SHORT-TIME SQUEEZING EFFECT IN HIGHER ORDER AMPLITUDE IN EIGHTH HARMONIC GENERATION

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ABSTRACT

The quantum effect of squeezing of the electromagnetic field is investigated in the fundamental mode in eighth harmonic generation with the approximation $|gt|^2 = 1$, *where g is coupling constant and 't' is the interaction time between waves during the process in nonlinear medium. Higher order amplitude squeezing in fundamental mode is found to be dependent on the selective phase values of the field amplitude. The effect of photon number on squeezing and signal to noise ratio in higher order field amplitude in fundamental mode has also been investigated.*

Keywords: *Harmonic generation, Nonlinear optics: Squeezing.*

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1. INTRODUCTION

Harmonic generation is a general feature of driven nonlinear system. A semi classical theory of high-order harmonic generation (HHG) has been given by Lewenstein *et al.* [1]. There are many recent developments in the area of high-order harmonic generation. Lytle *et al.* [2] studied experimental and theoretical work on the use of counter propagating light to enhance high order harmonic generation through all optical quasi-phase matching. Harmonic generation has been suggested as a possible generation scheme for squeezed states of light. Quantum systems are uncertain by nature. The quantum limit can be circumvented by using squeezed states of light, where fluctuations are reduced below the symmetric quantum limit in one quadrature at the expense of increased fluctuations in the canonically conjugate quadrature, while preserving Heisenberg uncertainty principle.

A large number of methods have been proposed in literature to generate and detect squeezed states theoretically and experimentally [3-13]. Due to low noise property of squeezed light, it leads to the applications in optical communication [14] as well as in the field of high precision measurement such as to enhance the sensitivity of interferometers for the detection of gravitational waves [15-16]. Recent work has highlighted the potential applications of squeezed light in the processing of quantum information, for example, for quantum teleportation [17-18], quantum cryptography [19] and quantum computation [20-21].

2. DEFINITION OF SQUEEZING AND HIGHER ORDER SQUEEZING

Squeezed states of an electromagnetic field are the states with reduced noise below the vacuum limit in one of the canonical conjugate quadrature. Higher order squeezing is defined in various ways. Hong and Mandel [22] and Hillery [23] have introduced the notion of higher order squeezing of quantized electromagnetic field as generalization of normal squeezing. Amplitude-squared squeezing is defined in terms of operators Y_1 and Y_2 as

$$Y_1 = \frac{1}{2} A^2 + A^{\dagger 2} \quad \text{and} \quad Y_2 = \frac{1}{2i} A^2 - A^{\dagger 2}$$

Where Y_1 and Y_2 are the real and imaginary parts of the square of field amplitude, respectively. A and $A^\dagger$ are slowly varying operators defined by $A = ae^{i\omega t}$ and $A^\dagger = a^\dagger e^{-i\omega t}$.

The operators Y_1 and Y_2 obey the commutation relation

$$\left[Y_1, Y_2\right] = i\ (2N + 1)$$

which leads to the uncertainty relation

$$\Delta Y_1 \Delta Y_2 \geq \left\langle \left(N + \frac{1}{2} \right) \right\rangle$$

where N is the usual number operator.

Amplitude-squared squeezing is said to exist in Y_i variable if

$$(\Delta Y_i)^2 < \left\langle \left(N + \frac{1}{2} \right) \right\rangle \quad \text{for } i = 1 \text{ or } 2$$

Amplitude-cubed squeezing is defined in terms of operators

$$Z_1 = \frac{1}{2}\ A^3 + A^{\dagger 3} \quad \text{and} \quad Z_2 = \frac{1}{2i}\ A^3 - A^{\dagger 3}$$

The operators Z_1 and Z_2 obey the commutation relation

$$\left[Z_1, Z_2\right] = \frac{i}{2}(9N^2 + 9N + 6)$$

which leads to the uncertainty relation

$$\Delta Z_1 \Delta Z_2 \geq \frac{1}{4}(9N^2 + 9N + 6)$$

Amplitude-cubed squeezing exists when

$$(\Delta Z_i)^2 < \frac{1}{4}\left\langle (9N^2 + 9N + 6) \right\rangle \quad \text{for } i = 1 \text{ or } 2$$

3. SQUEEZING OF FUNDAMENTAL MODE IN EIGHTH HARMONIC GENERATION

Eighth harmonic generation model is shown in Figure 1. In this model, the interaction is looked upon as a process which involves the absorption of eight photons, each having a frequency ω_1 going from state $|1\rangle$ to state $|2\rangle$ and emission of one photon of frequency ω_2, where $\omega_2 = 8\omega_1$.

