq \pi^2 \int_0^1|\phi|^2dz, \tag{15}$$

$$\int_0^1|Dh_z|^2dz \geq \pi^2 \int_0^1|h_z|^2dz. \tag{16}$$

Further,

$$\int_0^1(\,|D\theta|^2+a^2|\theta|^2)dz=\left|-\int_0^1\theta^*(D^2-a^2)\theta dz\right|\leq\int_0^1|\theta^*(D^2-a^2)\theta|\,dz$$

$$= \int_0^1 |\theta^*| \, |(D^2 - a^2)\theta| dz \le \int_0^1 |\theta| \, |(D^2 - a^2)\theta| dz$$

$$\le \left[\int_0^1 |\theta|^2 dz\right]^{\frac{1}{2}} \left[\int_0^1 |(D^2 - a^2)\theta|^2 dz\right]^{\frac{1}{2}} \quad (17)$$

(Using *Schwartz* inequality)

Also,

$$\int_0^1 (\,|D\theta|^2) dz = \left| - \int_0^1 \theta^* D^2 \theta dz \right| \le \int_0^1 |\theta^* D^2 \theta| dz \; = \int_0^1 |\theta^*| |D^2 \theta| \, dz$$

$$\le \int_0^1 |\theta| |D^2 \theta| \, dz$$

$$\le \left[\int_0^1 |\theta|^2 dz\right]^{\frac{1}{2}} \left[\int_0^1 |D^2 \theta|^2 dz\right]^{\frac{1}{2}}. \quad \text{(using Schwartz inequality)} \quad (18)$$

Using inequality (14), inequality (18) implies that

$$\int_0^1 |D^2 \theta|^2 dz \ge \pi^4 \int_0^1 |\theta|^2 dz, \quad (19)$$

Also proceeding analogously as in the derivation of the inequality (19), we can have

$$\int_0^1 |D^2 w|^2 dz \ge \pi^4 \int_0^1 |w|^2 dz, \quad (20)$$

$$\int_0^1 |D^2 h_z|^2 dz \ge \pi^4 \int_0^1 |h_z|^2 dz. \quad (21)$$

Combining inequalities (13) and (20), we have

$$\int_0^1 f(|D^2 w|^2 + 2a^2 |Dw|^2 + a^4 |w|^2) dz \ge f_{min} \, (\pi^2 + a^2)^2 \int_0^1 |w|^2 \, dz, \quad (22)$$

where, f_{min} is the minimum value of $f(z)$ in closed interval $[0,1]$.

Again, we have

$$\int_0^1 (\,|Dh_z|^2) dz = \left| - \int_0^1 h_z{}^* D^2 h_z dz \right|$$

$$\le \int_0^1 |h_z{}^* D^2 h_z| dz \; = \int_0^1 |h_z{}^*| |D^2 h_z| \, dz \; \le \int_0^1 |h_z| |D^2 h_z| \, dz$$

$$\le \left[\int_0^1 |h_z|^2 dz\right]^{\frac{1}{2}} \left[\int_0^1 |D^2 h_z|^2 dz\right]^{\frac{1}{2}}. \quad \text{(using Schwartz inequality)} \quad (23)$$

Using inequality (16), inequality (23) implies that

$$\int_0^1 |Dh_z|^2\,dz < \frac{1}{\pi^2}\int_0^1 |D^2h_z|^2 dz \quad (24)$$

Therefore, we can have

$$\int_0^1 |(D^2-a^2)h_z|^2 dz = \int_0^1 (|D^2h_z|^2 + a^2|Dh_z|^2 + a^2|Dh_z|^2 + a^4|h_z|^2)dz.$$

$$\geq (\pi^2+a^2)\frac{Q\sigma_1}{\sigma}\int_0^1 \left(|Dh_z|^2 + a^2|h_z|^2\right)dz$$

Further, multiplying equation (2) by its complex conjugate and integrating the various terms on left hand side of the resulting equation by parts for an appropriate number of times and making use of boundary condition $\theta(0)=0=\theta(1)$, we obtained

$$\int_0^1 \left(|D^2\theta|^2 + 2a^2|D\theta|^2 + a^4|\theta|^2\right)dz + 2p_r\int_0^1 \left(|D\theta|^2 + a^2|\theta|^2\right)dz + |p|^2\int_0^1 \left(|\theta|^2\right)dz = \int_0^1 \left(|w|^2\right)dz \quad (25)$$

