

---

**WEAKLY COMPATIBLE MAPS FOR THREE SELF MAPS IN  
COMPLEX VALUED METRIC SPACES**

Manoj Kumar\*

Pankaj Kumar\*\*

Sanjay Kumar\*\*\*

---

**ABSTRACT**

*In this paper, we prove some common fixed point theorems for three self maps in complex valued metric space. Also, we give suitable example in support of our results.*

**Mathematics Subject Classification:** (2001) AMS 47H10, 54H25.

**Keywords:** Complex valued metric space, partial order, weakly compatible maps.

---

\*Department of Mathematics, Delhi Institute of Technology and Management, Gannaur (Sonipat), India

\*\*Department of Mathematics, Guru Jambheshwar University of Science and Technology, Hisar, India

\*\*\*Department of Mathematics, Deenbandhu Chhotu Ram University of Science and Technology, Murthal, India

## 1. INTRODUCTION

In 2011, Azam et. al. [1] introduced the notion of complex valued metric space which is a generalization of the classical metric space. They established some fixed point results for mappings satisfying a rational inequality. The idea of complex valued metric spaces can be exploited to define complex valued normed spaces and complex valued Hilbert spaces; additionally, it offers numerous research activities in mathematical analysis.

A complex number  $z \in \mathbb{C}$  is an ordered pair of real numbers, whose first co-ordinate is called  $\text{Re}(z)$  and second coordinate is called  $\text{Im}(z)$ . Thus a complex-valued metric  $d$  is a function from a

set  $X \times X$  into  $\mathbb{C}$ , where  $X$  is a nonempty set and  $\mathbb{C}$  is the set of complex numbers.

Let  $\mathbb{C}$  be the set of complex numbers and  $z_1, z_2 \in \mathbb{C}$ . Define a partial order  $\lesssim$  on  $\mathbb{C}$  as follows:  $z_1 \lesssim z_2$  if and only if  $\text{Re}(z_1) \leq \text{Re}(z_2)$  and  $\text{Im}(z_1) \leq \text{Im}(z_2)$ , that is  $z_1 \lesssim z_2$ , if one of the following holds:

(C1)  $\text{Re}(z_1) = \text{Re}(z_2)$  and  $\text{Im}(z_1) = \text{Im}(z_2)$ ;

(C2)  $\text{Re}(z_1) < \text{Re}(z_2)$  and  $\text{Im}(z_1) = \text{Im}(z_2)$ ;

(C3)  $\text{Re}(z_1) = \text{Re}(z_2)$  and  $\text{Im}(z_1) < \text{Im}(z_2)$ ;

(C4)  $\text{Re}(z_1) < \text{Re}(z_2)$  and  $\text{Im}(z_1) < \text{Im}(z_2)$ .

In particular, we will write  $z_1 \approx z_2$  if  $z_1 \neq z_2$  and one of (C2), (C3), and (C4) is satisfied and we will write  $z_1 < z_2$  if only (C4) is satisfied.

**Remark 1.1.** We note that the following statements hold:

(i)  $a, b \in \mathbb{R}$  and  $a \leq b \Rightarrow az \lesssim bz \forall z \in \mathbb{C}$ .

(ii)  $0 \lesssim z_1 \approx z_2 \Rightarrow |z_1| < |z_2|$ ,

(iii)  $z_1 \lesssim z_2$  and  $z_2 < z_3 \Rightarrow z_1 < z_3$ .

**Definition 1.2.** Let  $X$  be a nonempty set. Suppose that the mapping  $d : X \times X \rightarrow \mathbb{C}$  satisfies the following conditions:

(i)  $0 \lesssim d(x, y)$ , for all  $x, y \in X$  and  $d(x, y) = 0$  if and only if  $x = y$ ;

(ii)  $d(x, y) = d(y, x)$  for all  $x, y \in X$ ;

(iii)  $d(x, y) \lesssim d(x, z) + d(z, y)$ , for all  $x, y, z \in X$ .

Then  $d$  is called a complex valued metric on  $X$  and  $(X, d)$  is called a complex valued metric space.

**Example 1.3.** Let  $X = \mathbb{C}$ . Define the mapping  $d : X \times X \rightarrow \mathbb{C}$  by

$d(z_1, z_2) = 2i |z_1 - z_2|$ , for all  $z_1, z_2 \in X$ .

Then  $(X, d)$  is a complex valued metric space.

