A COMMON FIXED POINT THEOREMS FOR A PAIR OF SELF MAPS SATISFYING A GENERAL CONTRACTIVE CONDITION OF INTEGRAL TYPE

R. A. Rashwan*

H. A. Hammad**

ABSTRACT

In this paper, we prove two common fixed point theorems for a pair of self maps satisfying a general contractive condition of integral type, which extend and improve the results of M. R. Singh, L. Sharmeswar Singh[1].

*Department of Mathematics, Faculty of Science, Assuit University, Assuit 71516, Egypt

**Department of Mathematics, Faculty of Science, Sohag University, Sohag 82524, Egypt

1. INTRODUCTION

The first well known result on fixed points for contractive map was Banach's fixed point theorems, published in 1922 (see [3], [6]). In general setting of complete metric space, Smart([13]) presented the following result.

Theorem 1.1. [1] Let (X,d) be a complete metric space, $c\in[0,1)$, and let $T:X\to X$ be a map such that for each $x,y\in X$,

$$d(Tx,Ty)\le c\,d(x,y)$$

Then, T has a unique fixed point $z\in X$ such that for each $x\in X$, $\lim_{n\to\infty}T^n(x)=z$.

After this classical result, many theorems dealing with maps satisfying various types of contractive inequalities have been established (see for details [4], [7]-[12], [14]).

In 2002, Branciari ([5]) obtained the following theorem.

Theorem 1.2. [1] Let (X,d) be a complete metric space, $c\in[0,1)$, and let $T:X\to X$ be a map such that for each $x,y\in X$,

$$\int_0^{d(Tx,Ty)}\varphi(t)dt\le c\int_0^{d(x,y)}\varphi(t)dt,$$

where $\varphi:R^+\to R^+$ is a Lebesgue - integrable map which is summable, non negative and such that

$$\int_0^{\in}\varphi(t)dt>0 \text{ for each } \varepsilon>0.$$

Keywords: Lebesgue-integrable map, Complete metric space, weakly compatible mappings, Common fixed point.
2010 Mathematics subject classification:47H10

Then, T has a unique fixed point $z\in X$ and for each $x\in X$, $\lim_{n\to\infty}T^n(x)=z$.

In 2007, Boikanyo [3] proved some fixed point theorems for a self map satisfying a general contractive condition of integral type as an extension of Branciari's theorem. In [5], it was mentioned that one can generalize other results related to contractive conditions of some kind, such as in [10].

The main purpose of our paper is to obtain some results of a common fixed point theorems for a pair of self maps satisfying a general contractive condition of integral type,which is extending and improve the results of M. R. Singh, L. Sharmeswar Singh[1].

Definition 1.3. [2] Let f and g be two mappings from a metric space (X,d) into itself, f and g is called weakly compatible if they commute at there coincidence point .

i.e $fx = gx \Rightarrow fgx = gfx.$ for some $x \in X$.

Definition 1.4. [2**]** Two self maps f and g of metric space (X,d) is called compatible if

$\lim_{n\to\infty} d(fgx_n, gfx_n) = 0$, whenever $\{x_n\}$ is a sequence such that

$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = t,$ for some t in X.

Definition 1.5. [2] Maps f and g are said to be commuting if $fgx = gfx \ \forall x \in X$.

Definition 1.6. [2] Let f and g are two mappings on a set X, if $fx = gx$ for some x in X, then x is called coincidence point of f and g .

Throughout this paper, N denotes the set of natural numbers.

2.MAIN RESULTS

Theorem 2.1. Let (X, d) be a complete metric space. Let $a_i (i = 1,2,3,4,5)$ be nonnegative real numbers satisfying $\sum_{i=1}^{5} a_i < 1$, T_1, T_2, f and g are four self maps of X satisfying the following conditions:

1- $T_1(X) \subseteq f(X)$ and $T_2(X) \subseteq g(X)$,

2-the pair (T_2, f) and (T_1, g) are weakly compatible,

3-
$$\int_0^{d(T_1x,T_2y)} \varphi(t)dt \le a_1 \int_0^{d(fx,gy)} \varphi(t)dt + a_2 \int_0^{d(fx,T_1x)} \varphi(t)dt$$
$$a_3 \int_0^{d(gy,T_2y)} \varphi(t)dt + a_4 \int_0^{d(fx,T_2y)} \varphi(t)dt + a_5 \int_0^{d(gy,T_1x)} \varphi(t)dt, \quad (2.1)$$

where $\varphi : R^+ \to R^+$ is a Lebesgue-integrable map which is summable, non-negative and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \phi(t)dt > 0$. Then T_1, T_2, f and g have a unique common fixed point $z \in X$.