If permissible, let $p_r \geq 0$. In view of this fact, equation (25) implies that:

$$\int_0^1 |(D^2-a^2)\theta|^2 dz = \int_0^1 (|D^2\theta|^2 + 2a^2|D\theta|^2 + a^4|\theta|^2)dz < \int_0^1 |w|^2 dz. \quad (26)$$

Further combining inequalities (13) and (19), we have

$$\int_0^1 (|D^2\theta|^2 + 2a^2|D\theta|^2 + a^4|\theta|^2)dz \geq (\pi^2+a^2)^2\int_0^1 |\theta|^2 dz. \quad (27)$$

So, inequality (17), together with inequalities (26) and (27), yields

$$\int_0^1 (|D\theta|^2 + a^2|\theta|^2)dz < \frac{1}{(\pi^2+a^2)}\int_0^1 |w|^2 dz. \quad (28)$$

We can also derive the following inequalities analogous to the derivation of the inequality (27);

$$\int_0^1 (\,|Dw|^2 + a^2|w|^2)dz \geq (\pi^2+a^2)\int_0^1 |w|^2 dz, \quad (29)$$

$$\int_0^1 (\,|D\phi|^2 + a^2|\phi|)dz \geq (\pi^2+a^2)\int_0^1 |\phi|^2 dz, \quad (30)$$

$$\int_0^1 (\,|Dh_z|^2 + a^2|h_z|^2)dz \geq (\pi^2+a^2)\int_0^1 |h_z|^2 dz.$$

Now, inequality (12), by using inequality (29), yields:

$$R_S a^2\int_0^1 |\phi|^2 dz > \frac{(\pi^2+a^2)}{\sigma}\int_0^1 |w|^2 dz - Q\frac{\sigma_1}{\sigma}\int_0^1 (\,|Dh_z|^2 + a^2|h_z|^2)dz \quad (31)$$

and

$$Q\int_0^1(|Dh_z|^2+a^2|h_z|^2)dz > \frac{(\pi^2+a^2)}{\sigma_1}\int_0^1|w|^2dz - R_S a^2\frac{\sigma}{\sigma_1}\int_0^1|\phi|^2dz. \quad (32)$$

Now, multiplying equation (11) by p_r and adding the resulting equation to equation (9), we have

$$\int_0^1 f(|D^2w|^2+2a^2|Dw|^2+a^4|w|^2)dz+\int_0^1 a^2(D^2f)|w|^2dz$$
$$+\quad \frac{2p_r}{\sigma}\int_0^1(|Dw|^2+a^2|w|^2)\,dz - R_T a^2\int_0^1(|D\theta|^2+a^2|\theta|^2)dz$$
$$+\quad R_S a^2\tau\int_0^1(|D\phi|^2+a^2|\phi|^2)dz+$$
$$Q\int_0^1(|D^2h_z|^2+2a^2|Dh_z|^2+a^4|h_z|^2)dz=0. \quad (33)$$

For $D^2f\geq o$ (which is true for the most of the temperature dependent viscosity variation laws), and for $p_r\geq 0$, equation (33) yields:

$$\int_0^1 f(|D^2w|^2+2a^2|Dw|^2+a^4|w|^2)dz+R_S a^2\tau\int_0^1(|D\phi|^2+a^2|\phi|^2)dz+$$
$$Q\int_0^1(|D^2h_z|^2+2a^2|Dh_z|^2+a^4|h_z|^2)dz<$$
$$R_T a^2\int_0^1(|D\theta|^2+a^2|\theta|^2)dz \quad (34)$$

Now, using inequalities (22), (28) and (31) in inequality (34), we have

$$\frac{(\pi^2+a^2)^3}{a^2}\left(f_{\min.}+\frac{\tau}{\sigma}\right)\int_0^1|w|^2dz+\frac{(\pi^2+a^2)^2}{a^2}Q\sigma_1\left(\frac{1}{\sigma_1}-\frac{\tau}{\sigma}\right)\int_0^1(|Dh_z|^2+a^2|h_z|^2)dz<R_T\int_0^1|w|^2dz$$
(35)

Again, using inequalities (22), (28) and (32) in inequality (34), we have

$$\frac{(\pi^2+a^2)^3}{a^2}\left(f_{\min.}+\frac{1}{\sigma_1}\right)\int_0^1|w|^2dz+R_S(\pi^2+a^2)^2\sigma\left(\frac{\tau}{\sigma}-\frac{1}{\sigma_1}\right)\int_0^1|\phi|^2dz<R_T\int_0^1|w|^2dz \quad (36)$$

From inequalities (35) and (36), we obtain

$$\frac{(\pi^2+a^2)^3}{a^2}(f_{\min.}+\delta_1)\int_0^1|w|^2dz<R_T\int_0^1|w|^2dz \quad (37)$$

where, $\delta_1 = \min.\left(\frac{\tau}{\sigma}, \frac{1}{\sigma_1}\right)$.

