

Atlas of Fixed Links and Equivalent Links in Mechanisms of 6 Links, 7 Joints, 1F Kinematic Chains Using [JJ] Matrix

Dr. Ali Hasan
Mechanical Engineering Department,
Jamia Millia Islamia, New Delhi.

ABSTRACT-Author's objective is to develop a new, easy, reliable, and efficient method to detect isomorphism and prepare a catalogue of fixed link and its corresponding equivalent links in the distinct mechanisms in kinematic chains of Group B. It will help the new researchers / designers to select the best mechanism kinematic chain and mechanism to perform the desired task at the conceptual stage of design. The proposed method is presented by comparing the structural invariants 'sum of the absolute values of the characteristic polynomial coefficients' [SCPC] and 'maximum absolute value of the characteristic polynomial coefficient' [MCPC] of [JJ] matrices. These invariants may be used to detect isomorphism in the mechanism kinematic chain having simple joints. The method is explained with the help of examples of planar kinematic chain having simple joints.

KEY WORDS: Kinematic Chain; Fixed Link; Equivalent Link; SCPC; MCPC

NOTATIONS USED: F.L.-Fixed Link, E.L.-Equivalent Link, n_5 n_4 n_3 n_2 -Pentagonal, Quaternary, Ternary, Binary Links

1. INTRODUCTION

Over the past several years much work has been reported in the literature on the structural synthesis of kinematic chains and mechanisms. Undetected isomorphism results in duplicate solutions and unnecessary effort. Therefore, the need for a reliable and efficient algebraic method for this purpose is necessary. Identifying isomorphism among kinematic chains using characteristic polynomials of adjacency matrices of corresponding kinematic chains are simple methods [Uicher and Raicu 1975, Mruthyunjaya and Raghavan 1979, Yan and Hall, 1981]. But the reliability of these methods was in questions as several counter examples were found by Mruthyunjaya [Mruthyunjaya, 1987]. The test proposed by Mruthyunjaya [Mruthyunjaya, 1987] is based on characteristic coefficients of the 'Degree matrix' of the graph of the kinematic chains. The elements of the degree matrix were sum of the degree of vertices (degree or type of links) or unity in a link-link adjacency matrix. Later on this test was also found unreliable. Krishnamurthy [Mruthyunjaya, 1987] proposed the representation polynomial for detecting isomorphism between two kinematic chains. The representation polynomial is the determinant of the generalized adjacency matrix, called representation matrix of the kinematic chain. But the representation matrix requires the use of a large number of symbols, the calculation and comparison of the representation polynomials is not as easy as that of the characteristic coefficients of the adjacency matrix. Balasubramanian and Parthasarthy [Balasubramanian and Parthasarthy 1981] proposed the procedure based on the concept of the permanent of a matrix for the purpose. Tang and Liu [Tang, Liu, Tyng, 1993] presented a method based on degree code as mechanism identifier. Several other methods like comments [Cubillo and Wan, Jinbao, 2005], [Hasan A., 2007, 2009, 2010, 2012] are also in use.

II. The Joint-Joint [JJ] Matrix

This matrix is based upon the connectivity of the joints through the links and defined, as a square symmetric matrix of size $n \times n$, where n is the number of joints in a kinematic chain.

$$[JJ] = \begin{Bmatrix} L_{ij} \end{Bmatrix}_{n \times n} \quad \text{-----} \quad (1)$$

Where

$$L_{ij} \begin{cases} = \text{Degree of link between } i^{\text{th}} \text{ and } j^{\text{th}} \text{ joints} \\ \text{those are directly connected} \end{cases}$$

Off course all the diagonal elements $L_{ii} = 0$

III. CHARACTERISTIC POLYNOMIAL OF [JJ] MATRIX

$D(\lambda)$ gives the characteristic polynomial of [JJ] matrix. The monic polynomial of degree n is given by equation (2).

$$|(JJ - \lambda I)| = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \text{-----} + a_{n-1}\lambda + a_n \quad \text{-----} \quad (2)$$

Where; n = number of simple joints in kinematic chain and

$1, a_1, a_2, a_{n-1}, a_n$ are the characteristic polynomial coefficients.

The two important properties of the characteristic polynomials are

(a) The sum of the absolute values of the characteristic polynomial coefficients (SCPC) is an invariant for a [JJ] matrix. i.e.

$$|1| + |a_1| + |a_2| + \text{-----} + |a_{n-1}| + |a_n| = \text{invariant}$$

(b) The maximum absolute value of the characteristic polynomial coefficient (MCPC) is another invariant for a [JJ] matrix.

