A Contribution to Universally Optimal One Sided Circular Neighbour Balanced design

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Abstract:

Specially in an agricultural design of experiment the treatment received by a plot may affect the other response on the neighbouring plots of a same block or it may happen to affect the response on the following plot. For example of the second condition, the tall varieties may affect the other crops grown on the neighbouring plots by their shades. Bailey (2003) has developed such design concerned with the study of one sided neighbour effect, under the above mentioned second condition. This paper gives a new series of Universally Optimal One-Sided Circular Neighbour Balanced designs.

***AMS subject classification*: 62K 05, 62K 10 (Primary).**

***Keywords*:** One Sided Circular Neighbour Balanced design, Universally Optimal, Equivalence class of sequences.

1. Introduction and preliminaries

Either in allied subjects or agricultural design of experiment where the treatment applied to one experimental plot may affect the response on the neighbouring plots and response on the plot to which, it is applied, Bailey *op.cit.* had proposed a particular type of designs for one sided neighbour effect which are concerned with the study of one-sided neighbour effect only. Such as the applications in the case of sunflower crops as well as in the case of cereal crops where tall varieties may shade the plot on their neighbouring and affect the response of the plot. Similarly, as in the case of pesticide or fungicide experiment where some portion of the treatment applied may spread to the neighbouring plot immediately down wind and spores from the untreated plots, one-sided neighbour effect of the preceding plot-treatment may occur to the following plot. The linear ridge is the form of the blocks of such design (Welham *et. al.* (1996)) the design where the plots are in 1-dimension and 2-dimension are studied. Azais, Bailey and Monod (1993) give a catalogue of circular neighbour balanced designs with t-1 blocks of size t or t blocks of size t-1, where t is the number of treatments. Many contributions are available in the literatures of Smart *et. al.* (1994), Langton

(1990), David and Kempton (1996). Later on Bailey and Druilhet (2004) have extended the work of Bailey *op. cit.* taking into account the effect of the treatment on the proceeding plot and following plot that on the concerned plot and the effect of the block. Under the name of Circular Neighbour Balanced design, both of such 1-dimensional and 2-dimensional designs are studied. Kumam and Meitei (2006) have contributed a construction-method of such design.

For the reference to the text of the paper, let us associate with some few definitions.

Definition1.1 One Sided Circular Neighbour Balanced design is an arrangement of v treatments in b linear blocks of size k (not necessarily distinct) such that (i) each treatment is replicated r times, (ii) every pair of distinct treatments has concurrence μ and (iii) every treatment is followed by each other treatment λ times assuming that in every block the last plot is followed by the first plot. It is denoted by One Sided Circular Neighbour Balanced design (v, b, r, k, μ, λ).

Such design is neighbour balanced as every treatment is followed by each other treatment λ times and also pairwise balanced in the sense that every pair of distinct treatments has concurrence μ. Clearly, vr=bk. These designs become circular, after having recommended to have a border plot before the first plot of each block, assuming that the treatment already applied to the last plot is applied to this border plot. But its response is not measured. It is only to get the neighbour effect of the treatment in the border plot to the last plot. So, in practical point of view, for conducting an experiment based on such designs of block size k, the planning of the design compels blocks to be of size k+1. It becomes Universally Optimal for estimation of the total effect (Bailey and Druilhet (2004) proposition 9, page 1657) if

(i) there are only s different type of treatments in every block,
(ii) out of s different type of treatments n_1 repeat m times in the block and each of the remaining n_2 occurs m+1 times, where $n_1+n_2=s$ and
(iii) all the occurrences of a treatment in a block must be in a single sequence of adjacent plots (possibly including both the last plot and the first plot).

For the future use in the sequel, Universally Optimal One Sided Circular Neighbour Balanced design (v, b, r, k, s, m, n_1, n_2, μ, λ.) denotes the design. Clearly, $n_1= s-n_2$, $n_2=k-sm$, $bs=v(v-1)\lambda$. The contribution of each block to the sum of the concurrences of all possible pair of treatments is $\theta/2$ where

$$\theta = n_1(n_1-1)m^2+ n_2(n_2-1)(m+1)^2+ 2n_1n_2m(m+1).$$

$$= sm(m+1) + k(k-2m-1).$$

Thus, $b\theta = v(v-1)\mu$.If a class Δ of competing designs contains a design D such that the information matrix C_D is completely symmetric and trace $(C_D)\geq$trace(C_d) for all d Є Δ, then the design D is said to be Universally Optimal. Two sequences of treatments on a block are equivalent if one sequence can be obtained from the other one by relevelling the treatments if we denote ξ the equivalence class of the sequence l on the block u of the design d the trace of C_d is given by (Bailey and Druilhet *op. cit.*)

$$C(\xi) = \text{trace}(C_{du}).$$

$$= (k - \frac{2}{k}\sum_{i=1}^{v} g_i^2 + \sum_{i=1}^{v} h_i)/2,$$

where g_i is the number of occurrences of treatment i in the sequence l and h_i is the number of time treatment i is on the left hand side of itself in sequence l.