**Definition 1.4.** Let  $(X, d)$  be a complex valued metric space,  $\{x_n\}$  be a sequence in  $X$  and  $x \in X$ .

(i) If for every  $c \in \mathbb{C}$ , with  $0 < c$  there is  $k \in \mathbb{N}$  such that for all  $n > k$ ,  $d(x_n, x) < c$ , then  $\{x_n\}$  is said to be convergent,  $\{x_n\}$  converges to  $x$  and  $x$  is the limit point of  $\{x_n\}$ . We denote this by  $\{x_n\} \rightarrow x$  as  $n \rightarrow \infty$  or  $\lim_{n \rightarrow \infty} x_n = x$ .

(ii) If for every  $c \in \mathbb{C}$ , with  $0 < c$  there is  $k \in \mathbb{N}$  such that for all  $n > k$ ,  $d(x_n, x_{n+m}) < c$ , where  $m \in \mathbb{N}$ , then  $\{x_n\}$  is said to be Cauchy sequence.

(iii) If every Cauchy sequence in  $X$  is convergent, then  $(X, d)$  is said to be a complete complex valued metric space.

**Lemma 1.5.** Let  $(X, d)$  be a complex valued metric space and let  $\{x_n\}$  be a sequence in  $X$ . Then  $\{x_n\}$  converges to  $x$  if and only if  $|d(x_n, x)| \rightarrow 0$  as  $n \rightarrow \infty$ .

**Lemma 1.6.** Let  $(X, d)$  be a complex valued metric space and let  $\{x_n\}$  be a sequence in  $X$ . Then  $\{x_n\}$  is a Cauchy sequence if and only if  $|d(x_n, x_{n+m})| \rightarrow 0$  as  $n \rightarrow \infty$ , where  $m \in \mathbb{N}$ . In 1996, Jungck [2] introduced the notion of weakly compatible maps as follows:

**Definition 1.7.** Two self maps  $f$  and  $g$  are said to be weakly compatible if they commute at coincidence points.

## 2. MAIN RESULTS

**Theorem 2.1.** Let  $S, T$  and  $f$  be three self maps of a complex valued metric space  $(X, d)$  satisfying the following:

$$(2.1) \quad SX \cup TX \subseteq fX,$$

$$(2.2) \quad d(Sx, Ty) \lesssim h d(fx, fy), \text{ for all } x, y \text{ in } X,$$

where  $0 \leq h < 1$ ,

$$(2.3) \quad fX \text{ is complete subspace of } X.$$

Then  $S, T$  and  $f$  have a unique coincidence point. Moreover, if  $(S, f)$  and  $(T, f)$  are weakly compatible, then  $S, T$  and  $f$  have a unique common fixed point.

**Proof.** Let  $x_0 \in X$ . Choose a point  $x_1$  in  $X$  such that  $fx_1 = Sx_0$ . This can be done, since  $SX \subseteq fX$ . Similarly, choose a point  $x_2$  in  $X$  such that  $fx_2 = Tx_1$ . Continuing this process and having chosen  $x_n$  in  $X$ , we obtain  $x_{n+1}$  in  $X$  such that

$$fx_{2k+1} = Sx_{2k}, \quad fx_{2k+2} = Tx_{2k+1}, \quad k = 0, 1, 2, \dots$$

From (2.2), we have

$$\begin{aligned} d(fx_{2k+1}, fx_{2k+2}) &= d(Sx_{2k}, Tx_{2k+1}) \\ &\lesssim h d(fx_{2k}, fx_{2k+1}). \end{aligned}$$

Similarly,

$$d(fx_{2k+2}, fx_{2k+3}) \lesssim h d(fx_{2k+1}, fx_{2k+2}).$$

Now, by induction, we obtain for each  $k = 0, 1, 2, \dots$ ,

$$d(fx_{2k+2}, fx_{2k+3}) \lesssim h d(fx_0, fx_1).$$

Let  $y_n = fx_n$ ,  $n = 0, 1, 2, \dots$

Now, for all  $n$ , we have

$$\begin{aligned} d(y_{n+1}, y_{n+2}) &\lesssim h d(y_n, y_{n+1}) \\ &\lesssim h^2 d(y_{n-1}, y_n) \lesssim \dots \lesssim h^{n+1} d(y_0, y_1). \end{aligned}$$

