Bulk viscous inflationary universe for barotropic fluid distribution in Bianchi Type III space time

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Abstract :

Inflationary cosmological models for barotropic fluid distribution with bulk viscosity in Bianchi Type III space time is investigated. To get the deterministic model of universe, we have also assumed the conditions $\theta = 3H$, $\rho = 3H^2$ and $\zeta = \rho^{1/2}$ as considered by Barrow[20] and Gron[24]. We find that the models represent anisotropic space-time in general but isotropizes at late time. The models also represent decelerating and accelerating phases of universe representing present day observations of universe. The presence of bulk viscosity prevents the matter density to vanish and the models have Point Type singularity at T = 0 and $\tau = 0$ respectively. We also find that one of the model (28) expands exponentially which has the similar characteristic due to repulsive gravitation of a dominating vacuum energy density.

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1 Introduction

The distribution of matter in our universe is satisfactorily described by perfect fluid due to large scale distribution of galaxies in our universe. Various cosmological models have perfect fluid behaviour because it is most simple to deal with this type of behaviour and these models are in good agreement with cosmological observations as pointed out by Riess et al.[1] and Spergel et al.[2]. On a more physical and realistic basis, we can replace energy-momentum tensor for perfect fluid by introducing cosmic viscosity. This modifies the equation of state of cosmic field. In hydrodynamics, viscosity is defined as a measure of the resistance to flow of a fluid and is related to viscosity gradient. There are two types of viscosity, namely, shear and bulk viscosity. The shear viscosity characterizes a change in shape of a fixed volume of the fluid while the bulk viscosity characterizes a change in volume of fluid of a fixed shape. Shear viscosity deals with anisotropy of space-time while isotropic cosmological models can have bulk viscosity.

It has been argued for a long time that the dissipative process in early stages of cosmic expansion may will account for the high degree of isotropy, we observe today. The first theory of relativistic viscous fluid was presented by Eckart[3]. But Eckart's theory deals with first order deviation from equilibrium while second order terms are necessary to prevent non-casual behaviour. Gron[4] and Maartens[5] have presented exhaustive reviews of research on cosmological models with non-causal and causal theories of viscous fluids respectively. Padmanabhan and Chitre[6] pointed out that the presence of bulk viscosity leads to inflationary like situations in general relativity. The effect of bulk viscosity on cosmological evolution has been investigated by many authors viz. Johri and Sudarshan[7], Romano and Pavon[8], Saha[9], Singh et al.[10], Sahni and Starobinsky[11], Bali et al.[12-14], Bevik et al.[15].

Inflation, the stage of exponential expansion of the universe was proposed by Guth[16] in the context of grand unified theory (GUT) which has been accepted as model of the early universe. There are two basic types of inflationary cosmological models: (i) the first one is due to appearance of flat potential e.g. GUT's phase transitions, (ii) the second one is due to scalar curvature-squared term. Barrow and Turner[17] pointed out that large anisotropy prevents transition into an inflationary era as per Guth's original inflationary scenario. Inflationary scenario for homogeneous and isotropic models has been studied by many authors viz. Linde[18], Wald[19], Barrow[20], La and Steinhardt[21]. In these models, the universe undergoes a phase transition characterized by the evolution of Higgs field (ϕ). The inflation will take place if potential $V(\phi)$ has flat region and in this region, the Higgs field (ϕ) evolves slowly but the universe expands in an exponential way due to vacuum field energy (Stein-Schabes[22]). Rothman and Ellis[23] have pointed out that we can have the solution of isotropic problem if we work with anisotropic metric that isotropizes in special case. Keeping in view of this investigation, many authors viz. Gron[24], (BT-I), Chakraborty[25] (BT-IX), Banerjee et al.[26] (BT-I), Bali et al.[27-29], (BT I,II) investigated inflationary cosmological models in anisotropic space-times under different conditions.

2 Metric and Field Equations

We consider Bianchi Type III line-element in orthogonal form as

$$ds^2 = -dt^2 + A^2 dx^2 + e^{-2\alpha x} B^2 dy^2 + C^2 dz^2 \tag{1}$$

where A, B, C are functions of t-alone.

