EFFECT OF BOUNDARY CONDITIONS AND STIFFENERS ON THE NATURAL FREQUENCIES OF RECTANGULAR PLATE

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ABSTRACT

The present work is aimed at vibration analysis of the rectangular plate with angle shaped stiffeners. The rectangular thin plate is considered and its natural frequencies are simulated by modal analysis using FEA software. In all the previous work the plate with rib stiffeners and beams having rectangular cross section was considered, here the analysis is performed for the rectangular plates with angle shaped stiffeners. The rigid coupling condition is assumed between the plate and stiffeners. The modal analysis is performed for different thickness and angles of stiffeners. Also two sets of boundary conditions are considered, All edges free (FFFF) and all edges clamped (CCCC).

Keywords: *Rectangular plate, Stiffeners, Boundary conditions, Dynamic analysis, FEA.*

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1. INTRODUCTION

The structures are subjected to dynamic loads in actual service which are periodic in nature, consequently the structures vibrate. If the operating frequency meets the natural frequency of the system the resonance occurs and it fails irrespective of its strength. Vibration also creates detrimental effect on relevant structures, human being and creates annoying noise. It is not possible to completely avoid such vibration, so design for vibration involves keeping operating frequency away from natural frequencies of the structure and minimizing displacement, velocity and acceleration of vibrating structure within permissible limits. One of the ways is the structural modification by tuning the stiffness, mass and damping of the structure.

Plates reinforced by stiffeners represent a class of structural components that are widely used in many applications such as ship decks, bridges, Automobile super structure, Aircraft, industrial structures and buildings etc. Addition of stiffener significantly affects the dynamic characteristics of the plate hence it is of direct interest in structural designs.

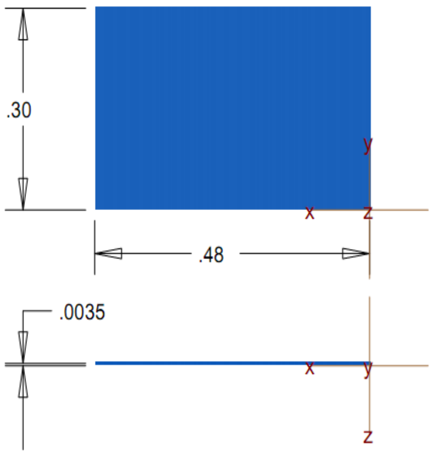
The free vibration analysis of the rectangular plate and plates with other geometry is well documented by Leissa W [1]. In his work the effect of plate's various boundary conditions on the modal parameters is studied and equations of natural frequency and mode shapes are given. The effect of Poisson's ratio and aspect ratio of the plate on natural frequencies was studied by Rossi *et al.* [2]. Manzanares B.*et al.*[3] carried out theoretical and experimental studies on the transverse vibration of rectangular plate with all edges free. They developed experimental technique using electromagnetic-acoustic transducer which is highly reliable for the lower normal modes.

Vibrations of stiffened plates have been extensively studied using various analytical and numerical techniques, as comprehensively reviewed in Refs. [4-5]. Luan Yu *et al.*[6] developed an improved smearing technique for modeling the vibration of cross stiffened thin rectangular plate. The smearing technique given by Szilard has inadequate accuracy of predicted natural frequencies whereas the improved smearing technique gives considerable accuracy comparatively. Hongan Xu *et al.* [7] presented an analytical method for the vibration analysis of plates reinforced by beams of arbitrary lengths and placement angles. The effects of various support conditions, general coupling conditions, and reinforcing arrangements with respect to the number, orientations, and lengths of attached beams on modal properties of combined system was studied. Dozio L.*et al.* [8] proposed analytical–numerical method for quick prediction of the modal characteristics of rectangular ribbed plates. The approach is suitable for low-frequency free vibration analysis of thin rectangular plates reinforced by a small number of light stiffeners. Zeng H.*et al.* [9] studied the vibration of rectangular stiffened plate by Differential Quadrature Method. The plate and the stiffeners are treated separately and their compatibility is modeled by their governing differential equations. Chen Z.*et al.*[10] studied the vibration localization in plates rib-stiffened in two orthogonal directions to see the effect of small misplacements of stiffeners, with various combinations of flexural and torsional rigidities, on both the free vibration and forced vibration responses of the plate. Holopainen T.P.*et al.* [11] proposed a new finite element model for free vibration analysis of eccentrically stiffened plates; using this model the mesh distortion does not have any significant effect on the accuracy of the results.