The Hamiltonian for this process is given as follows ($\hbar = 1$)

$$H = \omega_1 a^\dagger a + \omega_2 b^\dagger b + g\ \ a^8 b^\dagger + a^{\dagger 8} b \qquad (1)$$

in which g is a coupling constant for eighth harmonic generation.

The Heisenberg equation of motion for fundamental mode A is given as ($\hbar = 1$)

$$\frac{dA}{dt} = \frac{\partial A}{\partial t} + i\left[H, A\right] \tag{2}$$

Using Eq. (1) in Eq. (2), we obtain

$$\dot{A} = -8igA^{\dagger 7}B \tag{3}$$

Similarly, for harmonic mode B

$$\dot{B} = -igA^{8} \tag{4}$$

By assuming the short time interaction of waves with the medium and expanding $A(t)$ by using Taylors series expansion and retaining the terms up to g^2t^2 as

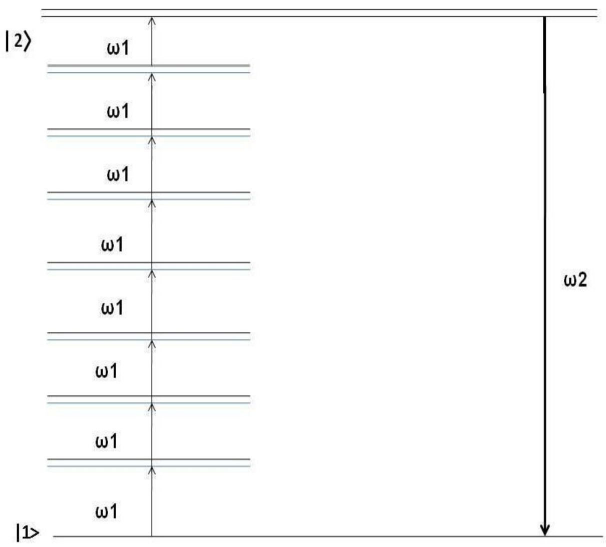

Fig. 1. Eighth harmonic generation model.

$$A\ t\ = A - 8igtA^{\dagger 7}B + 4g^2t^2\Big[56\ A^{\dagger 6}A^7 + 21A^{\dagger 5}A^6 + 210A^{\dagger 4}A^5 + 1050A^{\dagger 3}A^4 + 2520A^{\dagger 2}A^3 + 2520A^{\dagger}A^2 + 720A\ B^{\dagger}B - A^{\dagger 7}A^8\Big] \tag{5}$$

Initially, we consider the quantum state of the field amplitude as a product of coherent state for the fundamental mode A and the vacuum state for the harmonic mode B i.e.

$$\left|\psi\right\rangle = \left|a\right\rangle\left|0\right\rangle \tag{6}$$

Using Eq. (5) and (6), the second order amplitude in fundamental mode is expressed as

$$A^2\ t\ = A^2 - 8igt\ \ 2A^{\dagger 7}A + 7A^{\dagger 6}\ \ B - 4g^2t^2\ \ 2A^{\dagger 7}A^9 + 7A^{\dagger 6}A^8 \tag{7}$$ and

$$A^{\dagger 2}\ t\ = A^{\dagger 2} + 8igt\ \ 2A^{\dagger}A^{7} + 7A^{6}\ \ B^{\dagger} - 4g^{2}t^{2}\ \ 2A^{\dagger 9}A^{7}\ + 7A^{\dagger 8}A^{6} \tag{8}$$