Proof: Suppose $x_\circ$ is an arbitrary point of X and define the sequence $\{y_n\}$ in X such that

$$y_n = T_2x_n = gx_{n+1} \quad \text{and} \quad y_{n+1} = T_1x_{n+1} = fx_{n+2} \tag{2.2}$$

By interchanging x with y, T_1 with T_2 and f with g, we obtain

$$\int_0^{d(T_2y,T_1x)} \varphi(t)dt \le a_1 \int_0^{d(gy,fx)} \varphi(t)dt + a_2 \int_0^{d(gy,T_2y)} \varphi(t)dt + a_3 \int_0^{d(fx,T_1x)} \varphi(t)dt$$
$$+ a_4 \int_0^{d(gy,T_1x)} \varphi(t)dt + a_5 \int_0^{d(fx,T_2y)} \varphi(t)dt. \tag{2.3}$$

Now from (2.1) and (2.3) and using symmetric property, we have

$$\int_0^{d(T_1x,T_2y)} \varphi(t)dt \le a_1 \int_0^{d(fx,gy)} \varphi(t)dt + (\frac{a_2+a_3}{2}) \int_0^{d(fx,T_1x)} \varphi(t)dt + (\frac{a_2+a_3}{2}) \int_0^{d(gy,T_2y)} \varphi(t)dt$$
$$+ (\frac{a_4+a_5}{2}) \int_0^{d(fx,T_2y)} \varphi(t)dt + (\frac{a_4+a_5}{2}) \int_0^{d(gy,T_1x)} \varphi(t)dt \tag{2.4}$$

Using (2.4), for even n, we obtain

$$\int_0^{d(y_n,y_{n+1})} \varphi(t)dt = \int_0^{d(T_2x_n,T_1x_{n+1})} \varphi(t)dt \le a_1 \int_0^{d(fx_n,gx_{n+1})} \varphi(t)dt + (\frac{a_2+a_3}{2}) \int_0^{d(fx_n,T_1x_n)} \varphi(t)dt$$

$$+ (\frac{a_2+a_3}{2}) \int_0^{d(gx_{n+1},T_2x_{n+1})} \varphi(t)dt + (\frac{a_4+a_5}{2}) \int_0^{d(fx_n,T_2x_{n+1})} \varphi(t)dt + (\frac{a_4+a_5}{2}) \int_0^{d(gx_{n+1},T_1x_n)} \varphi(t)dt$$

From (2.2) we have

$$\int_0^{d(y_n,y_{n+1})} \varphi(t)dt \le a_1 \int_0^{d(y_{n-1},y_n)} \varphi(t)dt + (\frac{a_2+a_3}{2}) \int_0^{d(y_{n-1},y_n)} \varphi(t)dt + (\frac{a_2+a_3}{2}) \int_0^{d(y_n,y_{n+1})} \varphi(t)dt$$
$$+ (\frac{a_4+a_5}{2}) \int_0^{d(y_{n-1},y_{n+1})} \varphi(t)dt + (\frac{a_4+a_5}{2}) \int_0^{d(y_n,y_n)} \varphi(t)dt \tag{2.5}$$

Again using (2.4), for odd n, we obtain

$$\int_0^{d(y_{n+1},y_n)}\varphi(t)dt=\int_0^{d(T_1x_{n+1},T_2x_n)}\varphi(t)dt\le a_1\int_0^{d(fx_{n+1},gx_n)}\varphi(t)dt+(\frac{a_2+a_3}{2})\int_0^{d(fx_{n+1},T_1x_{n+1})}\varphi(t)dt+$$

$$(\frac{a_2+a_3}{2})\int_0^{d(gx_n,T_2x_n)}\varphi(t)dt+(\frac{a_4+a_5}{2})\int_0^{d(fx_{n+1},T_2x_n)}\varphi(t)dt+(\frac{a_4+a_5}{2})\int_0^{d(gx_n,T_1x_{n+1})}\varphi(t)dt$$