Now, for any  $m > n$ ,

$$\begin{aligned} d(y_m, y_n) &\lesssim d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \dots + d(y_{m-1}, y_m) \\ &\lesssim [h^n + h^{n+1} + \dots + h^{m-1}] d(y_0, y_1) \\ &\lesssim \frac{h^n}{1-h} d(y_0, y_1). \end{aligned}$$

Therefore, we have

$$|d(y_m, y_n)| \leq \frac{h^n}{1-h} |d(y_0, y_1)|.$$

Hence,

$$\lim_{n \rightarrow \infty} |d(y_m, y_n)| = 0,$$

which implies that  $\{y_n\}$  is a Cauchy sequence. Since  $fX$  is complete, there exists  $u, v$  in  $X$  such that  $y_n \rightarrow v = fu$ .

Choose a natural number  $N$  such that

$$d(y_n, v) < \frac{c}{2}, \text{ for all } n \geq N.$$

Hence, for all  $n \geq N$ , using triangle inequality and (2.2), we have

$$\begin{aligned} d(fu, Su) &\lesssim d(fu, y_{2n+2}) + d(y_{2n+2}, Su) \\ &= d(v, y_{2n+2}) + d(Tx_{2n+1}, Su) \\ &\lesssim d(v, y_{2n+2}) + h d(fx_{2n+1}, fu) \\ &\lesssim d(v, y_{2n+2}) + h d(y_{2n+1}, v) < \frac{c}{2} + \frac{c}{2} = c. \end{aligned}$$

Thus,  $d(fu, Su) \lesssim \frac{c}{m}$  for all  $m \geq 1$ , that is,

$$|d(fu, Su)| \leq \frac{c}{m}.$$

But, since  $m$  was arbitrary, so

$$|d(fu, Su)| = 0, \text{ implies that, } fu = Su.$$

Similarly, by using

$$d(fu, Tu) \lesssim d(fu, y_{2n+1}) + d(y_{2n+1}, Tu),$$

one can show that  $fu = Tu$ , it implies that,  $v$  is a common point of coincidence of  $S$ ,  $T$  and  $f$ , that is,  $v = fu = Su = Tu$ .

Now, we show that  $f$ ,  $S$  and  $T$  have a unique point of coincidence. For this, assume that there exists another point  $w$  in  $X$  such that  $w = fz = Sz = Tz$  for some  $z$  in  $X$ .

From (2.2), we have

$$\begin{aligned} d(v, w) &= d(Su, Tz) \\ &\lesssim h d(fu, fz) = h d(v, w), \text{ implies that, } v = w. \end{aligned}$$

Now, since  $(S, f)$  and  $(T, f)$  are weakly compatible, we have

$$Sv = Sfu = fSu = fv \text{ and } Tv = Tfu = fTu = fv.$$

It implies that  $Sv = Tv = fv = t$  (say). Then  $w$  is a point of coincidence of  $S$ ,  $T$  and  $f$ .

Therefore,  $v = t$ , by uniqueness.

Hence  $v$  is a common fixed point of  $S$ ,  $T$  and  $f$ .

**Theorem 2.2.** Let  $S$ ,  $T$  and  $f$  be three self maps of a complex valued metric space  $(X, d)$  satisfying (2.1), (2.3) and the following:

$$(2.4) \quad d(Sx, Ty) \lesssim h [d(fx, Sx) + d(fy, Ty)], \text{ for all } x, y \text{ in } X,$$

$$\text{where } 0 \leq h < \frac{1}{2}.$$

Then  $S$ ,  $T$  and  $f$  have a unique coincidence point. Moreover, if  $(S, f)$  and  $(T, f)$  are weakly compatible, then  $S$ ,  $T$  and  $f$  have a unique common fixed point.

**Proof.** Let  $x_0 \in X$ . Define a sequence of points in  $X$ , as in Theorem 2.1, given by the rule:

$$fx_{2k+1} = Sx_{2k}, \quad fx_{2k+2} = Tx_{2k+1}, \quad k = 0, 1, 2, \dots$$

From (2.4), we have

$$\begin{aligned} d(fx_{2k+1}, fx_{2k+2}) &= d(Sx_{2k}, Tx_{2k+1}) \\ &\lesssim h [d(fx_{2k}, Sx_{2k}) + d(fx_{2k+1}, Tx_{2k+1})] \\ &= h [d(fx_{2k}, fx_{2k+1}) + d(fx_{2k+1}, fx_{2k+2})], \text{ that is,} \end{aligned}$$

$$d(fx_{2k+1}, fx_{2k+2}) \lesssim \frac{h}{1-h} d(fx_{2k}, fx_{2k+1}).$$

Similarly, it can be shown that

$$\begin{aligned} d(fx_{2k+2}, fx_{2k+3}) &\lesssim \frac{h}{1-h} d(fx_{2k+1}, fx_{2k+2}) \\ &= p d(fx_{2k+1}, fx_{2k+2}), \quad p = \frac{h}{1-h} < 1. \end{aligned}$$

