Free Vibration Analysis of Skew Plate using Meshfree Method

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Abstract- The governing differential equations (GDEs) of skew plate utilizing first order shear deformation theory (FSDT) are derived using energy approach. The GDEs discretized by polynomial radial basis functions are used to predict the free vibration behavior of isotropic and orthotropic skew plates. Effect of orthotropic and span to thickness ratio on frequency parameter of simply supported skew plate is presented. Numerical results obtained are in good agreement with other published results. Some new results are also obtained.

Keywords: Skew plate, FSDT, Orthotropic, Meshfree, vibration.

INTRODUCTION

Plates are one of the important load carrying two dimensional structural elements being used in various high performance engineering structures. Depending on the boundaries, a plate may be polygonal, quadrilateral, triangular, circular, elliptic etc. Skew plates are among the important members of the quadrilateral family that are used in aerospace, naval, automotive and other engineering structures. Barton (1951) used Rayleigh-Ritz method and characteristic beam function to obtain the frequency parameters, nodal patterns and mode shape amplitude coefficients of cantilever skew plates. Conway and Farnham (1965) adopted the point matching method for the free flexural vibration of isotropic simply supported parallelogram plates and clamped rhombic plates. Ferreira et al. (2005), use the FSDT in the multiquadric radial basis function (MQRBF) procedure for predicting the free vibration behavior. Hasegawa (1957) proposed polynomial deflection functions in conjunction with Rayleigh-Ritz method for the vibration analysis of clamped skew plates. The excellent literature surveys (Leissa, 1969; Liew and Wang, 1993) and (Wang et al., 1992) show a great insight into the history and development of the analytical and numerical techniques for free vibration and buckling of skew plates, respectively. The exact solution for free vibration of simply supported isotropic skew plate was given by Seth (1947). Liew et. al. (2003, 2004) used reproducing kernel approximations and meshfree method for buckling analysis of isotropic circular and skew plates. In the present study, the free vibration analysis of skew plates using polynomial radial basis function and FSDT is presented.

MATHEMATICAL FORMULATION

The displacement field at any point in the plate is expressed as ignoring initial displacements in X and Y direction:

$$u = z\phi_x$$
$$v = z\phi_y \qquad (1)$$
$$w = w_0$$

The skew plate geometry is shown in Figure 1. Thickness h is along z axis whose mid plane is coinciding with x-y plane of the coordinate system is considered.

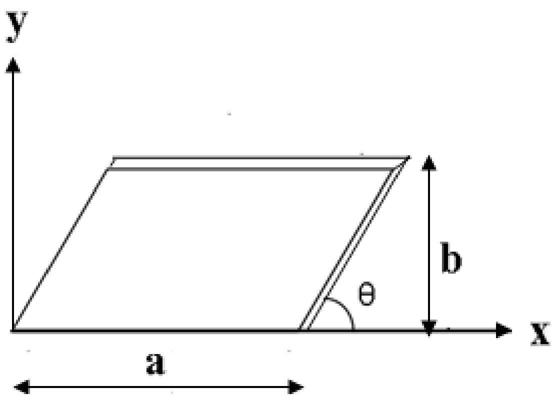

Figure1: Geometry of skew plate

The strain-displacement relations can be written as:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} z\dfrac{\partial \phi_x}{\partial x} \\ z\dfrac{\partial \phi_y}{\partial y} \\ z\dfrac{\partial \phi_x}{\partial y} + z\dfrac{\partial \phi_y}{\partial x} \end{Bmatrix} \qquad (2)$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} \phi_y + \dfrac{\partial w_0}{\partial y} \\ \phi_x + \dfrac{\partial w_0}{\partial x} \end{Bmatrix} \qquad (3)$$

The constitutive stress strain relation can be written as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 & 0 & 0 \\ 0 & 0 & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & 0 \\ 0 & 0 & 0 & 0 & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} \qquad (4)$$

Where, for isotropic plate

$$\bar{Q}_{11} = \bar{Q}_{22} = \frac{E}{\left(1-\nu^2\right)}, \quad \bar{Q}_{12} = \nu \bar{Q}_{11}, \ \bar{Q}_{44} = \bar{Q}_{55} = \bar{Q}_{66} = G$$

and for orthotropic plate

$$Q_{11} = \frac{E_{11}}{\left(1-\nu_{12}\nu_{21}\right)}, \ Q_{22} = \frac{E_{22}}{\left(1-\nu_{12}\nu_{21}\right)}, \ Q_{12} = \frac{\nu_{21}E_{11}}{\left(1-\nu_{12}\nu_{21}\right)},$$
$$Q_{44} = G_{23}, \quad Q_{55} = G_{13}, \quad Q_{66} = G_{12} \qquad (5)$$