From (2.2) we get

$$\int_0^{d(y_n,y_{n+1})}\varphi(t)dt\le a_1\int_0^{d(y_{n-1},y_n)}\varphi(t)dt+(\frac{a_2+a_3}{2})\int_0^{d(y_{n-1},y_n)}\varphi(t)dt+(\frac{a_2+a_3}{2})\int_0^{d(y_n,y_{n+1})}\varphi(t)dt$$

$$+(\frac{a_4+a_5}{2})\int_0^{d(y_{n-1},y_{n+1})}\varphi(t)dt+(\frac{a_4+a_5}{2})\int_0^{d(y_n,y_n)}\varphi(t)dt \qquad (2.6)$$

From (2.5) and (2.6) we observe that

$$\int_0^{d(y_n,y_{n+1})}\varphi(t)dt\le a_1\int_0^{d(y_{n-1},y_n)}\varphi(t)dt+(\frac{a_2+a_3}{2})\int_0^{d(y_{n-1},y_n)}\varphi(t)dt+(\frac{a_2+a_3}{2})\int_0^{d(y_n,y_{n+1})}\varphi(t)dt$$

$$+(\frac{a_4+a_5}{2})\int_0^{d(y_{n-1},y_{n+1})}\varphi(t)dt+(\frac{a_4+a_5}{2})\int_0^{d(y_n,y_n)}\varphi(t)dt$$

$$\le a_1\int_0^{d(y_{n-1},y_n)}\varphi(t)dt+(\frac{a_2+a_3}{2})\int_0^{d(y_{n-1},y_n)}\varphi(t)dt+(\frac{a_2+a_3}{2})\int_0^{d(y_n,y_{n+1})}\varphi(t)dt$$

$$+(\frac{a_4+a_5}{2})\int_0^{d(y_{n-1},y_n)}\varphi(t)dt+(\frac{a_4+a_5}{2})\int_0^{d(y_n,y_{n+1})}\varphi(t)dt.$$

It follows that

$$\int_0^{d(y_n,y_{n+1})}\varphi(t)dt\le(\frac{2a_1+a_2+a_3+a_4+a_5}{2-a_2-a_3-a_4-a_5})\int_0^{d(y_{n-1},y_n)}\varphi(t)dt$$

$$= r\int_0^{d(y_{n-1},y_n)}\varphi(t)dt \le .... \le r^n\int_0^{d(y_0,y_1)}\varphi(t)dt \to 0 \quad \text{as} \quad n\to\infty$$

Since $r<1$, owing to the assumption $\sum_{i=1}^{5}a_i<1$ .Therefore, $\lim_{n\to\infty} d(y_n,y_{n+1})=0$

(2.7)

Now we show that $\{y_n\}$ is a Cauchy sequence in X. Let $m>n$ where $m,n\in N$.

Without loss of generality , we consider two cases arise:

(I) m is even when n is odd

(II) m is odd when n is even

Case (I): We choose m and n to be odd and even respectively, by using (2.1) we have

$$\int_0^{d(y_n,y_m)}\varphi(t)dt = \int_0^{d(T_2x_n,T_1x_m)}\varphi(t)dt \le a_1\int_0^{d(gx_m,fx_n)}\varphi(t)dt + a_2\int_0^{d(gx_m,T_2x_m)}\varphi(t)dt$$

$$+\, a_3\int_0^{d(fx_n,T_1x_n)}\varphi(t)dt + a_4\int_o^{d(gx_m,T_1x_n)}\varphi(t)dt + a_5\int_0^{d(fx_n,T_2x_m)}\varphi(t)dt$$

By using (2.2) we have

$$\int_0^{d(y_n,y_m)}\varphi(t)dt \le a_1\int_0^{d(y_{m-1},y_{n-1})}\varphi(t)dt + a_2\int_0^{d(y_{m-1},y_m)}\varphi(t)dt + a_3\int_0^{d(y_{n-1},y_n)}\varphi(t)dt +$$

$$a_4\int_0^{d(y_{m-1},y_n)}\varphi(t)dt + a_5\int_0^{d(y_{n-1},y_m)}\varphi(t)dt .$$