Since, minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ with respect to a^2 is $\frac{27\pi^4}{4}$, therefore inequality (37) yields

$$\left[\frac{27\pi^4}{4}\left(f_{\min.}+\delta_1\right)-R_T\right]\int_0^1 |w|^2\,dz<0$$

From the above inequality, it is clear that if $R_T < \frac{27\pi^4}{4}(f_{min}+\delta_1)$, we have a contradiction. Hence we must have $p_r < 0$.

The above result clearly implies that the oscillatory modes of the system are stable, when $D^2 f \geq 0$ and $R_T < \frac{27\pi^4}{4}(f_{min}+\delta_1)$. Alternatively, the oscillatory modes of growing amplitude are not allowed for Veronis type Thermohaline convection problem with variable viscosity in the presence of magnetic field, if $R_T < \frac{27\pi^4}{4}(f_{min}+\delta_1)$, which clearly shows that the stability of the oscillatory motions depend upon the viscosity of the fluid and the magnetic Prandtl number.

In particular, when the viscosity is a linear, quadratic or exponential function of the temperature, the condition on viscosity i.e. $D^2 f \geq 0$ is automatically satisfied and $f_{min} = 1$.

It is remarkable to note that, when we consider the compliment of the above sufficient condition for the stability of the oscillatory motions, *i.e.* when $R_T \geq \frac{27\pi^4}{4}\left(f_{\min.}+\delta_1\right)$, the oscillatory modes of growing amplitude may exist. Hence it becomes important to prescribe the bounds for the growth rate of these motions.

b. Bounds for complex growth rate

In following analysis, we have derived the bounds, which arrest the complex growth rate of the arbitrary neutral or unstable ($p_r \geq 0$) oscillatory motions ($p_i \neq 0$) for VTHC problem.

Taking $p_r \geq 0$ in the equation (25) and using inequality (27) in the resulting inequality, we get the following inequality

$$\int_0^1 |\theta|^2 dz < \frac{1}{\left[(\pi^2+a^2)^2+|p|^2\right]} \int_0^1 |w|^2 dz. \tag{38}$$

Now, using inequalities (26) and (38) in inequality (17), we have

$$\int_0^1 (|D\theta|^2 + a^2|\theta|^2) dz < \frac{1}{(\pi^2+a^2)\left[1+\frac{|p|^2}{(\pi^2+a^2)^2}\right]^{\frac{1}{2}}} \int_0^1 |w|^2 dz. \tag{39}$$

So, using inequalities (15), (22), (24), (31) and (39), inequality (34) implies that

$$\frac{(\pi^2+a^2)^3}{a^2}\left(f_{\min.} + \frac{\tau}{\sigma}\right)\int_0^1 |w|^2 dz + \frac{(\pi^2+a^2)^2}{a^2} Q\sigma_1\left(\frac{1}{\sigma_1} - \frac{\tau}{\sigma}\right)\int_0^1 \left(|Dh_z|^2 + a^2|h_z|^2\right)dz$$

$$< \frac{R_T}{\left[1+\frac{|p|^2}{(\pi^2+a^2)^2}\right]^{1/2}} \int_0^1 |w|^2 dz \tag{40}$$

Upon using inequalities (15), (22), (24), (32) and (39), inequality (34) implies that

$$\frac{(\pi^2+a^2)^3}{a^2}\left(f_{\min.} + \frac{1}{\sigma_1}\right)\int_0^1 |w|^2 dz + R_S(\pi^2+a^2)^2\sigma\left(\frac{\tau}{\sigma} - \frac{1}{\sigma_1}\right)\int_0^1 |\phi|^2 dz$$

$$< \frac{R_T}{\left[1+\frac{|p|^2}{(\pi^2+a^2)^2}\right]^{1/2}} \int_0^1 |w|^2 dz \tag{41}$$

From inequalities (40) and (41), we obtain

$$\frac{(\pi^2+a^2)^3}{a^2}\left(f_{\min.} + \delta_1\right)\int_0^1 |w|^2 dz < \frac{R_T}{\left[1+\frac{|p|^2}{(\pi^2+a^2)^2}\right]^{1/2}} \int_0^1 |w|^2 dz \tag{42}$$