For amplitude squared squeezing, the real quadrature component for the fundamental mode is given as

$$Y_{1A}\ \ t\ \ = \frac{1}{2}\left[A^{2}\ \ t\ \ + A^{\dagger 2}\ \ t\ \right] \tag{9}$$

Using Eqs. (6), (7) and (9), we get the expectation values as

$$\begin{aligned}\left\langle \psi \left| Y_{1A}^{2}\ \ t\ \right| \psi \right\rangle = \frac{1}{4}\Big[&\alpha^{4} + \alpha^{*4} + 2\left|\alpha\right|^{4} + 4\left|\alpha\right|^{2} + 2 - 4g^{2}t^{2}(4\alpha^{4}\left|\alpha\right|^{14} \\ &+ 42\alpha^{4}\left|\alpha\right|^{12} + 168\alpha^{4}\left|\alpha\right|^{10} + 210\alpha^{4}\left|\alpha\right|^{8} + 4\alpha^{*4}\left|\alpha\right|^{14} \\ &+ 42\alpha^{*4}\left|\alpha\right|^{12} + 168\alpha^{*4}\left|\alpha\right|^{10} + 210\alpha^{*4}\left|\alpha\right|^{8} + 8\left|\alpha\right|^{18} + 36\left|\alpha\right|^{16})\Big]\end{aligned} \tag{10}$$

and

$$\begin{aligned}\left\langle \psi \left| Y_{1A}\ \ t\ \right| \psi \right\rangle^{2} = \frac{1}{4}\Big[&\alpha^{4} + \alpha^{*4} + 2\left|\alpha\right|^{4} - 4g^{2}t^{2}(4\alpha^{4}\left|\alpha\right|^{14} + 14\alpha^{4}\left|\alpha\right|^{12} \\ &+ 4\alpha^{*4}\left|\alpha\right|^{14} + 14\alpha^{*4}\left|\alpha\right|^{12} + 8\left|\alpha\right|^{18} + 28\left|\alpha\right|^{16})\Big]\end{aligned} \tag{11}$$

Therefore,

$$\begin{aligned}\left[\Delta Y_{1A}\ \ t\ \right]^{2} = \frac{1}{4}\Big[&4\left|\alpha\right|^{2} + 2 - 4g^{2}t^{2}(28\alpha^{4}\left|\alpha\right|^{12} + 168\alpha^{4}\left|\alpha\right|^{10} + 210\alpha^{4}\left|\alpha\right|^{8} \\ &+ 28\alpha^{*4}\left|\alpha\right|^{12} + 168\alpha^{*4}\left|\alpha\right|^{10} + 210\alpha^{*4}\left|\alpha\right|^{8} + 8\left|\alpha\right|^{16})\Big]\end{aligned} \tag{12}$$

Using Eqs. (5) and (6), number of photons in mode A may be expressed as

$$\begin{aligned}N_{1A}\ \ t\ &= A^{\dagger}\ \ t\ \ A\ \ t \\ &= A^{\dagger}A - 8g^{2}t^{2}A^{\dagger 8}A^{8}\end{aligned} \tag{13}$$

$$\begin{aligned}N_{1A}^{2}\ \ t\ &= N_{1A}\ \ t\ \ N_{1A}\ \ t \\ &= A^{\dagger 2}A^{2} + A^{\dagger}A - 8g^{2}t^{2}\ \ 2A^{\dagger 9}A^{9}\ + 8A^{\dagger 8}A^{8}\end{aligned} \tag{14}$$

Using Eqs. (6) and (13), we get

$$\left\langle N_{1A}\ \ t\ \ + \frac{1}{2}\right\rangle = \frac{1}{4}\left[4\left|\alpha\right|^{2} + 2 - 4g^{2}t^{2}(8\left|\alpha\right|^{16})\right] \tag{15}$$

Subtracting Eq. (15) from Eq. (12), we get

$$\left[\Delta Y_{1A}(t)\right]^2-\left\langle N_{1A}(t)+\frac{1}{2}\right\rangle=-2g^2t^2\left(28|\alpha|^{16}+168|\alpha|^{14}+210|\alpha|^{12}\right)\cos 4\theta \quad (16)$$

The right hand side of Eq. (16) is negative and thus shows the existence of squeezing in the second order of the fundamental mode for all values of θ for which $\cos 4\theta>0$.