Now from (2.7) and triangle inequality , one can find that

$$\int_0^{d(y_n,y_m)} \varphi(t)dt \le a_1 \int_0^{d(y_{m-1},y_m)} \varphi(t)dt + a_1 \int_0^{d(y_m,y_n)} \varphi(t)dt + a_1 \int_0^{d(y_{n-1},y_n)} \varphi(t)dt$$

$$a_2 \int_0^{d(y_{m-1},y_m)} \varphi(t)dt + a_3 \int_0^{d(y_{n-1},y_n)} \varphi(t)dt + a_4 \int_0^{d(y_{m-1},y_m)} \varphi(t)dt + a_4 \int_0^{d(y_m,y_n)} \varphi(t)dt$$

$$+ \ a_5 \int_0^{d(y_{n-1},y_n)} \varphi(t)dt + a_5 \int_0^{d(y_n,y_m)} \varphi(t)dt$$

$$\Rightarrow \int_0^{d(y_n,y_m)} \varphi(t)dt \le \left(\frac{a_1+a_2+a_4}{1-a_1-a_4-a_5}\right) \int_0^{d(y_{m-1},y_m)} \varphi(t)dt + \left(\frac{a_1+a_3+a_5}{1-a_1-a_4-a_5}\right) \int_0^{d(y_{n-1},y_n)} \varphi(t)dt$$

$$\le \left(\frac{a_1+a_2+a_4}{1-a_1-a_4-a_5}\right)^{m-1} \int_0^{d(y_0,y_1)} \varphi(t)dt + \left(\frac{a_1+a_3+a_5}{1-a_1-a_4-a_5}\right)^{n-1} \int_0^{d(y_0,y_1)} \varphi(t)dt$$

$$\le r^{m-1} \int_0^{d(y_0,y_1)} \varphi(t)dt + r^{n-1} \int_0^{d(y_0,y_1)} \varphi(t)dt \to 0 \quad \text{as n,m} \to \infty \ \text{ since } \ r<1.$$

Case(II): we choose m and n to be even and odd respectively.

From (2.1) and repeating the steps of case (I) also we have

$$\int_0^{d(y_n,y_m)} \varphi(t)dt \to 0 \quad \text{as } n,m \to \infty.$$

Then $\{y_n\}$ is a Cauchy sequence in the complete metric space X ,$\exists$ a point $z \in X$

Such that $\lim_{n\to\infty} y_n = z$

$$\Rightarrow \lim_{n\to\infty} T_2 x_n = \lim_{n\to\infty} g x_{n+1} = z \quad \text{and} \quad \lim_{n\to\infty} T_1 x_{n+1} = \lim_{n\to\infty} f x_{n+2} = z$$

i .e $\lim_{n\to\infty} T_2 x_n = \lim_{n\to\infty} g x_{n+1} = \lim_{n\to\infty} T_1 x_{n+1} = \lim_{n\to\infty} f x_{n+2} = z$

(2.8)

.Since $T_1(X) \subseteq f(X)$, $\exists$ a point $u \in X$ such that $z = fu$.

From (2.1) we get

$$\int_0^{d(z,T_2u)} \varphi(t)dt = \int_0^{d(Tx_{n+1},T_2u)} \varphi(t)dt \le a_1 \int_0^{d(fx_{n+1},gu)} \varphi(t)dt + a_2 \int_0^{d(fx_{n+1},T_1x_{n+1})} \varphi(t)dt$$

$$+ \; a_3 \int_0^{d(gu,T_2u)} \varphi(t)dt + a_4 \int_0^{d(fx_{n+1},T_2u)} \varphi(t)dt + a_5 \int_0^{d(gu,T_1x_{n+1})} \varphi(t)dt$$

By taking the limit as $n \to \infty$ and by (2.8) we have

$$\int_0^{d(z,T_2u)} \varphi(t)dt \le a_1 \int_0^{d(z,gu)} \varphi(t)dt + a_2 \int_0^{d(z,z)} \varphi(t)dt$$

$$+ \; a_3 \int_0^{d(gu,T_2u)} \varphi(t)dt + a_4 \int_0^{d(z,T_2u)} \varphi(t)dt + a_5 \int_0^{d(gu,z)} \varphi(t)dt$$

$$\le a_1 \int_0^{d(z,gu)} \varphi(t)dt$$

$$+ a_3 \int_0^{d(gu,z)} \varphi(t)dt + a_3 \int_0^{d(z,T_2u)} \varphi(t)dt + a_4 \int_0^{d(z,T_2u)} \varphi(t)dt + a_5 \int_0^{d(gu,z)} \varphi(t)dt$$

$$\Rightarrow \int_0^{d(z,T_2u)} \varphi(t)dt \le \left(\frac{a_1 + a_3 + a_5}{1 - a_3 - a_4}\right) \int_0^{d(z,gu)} \varphi(t)dt$$

$$\leq r \int_0^{d(z,gu)} \varphi(t)dt < \int_0^{d(z,gu)} \varphi(t)dt \text{ since } (r < 1). \tag{2.9}$$

If $z \neq T_2u$, so we have a contradiction in (2.9) $(\because T_2 \subseteq g)$.