IV. STRUCTURAL INVARIANTS [SCPC] AND [MCPC]

The values of characteristic polynomial coefficients are invariants for a [JJ] matrix. To make these [JJ] matrix characteristic polynomial coefficients as a powerful single number characteristic index, new composite invariants have been proposed. These invariants are ‘SCPC’ and ‘MCPC’. These invariants are unique for a [JJ] matrix and may be used as identification numbers to detect the isomorphism among simple jointed kinematic chains. The characteristic polynomial coefficients values are the characteristic invariants for the kinematic chains. Many investigators have reported co-spectral graph (non-isomorphic graph having same Eigen spectrum). But these Eigen spectra (Eigen values or characteristic polynomial) have been determined from (0, 1) adjacency matrices. The proposed [JJ] matrix provides distinct set of characteristic polynomial coefficients of the kinematic chains having co-spectral graphs. Therefore, it is verified that the structural invariants ‘SCPC’ and ‘MCPC’ are capable of

characterizing all kinematic chains and mechanisms uniquely. Hence, it is possible to detect isomorphism among all the given kinematic chains.

V. ISOMORPHISM OF KINEMATIC CHAINS.

Theorem: Two similar square symmetric matrices have the same characteristic polynomials.

Proof:

Let the two kinematic chains are represented by the two similar matrices A and B such that $B = P^{-1}AP$, taking into account that the matrix λI commutes with the matrix P and $|P^{-1}| = |P|^{-1}$. Since the determinant of the product of two square matrices equals the product of their determinants, we have

$$\begin{aligned} |B - \lambda I| &= |P^{-1}AP - \lambda I| \\ &= |P^{-1}(A - \lambda I)P| \\ &= |P^{-1}| |A - \lambda I| |P| = |A - \lambda I| \end{aligned}$$

Hence, D (λ) of 'A' matrix = D (λ) of 'B' matrix.

D (λ) = characteristic polynomial of the matrix.

It means that if D (λ) of two [JJ] matrices representing two kinematic chains is same, their structural invariants 'SCPC' and 'MCPC' will also be same and the two kinematic chains are isomorphic otherwise non-isomorphic chains.

VI. ILLUSTRATIVE EXAMPLE

The example concerns with 6-bars, 7-joints, 1-degree of freedom (dof) Stephenson's chain as shown in Figure 1

[JJ] Matrix

The [JJ] matrix representing the kinematic chain [Figure 1] using equation (1) is represented as:

$$JS = \begin{pmatrix} 0 & 3 & 0 & 0 & 2 & 0 & 3 \\ 3 & 0 & 2 & 0 & 0 & 0 & 3 \\ 0 & 2 & 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 3 & 0 & 2 & 3 & 0 \end{pmatrix}$$

Structural invariants of kinematic chain

The set of structural invariants derived from the [JJ] matrix using software MAT LAB for the kinematic chain shown in Figure 1 are:

$$\text{SCPC (JS)} = 9.7560\text{e}+003$$

$$\text{MCPC (JS)} = 3.6000\text{e}+003$$

Structural invariants of the mechanism

The structural invariants of the mechanisms derived from the kinematic chain shown in Figure 1 obtained from [JJ-a], [JJ-b], [JJ-c], etc are

$$\text{SCPC (JSa)} = 12280$$

$$\text{SCPC (JSb)} = 7.1960\text{e}+003$$

$$\text{SCPC (JSc)} = 8.0320\text{e}+003$$

$$\text{SCPC (JSd)} = 7.1960\text{e}+003$$

$$\text{SCPC (JSe)} = 1.2280\text{e}+004$$

$$\text{SCPC (JSf)} = 8.0320\text{e}+003$$

$$\text{MCPC (JSa)} = 4.8500\text{e}+003$$

$$\text{MCPC (JSb)} = 3.1920\text{e}+003$$

$$\text{MCPC (JSc)} = 3.1110\text{e}+003$$

$$\text{MCPC (JSd)} = 3.1920\text{e}+003$$

$$\text{MCPC (JSe)} = 4.8500\text{e}+003$$

$$\text{MCPC (JSf)} = 3.1110\text{e}+003$$

Identification of the distinct mechanisms

Observing the structural invariants for the above six mechanisms, it is found that the structural invariants of link 'a' and 'e' are same. Hence, they are treated as equivalent links and form only one distinct mechanism. Similarly, the structural invariants of link 'b' and 'd' are same, hence form second distinct mechanisms. In the similar way link 'c' and 'f' form third distinct mechanisms.