Although vibrations of stiffened plates have been extensively studied for decades, in most of the research work the rectangular plates with rib stiffeners and beams having rectangular cross section were considered. There is little attention paid to vibrations of rectangular plate with stiffeners having other cross sectional geometry. In the present work the analysis is performed for the rectangular plates with angle shaped stiffeners.

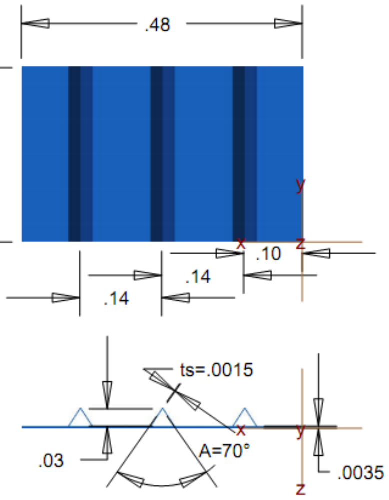
2. MODELING OF PLATES

A thin rectangular plate is considered as shown in figure 2.1 for the free vibration analysis of rectangular plate. The plate is assumed to follow the Classical Thin Plate Theory also known as the Kirchoff’s Plate Theory which is fundamental theory to model thin plates. The plate has length along x direction, width along y direction and thickness along z direction, hence the plate is assumed to lie in the x-y plane. The plate has three independent displacements i.e. displacement along x, y and z directions. The rectangular plate with three angle shaped stiffeners is as shown in figure 2.2. The stiffeners are lying on the plate such that length of the stiffeners is parallel to the width of the plate. The stiffeners and the plates are assumed to have rigid coupling at their connecting interface. The stiffener has an angle A and thickness ts, both Angle of stiffener (A) and thickness of stiffener (ts) are considered as variables.



Note: All dimensions are in m.

Figure 2.1 Bare rectangular plate.



Note: All dimensions are in m.

Figure 2.2 Rectangular plate with angle shaped stiffener.

The plates which are considered for modal analysis are tabulated as shown in table 2.1.

Table 2.1 Details of plates.

Plate No.	Base dimensions X × Y × Z (mm)	Angle of stiffener A (degree)	Thickness of stiffener ts (mm)	Material
1	480 × 300 × 3.5	Bare plate		Steel
2		70	1.5	
3		70	2	
4		70	3	
5		90	1.5	
6		90	2	
7		90	3	
8		120	1.5	
9		120	2	
10		120	3	

It is assumed that the plate and the stiffeners are made of the same material. The bare plate and the stiffened plates have following material characteristics;

Material: Steel.

Modulus of Elasticity: 2×10^{11} N/m^2.

Poisson's ratio: 0.3.

Density: 7800 kg/ m^3.

The finite element models of all the plates are generated in ANSYS software using the Solid45 element. This element is used for meshing the solid geometry; the element has eight nodes and three degree of freedom per each node. The meshed models are then used for FEA in ANSYS software. The modal analysis is performed for different boundary conditions. In the first set of boundary conditions all the edges of the plate are considered to be free, this set of boundary conditions is FFFF. In the second set of boundary conditions all the edges of the plate are considered to be clamped, this set of boundary conditions is CCCC.

3. RESULTS AND DISCUSSIONS

The modal analysis is performed on the bare plate and on the stiffened plates having all edges free. The results of first ten modes and corresponding natural frequencies for all the ten plates having all edges free (FFFF) are compared and shown in table 3.1.

The comparison of natural frequencies of the stiffened plates with 70°, 90° and 120° angle of stiffener and all edges free (FFFF) is shown in figure 3.1, it can be observed that plate with 120° angle of stiffener gives highest value of natural frequency for almost all modes.