We assume the co-ordinates to be comoving so that

$$v^1 = 0 = v^2 = v^3, v^4 = 1$$

In case of gravity minimally coupled to a scalar field V(ϕ) as given by Stein-Schabes[22], we have

$$S = \int \sqrt{-g}\left\{R - \frac{1}{2}g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\right\}d^4x \tag{2}$$

The Einstein's field equations (in geometrized units 8πG= c = 1) in case of massless scalar field ϕ with potential V(ϕ) and bulk viscosity are given by

$$R_{ij} - \frac{1}{2}Rg_i^j = -T_i^j \tag{3}$$

with

$$T_i^j = (\rho + p)v_i v^j + pg_i^j + \partial_i\phi\partial^j\phi - \left[\frac{1}{2}\partial_\rho\phi\partial^\rho\phi + V(\phi)\right]g_i^j - \zeta\theta(g_i^j + v_i v^j) \tag{4}$$

where ρ is the energy density, p the isotropic pressure, ϕ the Higgs field, V the potential, ζ the coefficient of bulk viscosity.

The conservation relation leads to

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\partial^\mu\phi) = -\frac{dV}{d\phi} \tag{5}$$

We assume the coordinates to be comoving so that $v^1 = 0 = v^2 = v^3$, $v^4 = 1$, $v^4 = -1$.

The field equations (3) with (4) for line-element (1) leads to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{B_4C_4}{BC} = p - \frac{1}{2}\dot{\phi}^2 + \zeta\theta + V(\phi) \tag{6}$$

$$\frac{A_{44}}{A} + \frac{C_{44}}{C} + \frac{A_4C_4}{AC} = p - \frac{1}{2}\dot{\phi}^2 + \zeta\theta + V(\phi) \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} - \frac{\alpha^2}{A^2} = p - \frac{1}{2}\dot{\phi}^2 + \zeta\theta + V(\phi) \tag{8}$$

$$\frac{A_4B_4}{AB} + \frac{A_4C_4}{AC} + \frac{B_4C_4}{BC} - \frac{\alpha^2}{A^2} = \rho + \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{9}$$

$$\frac{B_4}{B} - \frac{A_4}{A} = 0 \tag{10}$$

which leads to

$$A = mB \tag{11}$$

where m is a constant.

3 Solution of Field Equations

The equation (5) for scalar field (ϕ) leads to

$$\phi_{44} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\phi_4 = \frac{dV}{d\phi} \tag{12}$$

We are interested in inflationary solution so flat region is considered. Thus we have V(ϕ) is constant = k.

Now equation (12) leads to

$$\phi_{44} + \left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\phi_4 = 0 \tag{13}$$

where suffix '4' indicates ordinary partial derivative with respect to t.

From equation (13), we have

$$\phi_4 = \frac{\ell}{mB^2C} \tag{14}$$

where ℓ is constant of integration.

The average scale factor (R) for line-element (1) is given by

$$R^3 = ABC = m\,B^2C \tag{15}$$

Equation (6) and (9) lead to

$$\frac{B_{44}}{B} + \frac{C_{44}}{C} + \frac{3B_4C_4}{BC} + \frac{B_4^2}{B^2} - \frac{\alpha^2}{m^2B^2} = p + \rho + \zeta\theta + 2k \tag{16}$$

To find the deterministic model of the universe, we assume that

$$p = \gamma\rho;\ 0 \le \gamma \le 1, \text{ (barotropic fluid condition)} \tag{17}$$

$$\theta = 3H,\ \rho = 3H^2, \zeta = \rho^{1/2}$$

as considered in Barrow[20] and Gron[24] where p is isotropic pressure, ρ the matter density, θ the expansion, H the Hubble parameter and ζ the coefficient of bulk viscosity.

From equations (16) and (17), we have

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{3B_4C_4}{BC}+\frac{B_4^2}{B^2}-\frac{\alpha^2}{m^2B^2}=\left\{\frac{(\gamma+1)}{3}+\frac{1}{\sqrt{3}}\right\}\left(\frac{2B_4}{B}+\frac{C_4}{C}\right)^2+2k \qquad (18)$$