Therefore, 3 distinct mechanisms are obtained from kinematic chain shown in Figure 1

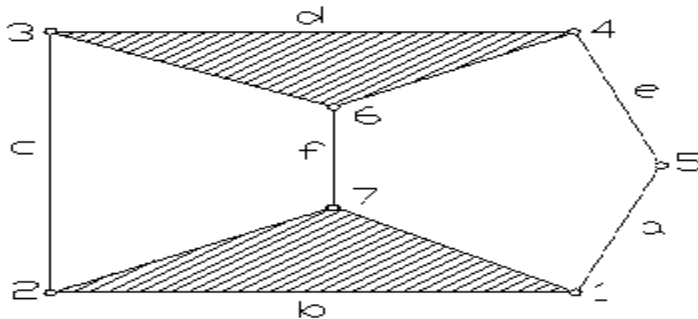


Figure 1: Six bar, Single dof, Stephenson chain

VII. RESULTS

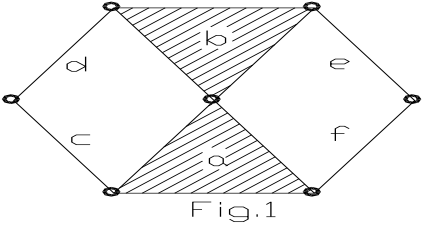
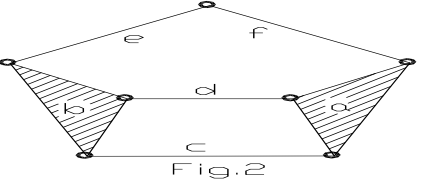
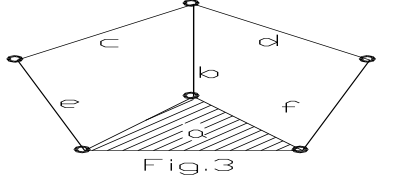
The proposed invariants [SCPC] and [MCPC] are used as the identification number of the kinematic chains having simple joints. The identification numbers of all 1-dof kinematic chains up to 10-Links are with the author. These invariants are also able to detect isomorphism among the kinematic chains with multiple joints also. Fixed Links and Equivalent Links in Distinct Mechanisms of 6 Links, 7 Joints, Single degree of freedom Kinematic Chains are listed in Table 1.

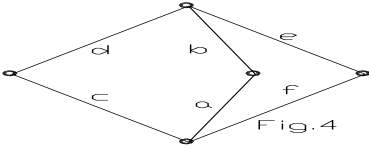
VIII. CONCLUSIONS

In this paper, a simple, efficient, and reliable method to identify isomorphism is proposed. By this method, the isomorphism of mechanisms kinematic chains can easily be identified. It incorporates all features of the kinematic chains and as such, violation of the isomorphism test is rather difficult. The method has been found to be successful in distinguishing all known 16 kinematic chain of 8-links, 230 kinematic chain of 10-links having 1-F. The advantage is that they are very easy to compute using MATLAB software. It is not essential to determine both the structural invariants to compare two chains, only in case the [SCPC] is same then it is needed to determine [MCPC] for both kinematic chains. The [JJ] matrices can be written with very little effort, even by mere inspection

of the chain. The proposed test is quite general in nature and can be used to detect isomorphism of not only planar kinematic chains of one degree of freedom, but also kinematic chains of multi degree of freedom.

Table-1: Distinct mechanisms of 6 link, 7 Joints, 1F Kinematic Chains

Kinematic Chain	$n_5 n_4 n_3 n_2$	F. L.	E. I.	D. M.
 <p>Fig.1</p> <p>SCPC(JW) = 5.2200e+003 MCPC(JW) = 3.1680e+003</p>	0024	a c	b d,e,f	2
 <p>Fig.2</p> <p>SCPC(JS) = 9.7560e+003 MCPC(JS) = 3.6000e+003</p>	0024	a c e	b d f	3
 <p>Fig.3</p> <p>SCPC(F3) = 1.2500e+003 MCPC(F3) = 528.0000</p>	0015	a b c e	- - d f	4

 <p>Fig. 4</p> <p>SCPC(F4) = 49.0000</p> <p>MCPC(F4) = 48.0000</p>	0006	a	b,c,d,e,f	1
TOTAL DISTINCT MECHANISMS OF 1-F, 6-LINK, 7-JOINT CHAINS = 10				