The natural frequencies of the stiffened plates with 70° angle and 1.5, 2, 3 mm thick stiffeners having all edges free (FFFF) is shown in figure 3.2, comparing the results it can be observed that all the three plates gives highest values of natural frequencies in different modes. Similar trend is observed in stiffened plate with 90° angle and 1.5, 2, 3 mm thick stiffeners having all edges free (FFFF) as shown in figure 3.3. In case of stiffened plate of 120° angle and 1.5, 2, 3 mm thick stiffener and all edges free (FFFF) as shown in figure 3.4, the stiffened plate with 3 mm thick stiffener gives highest value of natural frequencies.

Table 3.1 Comparison of Natural frequencies of all plates by FEA (FFFF)

ModE	Natural frequency (Hz.)									
	Plate 1	Plate 2	Plate 3	Plate 4	Plate 5	Plate 6	Plate 7	Plate 8	Plate 9	Plate 10
	Bare plate	A= 70 ts=1.5	A= 70 ts=2	A=70 ts=3	A= 90 ts=1.5	A=90 ts=2	A=90 ts=3	A=120 ts=1.5	A=120 ts=2	A=120 ts=3
1	79.428	90.429	89.793	88.258	98.379	97.515	95.462	87.928	93.855	105.19
2	79.716	178.82	179.87	178.86	218.78	220.2	219.05	239.83	251.37	272.92
3	182.2	248.2	246.7	243.4	264.9	262.8	258.4	337.7	348.4	364.1

	9	5	1		1		6	4	9	9
4	210.85	364.79	366.33	362.91	443.3	445.85	442.09	537.12	579.96	656.74
5	220.85	546.95	540.35	524.33	669.95	664.16	645.78	543.3	605.94	778.84
6	272.59	574.68	579.38	581.5	681.14	688.06	692.14	764.17	816.43	935.64
7	336.33	642.55	637.25	625.34	756.86	752.81	740.44	765.69	818.3	941.92
8	402.08	696.86	708.98	722.49	796.09	812.28	830.28	795.12	845.57	993.79
9	460.29	697.54	709.71	723.34	797.11	813.37	831.57	1129.9	1169.9	1231.9
10	554.51	820.76	812.42	796.09	911.63	904.61	888.91	1141	1177.1	1236

It is observed that with addition of stiffener of any size the natural frequencies of the plate increases from that of the plate without stiffener.

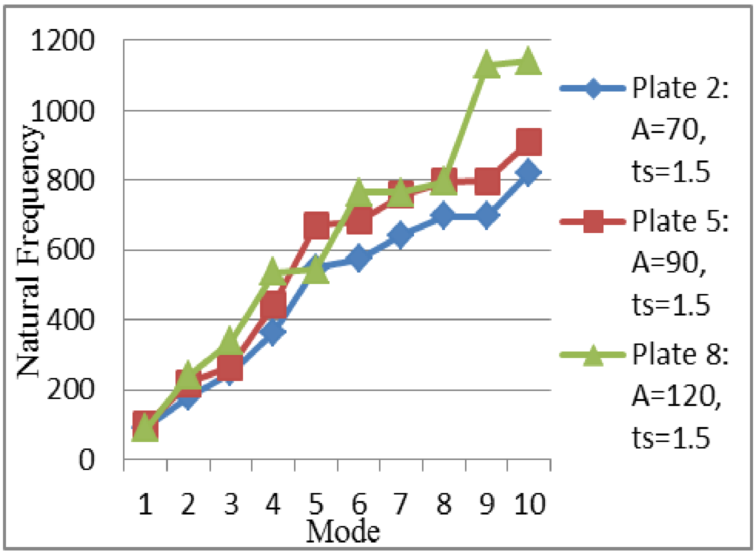


Figure 3.1 Comparison of natural frequency of plate 2, 5 and 8. (FFFF).

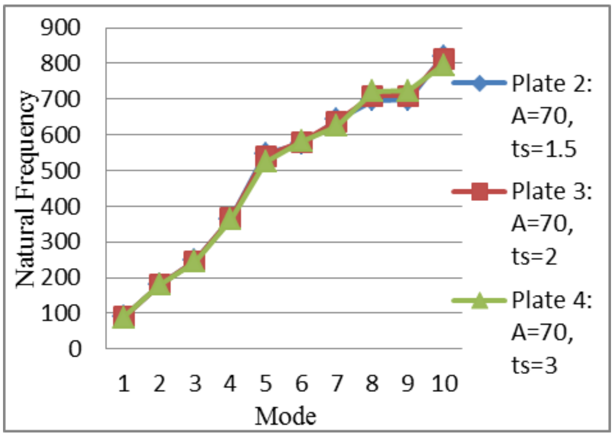


Figure 3.2 Comparison of natural frequency of plate 2, 3 and 4 FFFF).