2. Construction: In this paper as a new dimension in the sphere of construction of Universally Optimal One Sided Circular Neighbour Balanced design, a lemma of Praphullo and Meitei (2006) , using difference sets will be recalled such hereafter.

Given a set S of size k i.e. $\{i_1, i_2, \ldots, i_k\}$, the forward and the backward differences arising from this set are defined as follows:

$$F = (i_2 - i_1, i_3 - i_2, \ldots, i_k - i_{k-1}, i_1 - i_k) \text{ and}$$

$$B = (i_1 - i_2, i_2 - i_3, \ldots, i_{k-1} - i_k, i_k - i_1) \text{ respectively. Clearly, } B_k = -F_k.$$

Lemma 2.1: Let M be a module of v elements. Consider t be the initial blocks each containing k elements (not necessarily distinct) of M. These t blocks when developed module v generate an Universally Optimal One Sided Circular Neighbour Balanced design with the parameters v, b = tv, r = kt, k, s, m, n_1, n_2, μ, λ , if the following conditions are satisfied:

(i) there are only s different types of treatments in every initial block,

(ii) each of n_1 out of these s treatments occurs m times and each of the remaining $s-n_1=n_2$ (say) occurs (m+1) times in the block,

(iii) all the occurrences of a treatment in every initial block is in a single sequence of adjacent plots (assuming the last plot and the first plot are neighbour),

(iv) among the totality of forward (or backward) differences arising from the t initial blocks, every non-zero element of M occurs exactly λ times,

(v) among the totality of differences arising from the t initial blocks, every non-zero element of M occurs exactly μ times.

For, v=4t-1, be a prime or a prime power and let x be the primitive elements of GF (v=4t-1), then $x^{v-1}=x^0=1$, i.e.$x^{4t-1-1}=x^0=1$, i.e. $x^{4t-2}=1$, i.e. $x^{4t-2}-1=0$

i.e. $(x^{2t-1}+1)(x^{2t-1}-1)=0$ then $x^{2t-1}=-1$, since x is primitive element.

$$C_1 = \{x^0, x^2, x^4, \ldots, x^{4t-6}, x^{4t-4}\} \quad \ldots (2.1)$$

$$C_2 = \{x^1, x^3, x^5, \ldots, x^{4t-5}, x^{4t-3}\} \quad \ldots (2.2)$$

Consider the initial block as shown in (2.1), then the differences

arising from this block can be exhibited as follows :

1st type : $\pm(x^2-x^0)$, $\pm(x^4-x^2)$, … , $\pm(x^{4t-4}-x^{4t-6})$,

2nd type : $\pm(x^4-x^0)$, $\pm(x^6-x^2)$, … , $\pm(x^{4t-4}-x^{4t-8})$,

⋮ ⋮ ⋮ ⋮

ith. type : $\pm(x^{2i}-x^0)$, $\pm(x^{2i+2}-x^2)$, … , $\pm(x^{4t-4}-x^{4t-(2i+4)})$,

$\vdots \quad \vdots \quad \vdots \quad \vdots$

tth. type : $\pm(x^{2t}-x^0)$, $\pm(x^{2t+2}-x^2)$, … , $\pm(x^{4t-4}-x^{4t-(2t+4)})$,

$\vdots \quad \vdots \quad \vdots \quad \vdots$

[2t- (i+1)] : $\pm(x^{2[2t-(i+1)]}-x^0)$, $\pm(x^{2[2t-(i+1)]+2}-x^2)$, … , $\pm(x^{4t-4}-x^{4t-[2\{2t(i+1)\}+4]})$,

$\vdots \quad \vdots \quad \vdots$

(2t-4)th type : $\pm(x^{4t-8}-x^0)$, $\pm(x^{4t-6}-x^2)$, $\pm(x^{4t-4}-x^4)$,

(2t-3) th type: $\pm(x^{4t-6}-x^0)$, $\pm(x^{4t-4}-x^2)$,

(2t-2) th type : $\pm(x^{4t-4}-x^0)$,

Let, $x^{2i}-x^0=x^{q_i}$, for some q_i, combining the ith type differences with the {2t-(i+1)}th type differences, i=1, 2, … , (t-1) , remembering $x^{2t-1}= -1$.