Then $z = T_2u$, so $fu = T_2u = z$. Hence is coincidence point of f and T_2.

Since the pair of maps f and T_2 are weakly compatible,then

$$T_2 fu = fT_2u, \text{ i.e } T_2z = fz.$$

(2.10)

Again since $T_2(X) \subseteq g(X)$, there exists a point $\nu \in X$ such that $z = g\nu$.

Then by (2.1) and applied the same above steps, we can find that $T_1\nu = z$.

Therefore $T_1\nu = g\nu = z$, so ν is a coincidence point of T_1 and g.

Also the pair of maps T_1 and g are weakly compatible,

$$gT_1\nu = T_1g\nu \quad \text{i.e } gz=T_1z$$

(2.11)

Now we show that z is a fixed point of T_2, by using (2.1) we have

$$\int_0^{d(z,T_2z)} \varphi(t)dt = \int_0^{d(T_1x_{n+1},T_2z)} \varphi(t)dt \leq a_1 \int_0^{d(fx_{n+1},gz)} \varphi(t)dt + a_2 \int_0^{d(fx_{n+1},T_1x_{n+1})} \varphi(t)dt$$

$$+ a_3 \int_0^{d(gz,T_2z)} \varphi(t)dt + a_4 \int_0^{d(fx_{n+1},T_2z)} \varphi(t)dt + a_5 \int_0^{d(gz,T_1x_{n+1})} \varphi(t)dt$$

Taking the limit as $n \to \infty$ we get

$$\int_0^{d(z,T_2z)} \varphi(t)dt \leq a_1 \int_0^{d(z,gz)} \varphi(t)dt + a_2 \int_0^{d(z,z)} \varphi(t)dt + a_3 \int_0^{d(gz,T_2z)} \varphi(t)dt$$

$$+ a_4 \int_0^{d(z,T_2z)} \varphi(t)dt + a_5 \int_0^{d(gz,z)} \varphi(t)dt$$

$$\le a_1 \int_0^{d(z,gz)} \varphi(t)dt + a_3 \int_0^{d(z,gz)} \varphi(t)dt + a_3 \int_0^{d(z,T_2z)} \varphi(t)dt + a_4 \int_0^{d(z,T_2z)} \varphi(t)dt + a_5 \int_0^{d(z,gz)} \varphi(t)dt$$

$$= \left(\frac{a_1+a_3+a_5}{1-a_3-a_4}\right) \int_0^{d(z,gz)} \varphi(t)dt \le r \int_0^{d(z,gz)} \varphi(t)dt < \int_0^{d(z,gz)} \varphi(t)dt \quad (\text{ since } r<1)$$

If $z \ne T_2z$ we have a contradiction, hence $z = T_2z$

i.e from (2.10) we get $z = T_2z = fz$

(2.12)

Also by the same way we can show that z is a fixed point of T_1, hence $z = T_1z$

i.e from (2.11) we get $z = T_1z = gz$

(2.13)

From (2.12) and (2.13) we obtain that

$$z = T_2z = fz = T_1z = gz$$

Therefore z is a common fixed points of T_1, T_2, f and g.

For uniqueness of z let if possible that z and w are common fixed points of T_1, T_2, f and g

Such that ($w \ne z$), from (2.1) we have

$$\int_0^{d(z,w)} \varphi(t)dt = \int_0^{d(T_1z,T_2w)} \varphi(t)dt \le a_1 \int_0^{d(fz,gw)} \varphi(t)dt + a_2 \int_0^{d(fz,T_1z)} \varphi(t)dt$$

$$+ \ a_3 \int_0^{d(gw,T_2w)} \varphi(t)dt + a_4 \int_0^{d(fz,T_2w)} \varphi(t)dt + a_5 \int_0^{d(gw,T_1z)} \varphi(t)dt$$

$$\leq a_1 \int_0^{d(z,w)} \varphi(t)dt + a_2 \int_0^{d(z,z)} \varphi(t)dt + a_3 \int_0^{d(w,w)} \varphi(t)dt + a_4 \int_0^{d(z,w)} \varphi(t)dt + a_5 \int_0^{d(w,z)} \varphi(t)dt$$

$$\Rightarrow \int_0^{d(z,w)} \varphi(t)dt \leq (a_1 + a_4 + a_5) \int_0^{d(z,w)} \varphi(t)dt \leq r \int_0^{d(z,w)} \varphi(t)dt \quad (\text{ since } r < 1).$$

i.e z is a unique common fixed point of T_1, T_2, f and g. □

If we put $f = g$ in the above theorems we get the following corollary.