REFERENCES

- [Agrawal and Rao,1987] Agrawal V.P. and Rao J.S., “The Mobility Properties of Kinematic Chains”, Mech. Mach. Theory, Vol. 22, pp. 497-504, 1987.
- [Agrawal and Rao,1987] Agrawal V.P. and Rao J.S., “Structural Classification of Kinematic Chains and Mechanisms”, Mech. Mach. Theory, Vol. 22, pp. 489-496, 1987.
- [Ambeker, 1987(i)] Ambeker A. G. and Agrawal V. P., “Canonical Numbering of Kinematic Chains and Isomorphism Problem: Min. Code”, Mech. Mach theory, Vol. 22, No 4, pp 453-461, 1987.
- [Ambeker, 1987(ii)] Ambeker A. G. and Agrawal V. P., “Identification of Kinematic Chains, Mechanisms, Path Generators and Function Generators Using Min. Codes”, Mech. Mach theory, Vol. 22, No 4, pp 463-471, 1987.
- [Erdman and Sandor, 1988] Erdman A. G. and Sandor G. N., “Advanced Mechanism Design”, Vol. I and II, Prentice Hall of India, 1988.
- [Ghosh and Mallik, 1988] Amitabha Ghosh and Ashok Kumar Mallik, “Theory of Mechanisms and Machines”, East – West Press Pvt. Ltd., New Delhi, pp. II Edition 1988
- [Gibson and Marsh,1989] Gibson C.G. and Marsh D., “On the Linkage Varieties of Watt 6-Bar Mechanisms-I. Basic Geometry” Mech. Mach. Theory, Vol. 24, pp. 105-113, 1989.
- [Hain, 1967] Kurt Hain, “Applied Kinematics” McGraw-Hill, New York, 1967.
- [Hartenberg and Denavit, 1964] Hartenberg S. and Denavit J., “Kinematic Synthesis of Linkages” McGraw-Hill, New York, 1964.
- [Hasan A.,2009] “ Identification of Isomorphism among Kinematic Chains and Inversions Using Link Adjacency Values”, International J. of Mech. and Materials Engineering (IJMME), pp.309-315, No.3, Vol. 4(2009)
- [Hasan A.,2010] “ Application of Link Adjacency Values to Detect Isomorphism among Kinematic Chains”, Int. J. Mech. Mater. Design, ”, 6,157-162,2010
- [Hasan A.,2010] “ Isomorphism Identification of Compound Kinematic Chain and Their Mechanism”, i-manager’s Journal on Mechanical Engineering, Vol. 2 , No. 1, pp 7-15, Nov. 2011 – Jan. 2012. .
- [Hasan A., 2007] “ Systematic Development of Kinematic Chains and Mechanisms from a given Assortment of Links” , Journal of ‘Institution of Engineers (India), Vol. 88, pp.15-19, 2007