Using Eqs. (5) and (6) the third-order amplitude of the fundamental mode is expressed as

$$\begin{aligned}A^3(t)&=A^3-8igt\left(3A^{\dagger 7}A^2+21A^{\dagger 6}A+42A^{\dagger 5}\right)B-4g^2t^2\left(3A^{\dagger 7}A^{10}\right.\\&\left.+21A^{\dagger 6}A^9+42A^{\dagger 5}A^8\right)\end{aligned} \quad (17)$$

and

$$\begin{aligned}A^{\dagger 3}(t)&=A^{\dagger 3}+8igt\left(3A^{\dagger 2}A^7+21A^{\dagger}A^5+42A^5\right)B^{\dagger}-4g^2t^2(3A^{\dagger 10}A^7\\&+21A^{\dagger 9}A^6+42A^{\dagger 8}A^5\end{aligned} \quad (18)$$

Using Eq. (17) the real quadrature component for third-order squeezing in fundamental mode is expressed as

$$\begin{aligned}Z_{1A}(t)&=\frac{1}{2}\left[A^3(t)+A^{\dagger 3}(t)\right]\\&=\frac{1}{2}\left[A^3+A^{\dagger 3}-8igt\left(3A^{\dagger 7}A^2+21A^{\dagger 6}A+42A^{\dagger 5}\right)B+8igt\left(3A^{\dagger 2}A^7+21A^{\dagger}A^6\right.\right.\\&+42A^5\big)B^{\dagger}-4g^2t^2\left(3A^{\dagger 7}A^{10}+21A^{\dagger 6}A^9+42A^{\dagger 5}A^8+3A^{\dagger 10}A^7+21A^{\dagger 9}A^6\right.\\&\left.\left.+42A^{\dagger 5}A^8)\right]\end{aligned} \quad (19)$$

Using Eqs. (6) and (19), we get the expectation values as

$$\begin{aligned}\left\langle\psi\left|Z_{1A}^2(t)\right|\psi\right\rangle&=\frac{1}{4}\left[\alpha^6+\alpha^{*6}+2|\alpha|^6+9|\alpha|^4+18|\alpha|^2+6-4g^2t^2\left(6\alpha^6|\alpha|^{14}\right.\right.\\&+105\alpha^6|\alpha|^{12}+840\alpha^6|\alpha|^{10}+3150\alpha^6|\alpha|^8+5040\alpha^6|\alpha|^6+2520\alpha^6|\alpha|^4\\&+6\alpha^{*6}|\alpha|^{14}+105\alpha^{*6}|\alpha|^{12}+840\alpha^{*6}|\alpha|^{10}+3150\alpha^{*6}|\alpha|^8+5040\alpha^{*6}|\alpha|^6\\&\left.\left.+2520\alpha^{*6}|\alpha|^4+12|\alpha|^{20}+120|\alpha|^{18}+330|\alpha|^{16})\right]\end{aligned} \quad (20)$$

and

$$\begin{aligned}\left\langle\psi\left|Z_{1A}(t)\right|\psi\right\rangle^2&=\frac{1}{4}\left[\alpha^6+\alpha^{*6}+2|\alpha|^6-4g^2t^2(6\alpha^6|\alpha|^{14}+42\alpha^6|\alpha|^{12}\right.\\&+84\alpha^6|\alpha|^{10}+6\alpha^{*6}|\alpha|^{14}+42\alpha^{*6}|\alpha|^{12}+84\alpha^{*6}|\alpha|^{10}\\&\left.+12|\alpha|^{20}+84|\alpha|^{18}+168|\alpha|^{16})\right]\end{aligned} \quad (21)$$

Therefore,

$$\left[\Delta Z_{1A}\ \ t\ \ \right]^2 = \frac{1}{4}\Big[9|\alpha|^4 + 18|\alpha|^2 + 6 - 4g^2t^2\{(63|\alpha|^{12} + 756|\alpha|^{10} + 3150|\alpha|^8$$
$$+ 5040|\alpha|^6 + 2520|\alpha|^4)(\alpha^6 + \alpha^{*6}) + 36|\alpha|^{18} + 162|\alpha|^{16}\}\Big] \quad (22)$$