Corollary 2.2. Let (X,d) be a complete metric space suppose that the mappings T_1, T_2 and f are self maps satisfying the following conditions:

1- $T_1(X) \subseteq f(X)$ and $T_2(X) \subseteq f(X)$

2-the pair (T_2, f) and (T_1, g) are weakly compatible,

3- $$\int_0^{d(T_1x,T_2y)} \varphi(t)dt \leq a_1 \int_0^{d(fx,fy)} \varphi(t)dt + a_2 \int_0^{d(fx,T_1x)} \varphi(t)dt$$

$$+ \ a_3 \int_0^{d(fy,T_2y)} \varphi(t)dt + a_4 \int_0^{d(fx,T_2y)} \varphi(t)dt + a_5 \int_0^{d(fy,T_1x)} \varphi(t)dt$$

where $\varphi : R^+ \to R^+$ is a Lebesgue-integrable map which is summable, non-negative and such that $\int_0^{\varepsilon} \varphi(t)dt > 0$ for each $\varepsilon > 0$. Then T_1, T_2 and f have a unique common fixed point $z \in X$.

REMARK

(i) Theorem 2.1 (cf.[1]) is a special case of Theorem 2.1 by taking $f = g = I$ (I is the identity mapping). (ii)By taking $\varphi(t) = 1$ in Theorem 2.1, we obtain the contractive condition of the Theorems 2.1 not involving the integral.

(iii) Theorems 1.1 is a consequence of Theorem 2.1 If we take $f = g = I$, $T_1 = T_2 = T$,

$a_2 = a_3 = a_4 = a_5 = 0$ and $a_1 = c,\ 0 \leq c < 1$.

REFERENCES

1- M. R. Singh, L. Sharmeswar Singh, “Fixed point theorem for a pair of self maps satisfying a general contractive condition of integral type”, Kathmandu University Journal of Science Engineering and Technology, Vol. 6, pp. 20-27, 2010.

2- Vishal Gupta, Anil saini, Ravinder Kumar "common fixed points for weakly compatible Maps in 2-metric space"International Journal of Mathematical Archive-3(10), 2012,3670-3675.

3- S. Banach, 1922. *Sur les operations dans les ensembles abstraits et leur application aux equations integrales*, Fund. Math. 3: 133 – 181 (in French)

4- O. A. Boikanyo, 2007. *Some fixed point theorems for mappings satisfying a general contractive condition of integral types*, Far East J. Math. Sci., (FJMS) 26 (1)219 – 230.

5- A. Branciari, 2002. *A fixed point theorem for mappings satisfying a general contractive condition of integral type*, Int. J. Math. Sci. 29: 531 – 536.

6- R. Caccioppoli, 1930. *Un teorema generale sull' esistenza di elementi uniti in una transformazione funzionale*, Rend. Acad. Dei Lincei 11: 794 – 799 (in Italian).

7- S.K. Chatterjea, 1972. *Fixed – point theorems*, C.R. Acad. Balgare Sci. 25: 727 – 730.

8. G. E. Hardy and T. D. Rogers, 1973. *A generalization of a fixed point theorem of Reich*, Bull.Cand. Math. Soc. 16:201 –206.

9- R. Kannan, *Some results on fixed points*, Bull. Calcutta Math. Soc. 60 (1968), 71 – 76.

10- B. E. Rhoades, *A comparison of various definitions of contractive mappings*, Trans. Amer. Math. Soc. 226 (1977), 257 – 290.

11- B. E. Rhoades, *Two fixed point theorems for mappings satisfying a general contractive condition of integral type*, Int. J. Math. Math. Sci. 63 (2003), 4007 – 4013.

12- M. Sen Gupta (Mrs. Das Gupta), *On common fixed points of operators*, Bull. Cal. Math. Soc. 66 (1974) 149 –153.

13- D. R. Smart, *Fixed point theorems*, Cambridge University Press, London, 1974.

14- P. Vijayaraju, B.E. Rhoades and R. Mohanraj, *A fixed point theorem for a pair of maps satisfying a general contractive condition of integral type*, Int. J. Math. Math. Sci. 15 (2005), 2359 – 2364.