Now, by induction, we obtain for each  $k = 0, 1, 2, \dots$ ,

$$d(fx_{2k+1}, fx_{2k+2}) \lesssim p d(fx_{2k}, fx_{2k+1})$$

$$\lesssim p^2 d(fx_{2k-1}, fx_{2k}) \lesssim \dots \lesssim p^{2k+1} d(fx_0, fx_1).$$

Let  $y_n = fx_n$ ,  $n = 0, 1, 2, \dots$

Now, for all  $n$ , we have

$$\begin{aligned} d(y_{n+1}, y_{n+2}) &\lesssim p d(y_n, y_{n+1}) \\ &\lesssim p^2 d(y_{n-1}, y_n) \lesssim \dots \lesssim p^{n+1} d(y_0, y_1). \end{aligned}$$

Now, for any  $m > n$ ,

$$\begin{aligned} d(y_m, y_n) &\lesssim d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \dots + d(y_{m-1}, y_m) \\ &\lesssim [p^n + p^{n+1} + \dots + p^{m-1}] d(y_0, y_1) \\ &\lesssim \frac{p^n}{1-p} d(y_0, y_1). \end{aligned}$$

Therefore, we have

$$|d(y_m, y_n)| \leq \frac{p^n}{1-p} |d(y_0, y_1)|.$$

Hence,

$$\lim_{n \rightarrow \infty} |d(y_m, y_n)| = 0,$$

which implies that  $\{y_n\}$  is a Cauchy sequence.

Since  $fX$  is complete, there exists  $u, v$  in  $X$  such that  $y_n \rightarrow v = fu$ .

Choose a natural number  $N$  such that

$$d(y_{n+1}, y_n) < \frac{c(1-h)}{2h} \text{ and } d(y_{n+1}, v) < \frac{c(1-h)}{2}, \text{ for all } n \geq N.$$

Hence, for all  $n \geq N$ , using triangle inequality and (2.4), we have

$$\begin{aligned} d(fu, Su) &\lesssim d(fu, y_{2n+2}) + d(y_{2n+2}, Su) \\ &= d(v, y_{2n+2}) + d(Tx_{2n+1}, Su) \\ &\lesssim d(v, y_{2n+2}) + h [d(fu, Su) + d(fx_{2n+1}, Tx_{2n+1})], \text{ that is,} \end{aligned}$$

$$\begin{aligned} d(fu, Su) &\lesssim \frac{1}{1-h} d(v, y_{2n+2}) + \frac{h}{1-h} d(y_{2n+1}, y_{2n+2}) \\ &< \frac{c}{2} + \frac{c}{2} = c. \end{aligned}$$

Thus,  $d(fu, Su) \lesssim \frac{c}{m}$  for all  $m \geq 1$ , that is,

$$|d(fu, Su)| \leq \frac{c}{m}.$$

But, since  $m$  was arbitrary, so

$$|d(fu, Su)| = 0, \text{ implies that, } fu = Su.$$

Similarly, by using

$$d(fu, Tu) \lesssim d(fu, y_{2n+1}) + d(y_{2n+1}, Tu),$$

one can show that  $fu = Tu$ , it implies that,  $v$  is a common point of coincidence of  $S, T$  and  $f$ , that is,  $v = fu = Su = Tu$ .

Now, we show that  $f, S$  and  $T$  have a unique point of coincidence. For this, assume that there exists another point  $w$  in  $X$  such that  $w = fz = Sz = Tz$  for some  $z$  in  $X$ .

From (2.4), we have

$$\begin{aligned} d(v, w) &= d(Su, Tz) \\ &\lesssim h [d(fu, Su) + d(fz, Tz)] \\ &= h [d(v, v) + d(w, w)] = 0, \text{ that is,} \end{aligned}$$

$$|d(v, w)| \leq 0, \text{ implies that, } v = w.$$

Now, since  $(S, f)$  and  $(T, f)$  are weakly compatible, we have

$$Sv = Sfu = fSu = fv \text{ and } Tv = Tfu = fTu = fv.$$

It implies that  $Sv = Tv = fv = t$  (say). Then  $w$  is a point of coincidence of  $S, T$  and  $f$ .