To get the deterministic value of B and C, we assume that shear (σ) is proportional to expansion (θ) as considered by Thorne[30]. Thus, we have

$$B=C^n \qquad (19)$$

$$\frac{B_4}{B}=\frac{nC_4}{C}$$

and

$$\frac{B_{44}}{B}=\frac{nC_{44}}{C}+n\,(n-1)\frac{C_4^2}{C^2}$$

From equation (18), we have

$$2C_{44}+2\left\{2n-\left(\frac{\gamma+1}{3}+\frac{1}{\sqrt{3}}\right)\frac{(2n+1)^2}{(n+1)}\right\}\frac{C_4^2}{C}=\frac{2\alpha^2}{(n+1)m^2C^{2n-1}}+\frac{4kC}{(n+1)}$$

which leads to

$$2C_{44}+\frac{a}{C}C_4^2=\frac{b_1}{C^{2n-1}}+b_2C \qquad (20)$$

where

$$a=\left(\frac{\gamma+1}{3}+\frac{1}{\sqrt{3}}\right)\frac{(2n+1)^2}{n+1},\ b_1=\frac{2\alpha^2}{(n+1)m^2},\ b=\frac{4k}{n+1}$$

Let us consider $C_4=f(C)$

Thus

$$C_{44}=f\,f',\ f'/df/dC$$

Equation (20) leads to

$$\frac{df^2}{dC} + \frac{a}{C} f^2 = \frac{b_1}{C^{2n-1}} + b_2 C \qquad (21)$$

Equation (21) leads to

$$f^2 = \frac{b_1}{(a-2n+2)\, C^{2n-2}} + \frac{b_2}{(a+2)} C^2 + \frac{b_3}{C^a} \qquad (22)$$

After suitable transformation of coordinates, the metric (1) leads to the form

$$ds^2 = -\frac{T^{2n-2}}{a_1 + a_2 T^{2n} + b_3 T^{2n-2-a}} dT^2 + T^{2n} dX^2 + e^{-\frac{2\alpha}{m} X} T^{2n} dY^2 + T^2 dZ^2 \qquad (23)$$

where

$$a_1 = \frac{b_1}{a+2-2n}, a_2 = \frac{b_2}{a+2}$$

mx = X

y = Y

z = Z

C = T

Special Case. To get the solution in terms of cosmic time t, we assume $b_3 = 0$.

If $b_3 = 0$, then from equation (22), we have

$$f^2 = \frac{a_1}{C^{2n-2}} + a_2 C^2$$

which leads to

$$\frac{C^{n-1} dC}{\sqrt{a_1 + a_2 C^{2n}}} = dt$$

Thus, we have

$$\frac{d\xi}{\sqrt{\beta^2+\xi^2}} = n\sqrt{a_2}dt \qquad (24)$$

where $\frac{a_1}{a_2} = \beta^2, C^n = \xi$. Equation (24) leads to

$$\xi = \beta \sinh(n\sqrt{a_2}t + a_3)$$

Therefore, we have

$$C^n = \beta \sinh \tau \qquad (25)$$

where

$$n\sqrt{a_2}t + a_3 = n\sqrt{a_2}\tau.$$

Also

$$B = C^n = \beta \sinh \tau \qquad (26)$$

$$A = mB = m\beta \sinh \tau \qquad (27)$$

After suitable transformation of coordinates, the metric (1) leads to the form

$$ds^2 = -d\tau^2 + \sinh^2\tau \left\{ dX^2 + e^{-\frac{2\alpha}{m\beta}X} dY^2 \right\} + \sinh^{2/n}\tau\, dZ^2 \qquad (28)$$

where

$m\beta x = X$

$\beta y = Y$

$\beta^{1/n} = Z$

4 Physical and Geometrical Aspects

The rate of Higgs field (ϕ), the spatial volume (R^3), the expansion (θ), the shear (σ), the Hubble parameter (H), the deceleration parameter (q) for the model (23) are given by

$$\phi_4 = \frac{\ell}{mC^{2n+1}} = \frac{\ell}{m\,T^{2n+1}} \tag{29}$$

which leads to

$$\phi = \frac{\ell}{m}\int \frac{1}{T^{n+2}\sqrt{a_1 + a_2 T^{2n} + b_3\, T^{2n-2-a}}}\, dT + M \tag{30}$$

where M is constant of integration.