Using Eqs. (13) and (14), we get

$$\frac{1}{4}\left\langle 9N_{1A}^2\ \ t\ \ + 9N_{1A}\ \ t\ \ + 6\right\rangle = \frac{1}{4}[9|\alpha|^4 + 18|\alpha|^2 + 6 - 4g^2t^2(36|\alpha|^{18}$$
$$+ 162|\alpha|^{16})] \quad (23)$$

Subtracting Eq. (23) from Eq. (22), we obtain

$$\left[\Delta Z_{1A}\ \ t\ \ \right]^2 - \frac{1}{4}\left\langle 9N_{1A}^2\ \ t\ \ + 9N_{1A}\ \ t\ \ + 6\right\rangle$$
$$= -2g^2t^2(63|\alpha|^{18} + 756|\alpha|^{16} + 3150|\alpha|^{14} + 5040|\alpha|^{12} + 2520|\alpha|^{10})\cos 6\theta \quad (24)$$

The right hand side of Eq. (24) is negative, indicating that squeezing occurs in cubed amplitude for all values of θ for which $\cos 6\theta > 0$ in the fundamental mode of eighth harmonic generation.

4. SIGNAL-TO-NOISE RATIO

Signal to noise ratio is defined as ratio of the magnitude of the signal to the magnitude of the noise. With the approximations $\theta = 0$ and

$|gt|^2 \ll 10^{-4}$, the maximum signal to noise ratio (in decibels) in higher orders of field amplitude, is given below.

Using Eqs. (11) and (12), SNR in amplitude-squared squeezing is given as

$$SNR_2 = 20^* \log_{10} \frac{(2|\alpha|^6 + 7|\alpha|^4)}{(8|\alpha|^4 + 42|\alpha|^2 + 52.5)} \quad (25)$$

Using Eqs. (21) and (22), SNR in amplitude-cubed squeezing is expressed as

$$SNR_3 = 20^* \log_{10} \frac{(24|\alpha|^{10} + 168|\alpha|^8 + 336|\alpha|^6)}{(162|\alpha|^8 + 1674|\alpha|^6 + 6300|\alpha|^4 + 10080|\alpha|^2 + 5040)} \quad (26)$$

5. RESULTS

The results show the presence of squeezing in second and third order of field amplitude in fundamental mode in eighth harmonic generation. We denote right hand side of Eqs. (16) and (24) by S_Y and S_Z respectively, which shows the presence of squeezing in amplitude-squared and amplitude-cubed states of the field. Taking $|gt|^2 = 10^{-4}$ and $\theta = 0$ for maximum squeezing, the variations of S_Y and S_Z are shown in Figures 2 and 3 respectively. Squeezing increases non-linearly with $|\alpha|^2$ and this confirms that the squeezed states are associated with the photon number in fundamental mode. The variation of SNR in higher orders of field amplitude for a squeezed state with photon number has also been shown in Figure 4.

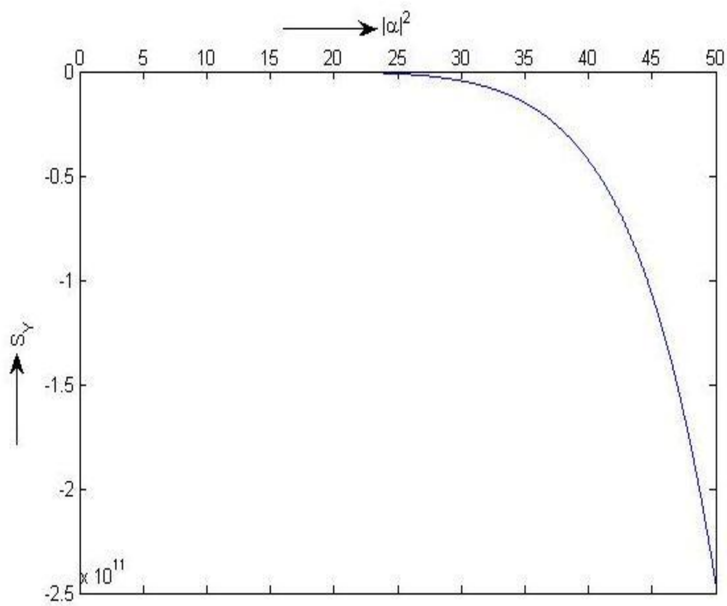

Fig. 2. Dependence of amplitude-squared squeezing on $|\alpha|^2$.