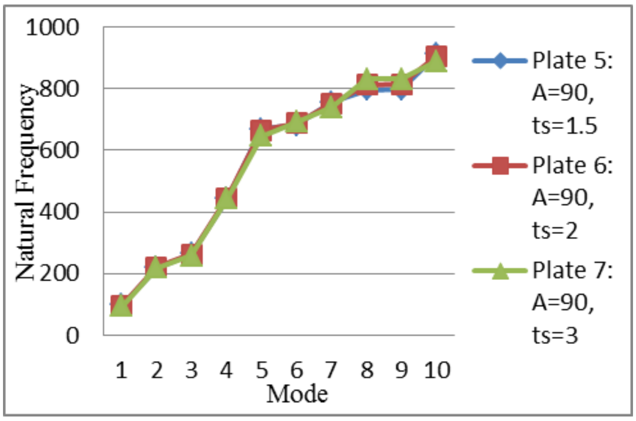


Figure 3.3 Comparison of natural frequency of plate 5, 6 and 7 (FFFF).

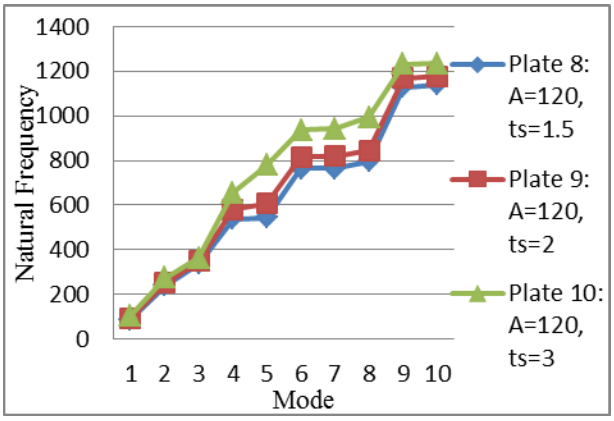


Figure 3.4 Comparison of natural frequency of plate 8, 9 and 10 (FFFF).

Similarly the modal analysis is performed on the bare plate and on the stiffened plates having all edges clamped. The results of first ten modes and corresponding natural frequencies for all the ten plates having all edges clamped (CCCC) are compared and shown in table 3.2.

Table 3.2 Comparison of Natural frequencies of all plates by FEA (CCCC)

Mode	Natural frequency (Hz.)									
	Plate 1	Plate 2	Plate 3	Plate 4	Plate 5	Plate 6	Plate 7	Plate 8	Plate 9	Plate 10
	Bare plate	A= 70 ts=1.5	A= 70 ts=2	A= 70 ts=3	A= 90 ts=1.5	A=90 ts=2	A=90 ts=3	A=120 ts=1.5	A=120 ts=2	A=120 ts=3
1	252.82	742.67	792.76	855.19	784.49	838.55	906.47	750.75	804.56	888.3
2	379.55	864.66	911.2	966.93	920.04	968.12	1025.5	790.68	848.64	960.86
3	601.53	1022	1060.3	1101.1	1076.1	1114.1	1154.4	796.25	864.89	1017.5
4	637.48	1511.2	1572.5	1648.4	1766.8	1862.9	1987.8	1491.9	1599	1815.9
5	765.29	1855.4	1945.1	2051.2	2095.5	2228.5	2389.2	1525.6	1628.6	1834
6	922.91	1936.5	2009.4	2097.7	2226.1	2325.3	2448.7	1557.1	1656.6	1851.1
7	987.68	1963.9	2027	2099.4	2245	2373.6	2528.7	1741.8	1866.4	2070.6
8	1235	2553	2630.9	2723.3	2578	2736.6	2920.3	1843.6	1947.1	2151.9
9	1313.9	2569.4	2702.3	2815.7	2727	2831.4	2963.2	2011.6	2080.7	2251.6
10	1353.7	2644.1	2750.4	2849.1	2874.2	3002.2	3154.6	2241.8	2357.3	2564