Since, minimum value of $\frac{(\pi^2+a^2)^3}{a^2}$ with respect to a^2 is $\frac{27\pi^4}{4}$, therefore inequality (42) yields

$$\left[\frac{27\pi^4}{4}(f_{min} + \delta_1) - \frac{R_T}{\left[1+\frac{|p|^2}{(\pi^2+a^2)^2}\right]^{\frac{1}{2}}}\right]\int_0^1 |w|^2 dz < 0. \qquad (43)$$

Inequality (43), clearly implies that

$$|p| < (\pi^2 + a^2)\sqrt{\Omega^2 - 1} \qquad (44)$$

where, $\Omega = \frac{4R_T}{27\pi^4(f_{min}+\delta_1)}$.

Also, from inequality (37), we have

$$(\pi^2 + a^2)\frac{(\pi^2+a^2)^2}{a^2}(f_{min} + \delta_1) < R_T. \qquad (45)$$

The minimum value of $\frac{(\pi^2+a^2)^2}{a^2}$ with respect to a^2 is $4\pi^2$, therefore inequality (45) yields

$$(\pi^2 + a^2) < \frac{R_T}{4\pi^2(f_{min}+\delta_1)}. \qquad (46)$$

Now, using inequality (46) in inequality (44), we get

$$|p| < \frac{R_T}{4\pi^2(f_{min}+\delta_1)}\sqrt{\Omega^2 - 1}.$$

The above result can be stated in the following Theorem.

Theorem 1. If $(p, w, \theta, \phi, h_z)$, $p = p_r + ip_i$, $p_i \neq 0$, $p_r \geq 0$, $R_T > 0$ and $R_S > 0$, $R_T \geq \frac{27\pi^4}{4}(f_{min} + \delta_1)$ is solution of equations (1)-(4) together with either of boundary conditions (5)-(8), then

$$|p| < \frac{R_T}{4\pi^2(f_{min}+\delta_1)}\sqrt{(\Omega^2 - 1)},$$

where, $\Omega = \frac{4R_T}{27\pi^4(f_{min}+\delta_1)}$.

In terminology of hydrodynamic stability theory, the above result can be stated as;

"The complex growth rate $p = p_r + ip_i$ of an arbitrary oscillatory $(p_i \neq 0)$ perturbation of growing amplitude ($p_r \geq 0$), in a Veronis' Type Thermohaline Convection with temperature dependent viscosity in the presence of magnetic field, when, $D^2 f \geq 0$ and $\Omega^2 > 1$, lies inside a semicircle in the right half of the $p_r p_i$-plane, whose centre is at the origin and whose radius is $\frac{R_T}{4\pi^2(f_{min} + \delta_1)}\sqrt{\Omega^2 - 1}$. Further, this result is uniformly valid for all combinations of rigid and dynamically free boundary conditions.

We observed from Theorem 1 that the radius of the growth rate for these oscillatory perturbations clearly depends upon the viscosity of the fluid and the and hence by modulating the viscosity through temperature, one can control the growth rate.

Also, Theorem 1 implies that when $\Omega^2 \leq 1$ and $R_T \leq \frac{27\pi^4}{4}(f_{min} + \delta_1)$, the oscillatory motions of growing amplitude are not allowed.

In particular, when the viscosity is a linear, quadratic or exponential function of temperature, $D^2 f \geq 0$ and $f_{min} = 1$ in the interval $z \in [0,1]$.

Therefore, from the above result we have

$$|p| < \frac{R_T}{4\pi^2(1+\delta_1)}\sqrt{\Omega^2 - 1},$$

where, $\Omega = \frac{4R_T}{27\pi^4(1+\delta_1)}$.

This is the same bound as obtained for Veronis' Type Thermohaline Convection problem with constant viscosity in the presence of magnetic field. Thus, the obtained result is a generalization of the result obtained for VTHC in the presence of magnetic field with constant viscosity.

Further, from the expression for the growth rate given in Theorem 1, we can see that the radius of arbitrary oscillatory perturbation decrease as the minimum value $f(T)$ increases, which means that more the viscosity of the fluid smaller is the region for the growth rate of the oscillatory perturbation which may be neutral or unstable. Hence, we conclude that by modulating the viscosity of the fluid with temperature, one can control the oscillation in Magnetothermohaline convection.

Following the analysis adopted in the derivation of the results for the case of VTHC problem, analogous results can be easily derived for the case of STHC problem in the presence of the magnetic field with variable viscosity, just by replacing R_T and R_S, respectively with $-|R_T|$ and $-|R_S|$.

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