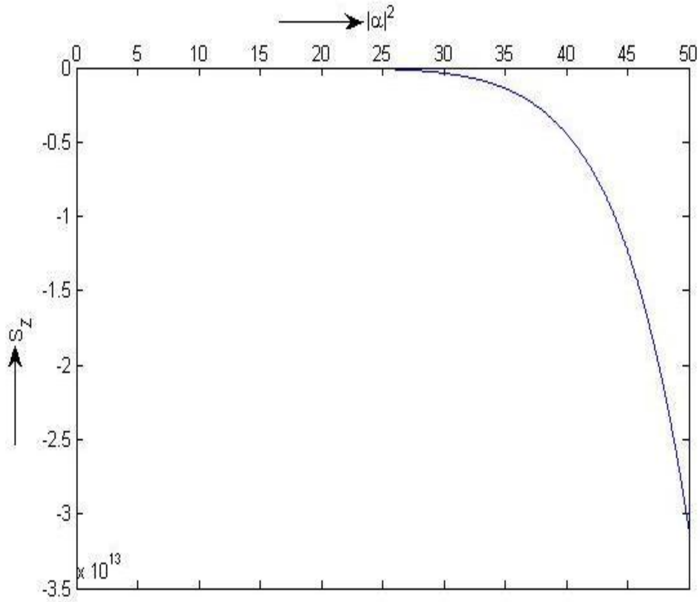

Fig. 3. Dependence of amplitude-cubed squeezing on $|\alpha|^2$.

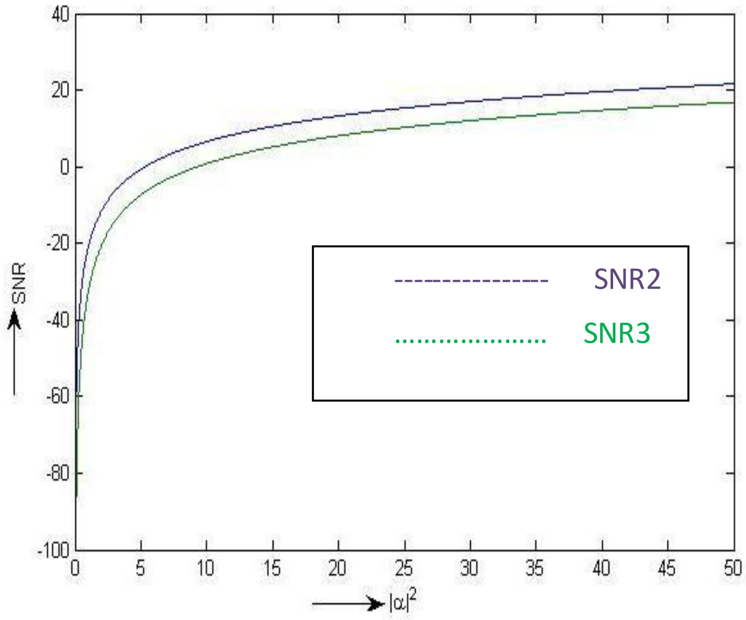

Fig. 4. Signal to noise ratio for different order squeezing.

6. CONCLUSION

This paper presents one of the possible ways of generating higher order squeezed states in fundamental mode in eighth harmonic generation under short time approximation. The degree of higher order squeezing varies with the phase of the input coherent light θ, initial photon number $|\alpha|^2$ and the interaction time t. Further, Figures 2 and 3, show that the degree of squeezing increases with increase in the order of field amplitude of the fundamental mode.

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