The spatial volume

$$(R^3) = mC^{2n+1} = m\,T^{2n+1} \tag{31}$$

$$(\theta) = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = (2n+1)\frac{C_4}{C}$$

$$= \frac{(2n+1)\sqrt{a_1 + a_2 T^{2n} + b_3 T^{2n-2-a}}}{T^n} \tag{32}$$

$$(\sigma) = \frac{1}{\sqrt{3}}\left(\frac{B_4}{B} - \frac{C_4}{C}\right) = \frac{(n-1)C_4}{\sqrt{3}\,C} = \frac{(n-1)\sqrt{a_1 + a_2 T^{2n} + b_3 T^{2n-2-a}}}{\sqrt{3}\,T^n} \tag{33}$$

The Hubble parameter $(H) = \frac{\theta}{3}$

$$= \frac{(2n+1)\sqrt{a_1 + a_2 T^{2n} + b_3\, T^{2n-2-a}}}{3T^n} \tag{34}$$

The deceleration parameter $q = \frac{-R_{44}/R}{R_4^2/R^2}$

$$= \frac{2n + 3a - 8}{2(2n+1)} \tag{35}$$

$$\frac{\sigma}{\theta} = \frac{(n-1)}{\sqrt{3}(2n+1)} \tag{36}$$

The rate of Higgs field (ϕ), the spatial volume (R^3), the expansion (θ), the shear (σ), the Hubble parameter (H), the deceleration parameter (q) for the model (28) are given by

$$\phi_4 = \frac{\ell}{mC^{2n+1}} = \frac{\ell}{m\beta^{\frac{2n+1}{n}} \sinh^{\frac{2n+1}{n}} \tau} \tag{37}$$

which leads to

$$\phi = \frac{-\ell\left\{\operatorname{cosech}^{1/n}\tau \coth\tau + \frac{1}{n}\int \operatorname{cosech}^{1/n} d\tau\right\}}{m(n+1)\,\beta^{\frac{2n+1}{n}}\sqrt{a_2}} + N$$

where N is constant of integration.

For n = 1/2

$$\phi = \frac{-2\ell}{3m\,\beta^4\sqrt{a_2}}\{\operatorname{cosech}^2\tau \coth\tau - 2\coth\tau\} + N$$

The spatial volume

$$(R^3) = m\beta^{\frac{2n+1}{n}} \sinh^{\frac{2n+1}{n}} \tau \tag{39}$$

The expansion $(\theta) = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C} = (2n+1)\frac{C_4}{C}$

$$= (2n+1)\coth\tau \tag{40}$$

$$(\sigma) = \frac{(n-1)C_4}{\sqrt{3}\,C} = \frac{(n-1)\coth\tau}{\sqrt{3}} \tag{41}$$

$$(q) = \frac{-R_{44}/R}{R_4^2/R^2}$$

$$= -\frac{3n\tanh^2\tau}{(2n+1)} + \frac{(n-1)}{(2n+1)} \tag{42}$$

$$(H) = \frac{R_4}{R} = \frac{(2n+1)\coth\tau}{3n} \tag{43}$$

and

$$\rho = 3H^2 = \frac{(2n+1)^2}{3n^2}\coth^2\tau \tag{44}$$

$$\zeta = \rho^{1/2} = \sqrt{3}\,H\,\frac{(2n+1)}{n\sqrt{3}}\coth\tau \tag{45}$$

$$\frac{\sigma}{\theta} = \frac{(n-1)}{\sqrt{3}(2n+1)} \tag{46}$$

The average expansion anisotropy defined by Caderni and Fabbri[33] is given by

$$\hat{A} = \frac{1}{3}\left[\left(\frac{H_i - H}{H}\right)^2\right], \; i = 1,2,3 \tag{47}$$

which leads to

$$\hat{A} = \frac{2(n-1)^2}{(2n+1)^2} \tag{48}$$

5 Summary and Conclusion

The spatial volume (R^3) for the models (23) and (28) increases with time and exponentially with time representing inflationary scenario. The models in general, represent anisotropic space-time but isotropic at late time. The models also isotropize for n = 1. The Hubble parameter is initially large but decreases with time. The model (23) represent decelerating and accelerating phases of universe if 2n+3a > 8 and 2n+3a < 8 respectively. However, the model (28) represents accelerating universe for n = 1. Both the models have Point Type singularity at T = 0 and τ = 0 respectively[31]. The presence of bulk viscosity prevents the matter density to vanish. We also find that the model (28) expands exponentially which has the similar characteristic due to repulsive gravitation of a dominating vacuum energy density. This vacuum energy is due to Higgs field which produces a large cosmological constant. As pointed out by Gron[32], there is radiation dominated period before inflationary era while the radiation density diminishes during expansion. The model (28) indicates that energy density (ρ) is large initially and decreases during expansion. In general, the model (28) represents anisotropic space-time. However, for n = 1, the model isotropizes and represents accelerating universe.

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