Now, [i th type differences; {2t - (i+1)} th type differences]

$\pm(x^{2i}-x^0)$, $\pm(x^{2i+2}-x^2)$, … , $\pm(x^{4t-4}-x^{4t-(2i+4)})$; $\pm(x^{2\{2t-(i+1)\}}-x^0)$,

$\pm(x^{2\{2t-(i+1)+2}-x^2)$, … , $\pm(x^{4t-4}-x^{4t-2\{2t-(i+1)\}+4})$

i.e. $\pm(x^{2i}-x^0)$, $\pm(x^{2i+2}-x^2)$, … , $\pm(x^{4t-4}-x^{4t-2i-4})$; $\pm(x^{4t-2i-2}-x^{4t-2})$,

$\pm(x^{4t-2i-2+2}-x^2)$, … , $\pm(x^{4t-4}-x^{4t-4t+2i+2-4})$

i.e. $\pm(x^{2i}-x^0)$, $\pm(x^{2i+2}-x^2)$, … , $\pm x^{4t-2i-4}(x^{2i}-x^0)$; $\pm x^{4t-2i-2}(x^{2i}-x^0)$,

$\pm x^2 (x^{4t-2i-2}-x^0)$, …, $\pm x^{4t-4}(x^0-x^{-4t+2i+2})$

i.e. $\pm x^{q_i}$, $\pm x^{q_{i+2}}$, $\pm x^{q_{i+4}}$, … , $\pm x^{q_{i+4t-2i-4}}$; $\pm x^{q_{i+4t-2i-2}}$, $\pm x^{q_{i+4t-2i}}$, … , $\pm x^{q_{i+4t-4}}$ …. (2.3)

All the positive terms from (2.3)

x^{q_i}, $x^{q_{i+2}}$, $x^{q_{i+4}}$, … , $x^{q_{i+4t-2i-4}}$; $x^{q_{i+4t-2i-2}}$, $x^{q_{i+4t-2i}}$, … , $x^{q_{i+4t-4}}$ … (2.4)

And all the negative terms can be exhibited from (2.3)

(-1) x^{q_i}, (-1) $x^{q_{i+2}}$, …, (-1) $x^{q_{i+4t-2i-4}}$; (-1) $x^{q_{i+4t-2i-2}}$, (-1) $x^{q_{i+4t-2i}}$, … , (-1) $x^{q_{i+4t-4}}$.

i.e. $x^{q_{i+2t-1}}$, $x^{q_{i+2t+1}}$, …, $x^{q_{i+6t-2i-5}}$; $x^{q_{i+6t-2i-3}}$, $x^{q_{i+6t-2i-1}}$,…, $x^{q_{i+6t-5}}$ … (2.5)

Similarly all the possible differences arising from (2.2) can be written as follows.

$\pm \mathbf{x}^{q_{i+1}}$, $\pm \mathbf{x}^{q_{i+3}}$, … , $\pm \mathbf{x}^{q_{i+4t-2i-3}}$; $\pm \mathbf{x}^{q_{i+4t-2i-1}}$, $\pm x^{q_{i+4t-2i+1}}$, … , $\pm x^{q_{i+4t-3}}$, … (2.6)

All the positive terms from (2.6)

$\mathbf{x}^{q_{i+1}}, \mathbf{x}^{q_{i+3}}, \ldots, \mathbf{x}^{q_{i+4t-2i-3}}$; $\mathbf{x}^{q_{i+4t-2i-1}}$, $x^{q_{i+4t-2i+1}}, \ldots, x^{q_{i+4t-3}}$, ... (2.7)

and all the negative terms can be exhibited as follows.

$(-1)\,\mathbf{x}^{q_{i+1}}, (-1)\,\mathbf{x}^{q_{i+3}}, \ldots, (-1)\,\mathbf{x}^{q_{i+4t-2i-3}}$; $(-1)\,\mathbf{x}^{q_{i+4t-2i-1}}, (-1)\,x^{q_{i+4t-2i+1}}, \ldots, (-1)\,x^{q_{i+4t-3}}$,

i.e. $x^{q_{i+2t}}$, $x^{q_{i+2t+2}}, \ldots, x^{q_{i+6t-2i-4}}$; $x^{q_{i+6t-2i-2}}$, $x^{q_{i+6t+2i}}, \ldots, x^{q_{i+6t-4}}$, ... (2.8)

From (2.3) and (2.6), it is learnt that every non-zero elements of GF(4t-1) occurs exactly twice in (2.3) and occurs exactly twice in (2.6) as i=1, 2, ... , t-1, among the totality of all possible differences arisen from C_1 and C_2 every non-zero elements of GF(4t-1) exactly occurs 2(t-1) times.