- 12.[Hasan A., 2007]“ Isomorphism in Kinematic Chains Using Path Matrix”,Journal of Institution of Professional Engineers (IPENZ), New Zealand,2007
- 13.[Hasan A.,2009] “ Isomorphism and Inversions of Kinematic Chains up to 10 Links ”,Journal of ‘Institution of Engineers (India), Vol. 90, pp.10-14, 2009
- 14.[**Hunt, 1978**] Hunt K.H., “Kinematic Geometry of Mechanisms” Oxford Engineering Science Series, 1978.
- 15.[**Hwang and Hwang, 1992**] Wen – Min Hwang and Yii – Wen Hwang, “Computer Aided Structural Synthesis of Planar Kinematic Chains with Simple Joints”, Mech. Mach. Theory, Vol. 27, pp. 189 – 199, 1992
16. [**Mohd,1998**] Mohd A. “Kinematic chains structural modelling and correlation with machine performance”, M.Tech. Dissertation, IIT Delhi,1998
- 17.[**Mruthyunjaya, 1984(i)**] Mruthyunjaya T. S., “A Computerized Methodology for Structural Synthesis of Kinematic Chains, Part – 1, Formulation”, Mech. Mach. Theory, Vol. 19, No. 6, pp. 487 – 495, 1984
- 18.[**Mruthyunjaya, 1984(ii)**] Mruthyunjaya T. S., “A Computerized Methodology for Structural Synthesis of Kinematic Chains, Part – 2, Application to several fully or Partial Known Cases”, Mech. Mach. Theory, Vol. 19, No. 6, pp. 497 – 505, 1984
- 19.[**Mruthyunjaya and Balasubramaniam, 1987**] Mruthyunjaya T.S. and Balasubramaniam H.R., “In Quest of Reliable and Efficient Computational Test for Detection of Isomorphism in Kinematic Chains”, Mech. Mach theory, Vol. 22, No 4, pp 131-139, 1987.
- 20.[**Mruthyunjaya and Raghvan, 1979**] Mruthyunjaya T. S. and Raghvan M. R., “Structural Analysis of Kinematic Chains and Mechanisms based on Matrix representation”, Transaction of the ASME, Journal of Mechanical Design, Vol. 101, pp. 488 – 518, 1979.
- 21.[**Preben W. Jensen, 1991**] Preben W. Jensen, “Classical and Modern Mechanism for Engineers and Inventors”, Marcel Decker, Inc, New York (1991)
- 22.[**Raicu, 1974**] Raicu A., “Matrices Associated with Kinematic chains from 3 to 5 Members”, Mech. Mach. Theory, Vol. 9, pp. 123 – 129, 1974.
- 23.[**Rao, 1989**] Rao A.C, “Selection of Ground, Input and Output Link in Mechanisms, pp. A Quantitative Approach”, Trans, CSME, Vol. 13, No. 1/2, pp 23-30, 1989.
- 24.[**Rao, 1997**] Rao A. C., “Hamming Number Technique – 2, Generation of n planer kinematic Chains’, Mech. Mach. Theory, Vol. 32, No. 4, pp. 489-499, 1997.
25. [**Rao, 2000**] Rao A. C., “Application of Fuzzy Logic for the Study of Isomorphism, Inversions, Symmetry, Parallelism and Mobility in Kinematic Chains”, Mech. Mach. Theory, Vol. 35, pp. 1103-1116, 2000.
- 26.[**Rao,et.al.,2006**] Rao a.b.s., Srinath A. and Rao A.C., “Synthesis of Planar Kinematic Chains”, IE (I), vol.86, pp 195-201, 2006.
- 27.[**Rao and Rao, 1993(i)**] Rao A. C. and Rao C. N, “Loop Based Pseudo Hamming Value-I, Testing Isomorphism And Rating Kinematic Chain.”, Mech. Mach. Theory, Vol. 28, No. 1, pp. 113- 127, 1993.
- 28.[**Rao and Rao, 1993(ii)**] Rao A. C. and Rao C. N, “Loop Based Pseudo Hamming Value-II, Inversions, Preferred Frames and Actuators”, Mech. Mach. Theory, Vol. 28, No. 1, pp. 129- 143, 1993.
- 29.[**Rao et al, 2001**] Rao A. C., Pratap and B. Deshmukh, “Computer Aided Structural Synthesis of Planer Kinematic Chains Obviating the Test of Isomorphism”, Mech. Mach. Theory, Vol. 36, pp. 489-506, 2001.
- 30.[**Tuttle,1975**] Tuttle E.R, “Generation of Planar Kinematic Chains”, Mech. Mach. Theory, Vol. 31, pp. 729 – 748, 1996.
-

- 31.[Uicker and Raicu, 1975]** Uicker J. J. and Raicu A., “A method for the Identification and Recognition of Equivalence of Kinematic Chains”, Mech. Mach. Theory, Vol. 10, pp. 375 – 383, 1975.
- 32. [Yadav, 1996(i)]** Yadav J. N. , Pratap C. R.and Agrawal V. P., “Mechanisms of a kinematic chain and degree of structural similarity based on the concept of link – path code”, Mech. Mach. Theory, Vol. 31, pp. 865 – 871, 1996.
- 33. [Yan and Hall, 1982]** Yan H. S. and Hall A. S., “Linkage Characteristic Polynomials: Assembly Theorems, Uniqueness” ASME, Journal of Mechanical Design, Vol. 104, pp. 11-20, 1982.
- 34.[Yan and Hwang, 1991]** Yan and Hwang, “Specialization of Mechanisms”, Mech. Mach. Theory, Vol. 26, No. 6, pp. 341-351, 1991.
- 34. [Zhang and Li, 1999]** Zhang W. J. and Li Q., “On A New Approach to Mechanism Topology Identification” ASME, Journal of Mechanical Design, Vol. 121, pp. 57-64, 1999.