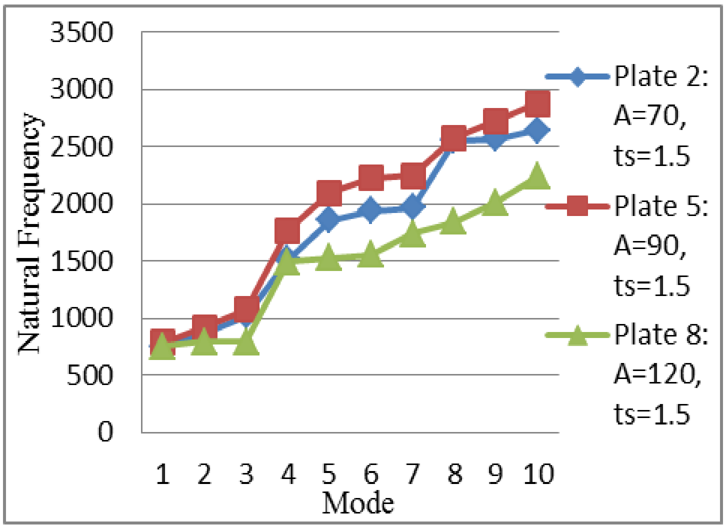


Figure 3.5 Comparison of natural frequency of plate 2, 5 and 8 (CCCC).

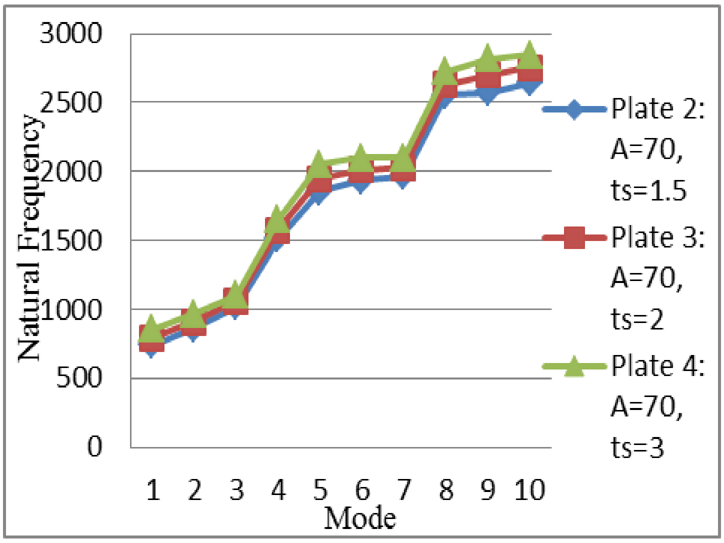


Figure 3.6 Comparison of natural frequency of plate 2, 3 and 4 (CCCC).

Similar to the plate with free boundary conditions, it is observed that with addition of stiffener of any size the natural frequencies of the plate increases from that of the plate without stiffener.

The natural frequencies of the stiffened plates with 70°, 90° and 120° angle of stiffener and all edges clamped (CCCC) are compared as shown in figure 3.5, it can be observed that plate with 90° angle shaped stiffener gives highest value of natural frequencies in all modes.

The natural frequencies of the stiffened plates with 70° angle and 1.5, 2, 3 mm thick stiffener and all edges free (CCCC) is shown in figure 3.6, it can be observed that the plate with 3 mm thick stiffener gives highest value of natural frequencies in all modes. Similar trend can be observed in 90° and 120° angle stiffened plates.

4. CONCLUSION

The vibration analysis of the bare plate and stiffened plates is performed. Results of the analysis give the following information regarding vibration of plates.

1. Addition of the stiffener to the plate has an effect of shifting natural frequencies towards higher side.
2. The plate with all edges clamped gives higher natural frequencies than the plate with all edges free for all the cases.
3. The increasing thickness of the stiffener increases the natural frequencies of the plate for all the boundary conditions.
4. The increasing angle of stiffener does not mean increasing value of the natural frequency, out of three angle shaped stiffener 70°, 90°, 120°; the 90° angle shaped stiffener gives the highest value in natural frequency for clamped boundary condition but for free boundary condition that’s not true.

Further investigation is going on by suitably designing the experimentations for the validation.

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