Let $x^p = x^2 - x^0$, for some p, the forward differences arisen from the C_1 and the C_2 are given as follows.

F.D.=$(x^2 - x^0), (x^4 - x^2), \ldots, (x^{4t-4} - x^{4t-6}), (x^0 - x^{4t-4})$.

i.e. $x^p, x^{p+2}, \ldots, x^{p+2t-6}, x^{p+4t-4}$. ... (2.9)

Similarly the forward differences arisen from C_2

we get, $x^{p+1}, x^{p+3}, x^{p+5}, \ldots, x^{p+4t-5}, x^{p+4t-3}$ ... (2.10)

Combining the forward differences arisen from the C_1 and the C_2 by using (2.9) and (2.10) can be exhibited as follows,

$x^p, x^{p+1}, x^{p+2}, x^{p+3}, \ldots, x^{p+4t-6}, x^{p+4t-5}, x^{p+4t-4}, x^{p+4t-3}$ ...(2.11)

From (2.11), we clearly seen that the forward differences are the power of primitive element x, which are increasing from p to p+4t-3. All the 2t-2 differences are nothing but all 2t-2 non-zero elements of GF (4t-1). Thus among the totality of the forward or backward differences arising out of the C_1 and the C_2 every non-zero elements of GF(4t-1), occurs exactly once. Thus λ=1, developing the initial blocks C_1 and C_2 under the reduction modulo of GF(4t-1), the Universally Optimal One Sided Circular Neighbour Balanced design given in the following theorem can be constructed.

Theorem: 2.1. Developing the (2.1) and the (2.2) Under mod (v), where x is a primitive element of GF(v); v(=4t-1, prime for some t) a construction of Universally Optimal One Sided Circular Neighbour Balanced design with parameters v=4t-1, b=2v, r=2(2t-1), k=2t-1=s=n_1, m=1, n_2=0, μ=2(t-1), λ=1 is always guarantee.

Proof: Obviously, all the (2t-1) elements in C_1 and (2t-1) elements in C_2 are distinct. Hence, k=s=2t-1=n_1, m=1, n_2=0.

Example: 2.1. An illustrative example for a construction of Universally Optimal One Sided Circular Neighbour Balanced design is made here below with parameters v=4t-1, b=2v, r=2(2t-1), k=2t-1=s=n_1, m=1, n_2=0, μ=2(t-1), λ =1, when taking t=3, since the primitive element of 11 is 2, from the C_1 and the C_2, we get,

$C_1=\{x^0, x^2, x^4, \ldots, x^{4t-4}\}=\{2^0, 2^2, 2^4, \ldots, 2^8\}=\{1, 4, 5, 9, 3\}$

$C_2=\{x^1, x^3, x^5, \dots, x^{4t-3}\}=\{2, 2^3, 2^5, \dots, 2^9\}=\{2, 8, 10, 7, 6\}$, as i=1, 2, ... , (t-1).

Developing C_1 and C_2 under the reduction modulo of 11, a solution of an Universally Optimal One Sided Circular Neighbour Balanced design with the parameters becomes, v=11, b=22, r=10, $k=5=s=n_1$, $n_2=0$, m=1, μ=4, λ=1.

(1, 4, 5, 9, 3), (2, 5, 6, 10, 4), (3, 6, 7, 0, 5), (4, 7, 8, 1, 6)

(5, 8, 9, 2, 7), (6, 9, 10, 3, 8), (7, 10, 0, 4, 9), (8, 0, 1, 5, 10),

(9, 1, 2, 6, 0), (10, 2, 3, 7, 1), (0, 3 , 4, 8, 2), (2, 8, 10, 7, 6)

(3, 9, 0, 8, 7), (4, 10, 1, 9, 8), (5, 0, 2, 10, 9), (6, 1, 3, 0, 10) ,

(7, 2, 4, 1, 0), (8, 3, 5, 2, 1), (9, 4, 6, 3, 2), (10, 5, 7, 4, 3),

(0, 6, 8, 5, 4), (1, 7, 9, 6, 5).

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