Therefore,  $v = t$ , by uniqueness.

Hence  $v$  is a common fixed point of  $S, T$  and  $f$ .

**Theorem 2.3.** Let  $S, T$  and  $f$  be three self maps of a complex valued metric space  $(X, d)$  satisfying (2.1), (2.3) and the following:

$$(2.5) \quad d(Sx, Ty) \lesssim h [d(fy, Sx) + d(fx, Ty)], \text{ for all } x, y \text{ in } X,$$

$$\text{where } 0 \leq h < \frac{1}{2}.$$

Then  $S, T$  and  $f$  have a unique coincidence point. Moreover, if  $(S, f)$  and  $(T, f)$  are weakly compatible, then  $S, T$  and  $f$  have a unique common fixed point.

**Proof.** Let  $x_0 \in X$ . Define a sequence of points in  $X$ , as in Theorem 2.1, given by the rule:

$$fx_{2k+1} = Sx_{2k}, fx_{2k+2} = Tx_{2k+1}, k = 0, 1, 2, \dots$$

From (2.5), we have

$$\begin{aligned} d(fx_{2k+1}, fx_{2k+2}) &= d(Sx_{2k}, Tx_{2k+1}) \\ &\lesssim h [d(fx_{2k+1}, Sx_{2k}) + d(fx_{2k}, Tx_{2k+1})] \\ &= h [d(fx_{2k+1}, fx_{2k+1}) + d(fx_{2k}, fx_{2k+2})] \\ &= h d(fx_{2k}, fx_{2k+2}) \\ &\lesssim h [d(fx_{2k}, fx_{2k+1}) + d(fx_{2k+1}, Tx_{2k+2})], \text{ that is,} \end{aligned}$$

$$d(fx_{2k+1}, fx_{2k+2}) \lesssim \frac{h}{1-h} d(fx_{2k}, fx_{2k+1}).$$

Similarly, it can be shown that

$$\begin{aligned} d(fx_{2k+2}, fx_{2k+3}) &\lesssim \frac{h}{1-h} d(fx_{2k+1}, fx_{2k+2}) \\ &= p d(fx_{2k+1}, fx_{2k+2}), \quad p = \frac{h}{1-h} < 1. \end{aligned}$$

Now, by induction, we obtain for each  $k = 0, 1, 2, \dots$ ,

$$\begin{aligned} d(fx_{2k+1}, fx_{2k+2}) &\lesssim p d(fx_{2k}, fx_{2k+1}) \\ &\lesssim p^2 d(fx_{2k-1}, fx_{2k}) \lesssim \dots \lesssim p^{2k+1} d(fx_0, fx_1). \end{aligned}$$

Let  $y_n = fx_n, n = 0, 1, 2, \dots$

Now, for all  $n$ , we have

$$\begin{aligned} d(y_{n+1}, y_{n+2}) &\lesssim p d(y_n, y_{n+1}) \\ &\lesssim p^2 d(y_{n-1}, y_n) \lesssim \dots \lesssim p^{n+1} d(y_0, y_1). \end{aligned}$$

Now, for any  $m > n$ ,

$$\begin{aligned} d(y_m, y_n) &\lesssim d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \dots + d(y_{m-1}, y_m) \\ &\lesssim [p^n + p^{n+1} + \dots + p^{m-1}] d(y_0, y_1) \\ &\lesssim \frac{p^n}{1-p} d(y_0, y_1). \end{aligned}$$

Therefore, we have

$$|d(y_m, y_n)| \leq \frac{p^n}{1-p} |d(y_0, y_1)|.$$

Hence,

$$\lim_{n \rightarrow \infty} |d(y_m, y_n)| = 0,$$

which implies that  $\{y_n\}$  is a Cauchy sequence.

Since  $fX$  is complete, there exists  $u, v$  in  $X$  such that  $y_n \rightarrow v = fu$ .

Choose a natural number  $N$  such that

$$d(y_{n+1}, y_n) < \frac{c(1-h)}{3}, \text{ for all } n \geq N.$$

Hence, for all  $n \geq N$ , using triangle inequality and (2.5), we have

$$\begin{aligned} d(fu, Su) &\lesssim d(fu, y_{2n+1}) + d(y_{2n+1}, Su) \\ &= d(v, y_{2n+1}) + d(Tx_{2n+1}, Su) \\ &\lesssim d(v, y_{2n+1}) + h [d(fu, Tx_{2n+1}) + d(fx_{2n+1}, Su)] \\ &= d(v, y_{2n+1}) + h [d(v, y_{2n+2}) + d(y_{2n+1}, Su)], \text{ that is,} \\ d(fu, Su) &\lesssim d(v, y_{2n+1}) + h [d(v, y_{2n+2}) + d(y_{2n+1}, v) + d(v, Su)], \text{ implies that,} \end{aligned}$$

$$d(fu, Su) \lesssim \frac{1}{1-h} d(v, y_{2n+1}) + \frac{h}{1-h} [d(v, y_{2n+2}) + d(y_{2n+1}, v)]$$

$$\prec \frac{c}{3} + \frac{c}{3} + \frac{c}{3} = c.$$

Thus, we have

$$d(fu, Su) \lesssim \frac{c}{m}, \text{ for all } m \geq 1, \text{ that is,}$$

$$|d(fu, Su)| \leq \frac{c}{m}.$$

But, since  $m$  was arbitrary, so

$$|d(fu, Su)| = 0, \text{ implies that, } fu = Su.$$

Similarly, by using

$$d(fu, Tu) \lesssim d(fu, y_{2n+1}) + d(y_{2n+1}, Tu),$$

one can show that  $fu = Tu$ , it implies that,  $v$  is a common point of coincidence of  $S$ ,  $T$  and  $f$ , that is,  $v = fu = Su = Tu$ .

Now, we show that  $f$ ,  $S$  and  $T$  have a unique point of coincidence. For this, assume that there exists another point  $w$  in  $X$  such that  $w = fw = Sw = Tw$  for some  $w$  in  $X$ .

From (2.5), we have

$$d(v, w) = d(Su, Tw)$$

$$\lesssim h [d(fz, Su) + d(fu, Tw)]$$

$$= h [d(w, v) + d(v, w)] = 2h d(v, w), \text{ that is,}$$

$$|d(v, w)| \leq 2h |d(v, w)|, \text{ implies that, } v = w.$$

Now, since  $(S, f)$  and  $(T, f)$  are weakly compatible, we have

$$Sv = Sfu = fSu = fv \text{ and } Tv = Tfu = fTu = fv.$$

It implies that  $Sv = Tv = fv = t$  (say). Then  $w$  is a point of coincidence of  $S$ ,  $T$  and  $f$ .

Therefore,  $v = t$ , by uniqueness.

Hence  $v$  is a common fixed point of  $S$ ,  $T$  and  $f$ .

**Example 2.4.** Let  $X = [0, 1]$  and let  $d : X \times X \rightarrow \mathbb{C}$  by  $d(x, y) = i|x - y|$ , for all  $x, y \in X$ .

Then  $(X, d)$  is a complex valued metric space.

Define the functions  $S, T, f : X \rightarrow X$  by

$$Sx = \frac{x}{4} = Tx \text{ and } fx = \frac{x}{2}.$$

$$\text{Clearly } SX \cup TX = [0, \frac{1}{4}] \subseteq [0, \frac{1}{2}] = fX.$$

Also  $(S, f)$  and  $(T, f)$  are weakly compatible.

Now,

$$d(Sx, Ty) = \frac{i}{4}|x - y|, \quad d(fx, fy) = \frac{i}{2}|x - y|.$$

$$\text{Clearly, for } h = \frac{2}{3} < 1,$$

$$d(Sx, Ty) \lesssim h d(fx, fy).$$

Also  $0$  is the common fixed point of  $S$ ,  $T$  and  $f$ .

Hence all the conditions of Theorem 2.1 are satisfied.

**REFERENCES**

1. Azam A., Fisher B., Khan M., Common fixed point theorems in complex valued metric spaces, Numer. Funct. Anal. Optim. 3 (3) (2011) 243–253.
2. Banach, S: Sur les opérations dans les ensembles abstraits et leurs applications aux équations intégrales. Fund Math. 3, (1922), 133–181.
3. Jungck G., Common fixed points for non-continuous non-self mappings on non-metric spaces, Far East J. Math. Sci. 4(2), (1996), 199-212.