The governing differential equations of plate are obtained using Hamilton's principle and expressed as:

$$\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_2 \frac{\partial^2 \phi_x}{\partial t^2}$$
$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_{yy}}{\partial y} - Q_y = I_2 \frac{\partial^2 \phi_y}{\partial t^2} \qquad (6)$$
$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = I_0 \frac{\partial^2 w}{\partial t^2}$$

Where,

$$M_{xx} = D_{11}\frac{\partial \phi_x}{\partial x} + D_{12}\frac{\partial \phi_y}{\partial y}$$

$$M_{yy} = D_{12}\frac{\partial \phi_x}{\partial x} + D_{22}\frac{\partial \phi_y}{\partial y} \qquad (7)$$

$$M_{xy} = D_{16}\frac{\partial \phi_x}{\partial x} + D_{26}\frac{\partial \phi_y}{\partial y}$$

$$Q_x, Q_y = \int_{-h/2}^{h/2} \left(\sigma_{xz}, \sigma_{yz}\right) dz$$

$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = K \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial w_0}{\partial y} + \phi_y \\ \dfrac{\partial w_0}{\partial x} + \phi_x \end{Bmatrix} \qquad (8)$$

$$Aij, D_{ij} = \bar{Q}_{ij} \int_{-h/2}^{h/2} \left(1, z^2\right) dz \qquad (9)$$

$$I_0, I_2 = \rho \int_{-h/2}^{h/2} \left(1, z^2\right) dz \qquad (10)$$

K=shear correction factor taken here as 5/6.

The boundary conditions for an arbitrary edge with simply supported conditions are as follows:

$$\phi^s, w, M_{nn} = 0 \quad (11)$$

Where,

$$\phi^s = -n_y.\phi^x + n_x.\phi^y$$

$$M_{nn} = n_x^2 \mathrm{M_{xx}} + 2\mathrm{n_x n_y M_{xy}} + \mathrm{n}_y^2 \mathrm{M_{yy}}$$

$$n_x = \cos(\theta), \quad n_y = \sin(\theta)$$

The Governing Differential Equation in form of displacement variables (assuming $\bar{Q}_{16} = \bar{Q}_{26} = 0$) can be written as:

$$\left(D_{11}\frac{\partial^2 \phi_x}{\partial x^2} + D_{66}\frac{\partial^2 \phi_x}{\partial y^2} - KA_{55}\left(\phi_x + \frac{\partial w_0}{\partial x} \right) \right) + \left(D_{12}\frac{\partial^2 \phi_y}{\partial x \partial y} + D_{22}\frac{\partial^2 \phi_y}{\partial y^2} + D_{66}\frac{\partial^2 \phi_y}{\partial x \partial y} \right) = I_2 \frac{\partial^2 \phi_x}{\partial t^2} \quad (12)$$

$$\left(D_{66}\frac{\partial^2 \phi_x}{\partial x \partial y} + D_{12}\frac{\partial^2 \phi_x}{\partial x \partial y} \right) + 2D_{26}\frac{\partial^2 \phi_y}{\partial x \partial y} + D_{66}\frac{\partial^2 \phi_y}{\partial x^2} + D_{22}\frac{\partial^2 \phi_y}{\partial y^2} - KA_{44}\left(\phi_y + \frac{\partial w_0}{\partial y} \right) = I_2 \frac{\partial^2 \phi_y}{\partial t^2} \quad (13)$$

$$K\left(A_{55}\frac{\partial \phi_x}{\partial x} + A_{44}\frac{\partial \phi_y}{\partial y} + A_{55}\frac{\partial^2 w_0}{\partial x^2} + A_{44}\frac{\partial^2 w_0}{\partial y^2} \right) = I_0 \frac{\partial^2 w_0}{\partial t^2} \quad (14)$$

SOLUTION METHODOLOGY

The governing differential equations (11-14) are expressed in terms of displacement functions. Radial basis function based formulation works on the principle of interpolation of scattered data over entire domain. The unknown field variables w_0, ϕ_x and ϕ_y appearing in governing differential equations is assumed in terms of radial basis function as:

$$w_0(x, y) = \sum_{j=1}^{N} \alpha_j^{w_0} g\left(\left\| X - X_j \right\|, m \right)$$

$$\phi_x(x, y) = \sum_{j=1}^{N} \alpha_j^{\varphi_x} g\left(\left\| X - X_j \right\|, m \right)$$

$$\phi_y(x, y) = \sum_{j=1}^{N} \alpha_j^{\varphi_y} g\left(\left\| X - X_j \right\|, m \right)$$

For free vibration problems, harmonic solution in terms of displacements is assumed as:

$$w_0(x, y, t) = W_0(x, y)e^{i\omega t}$$

$$\phi_x(x, y, t) = \phi\phi_x(x, y)e^{i\omega t}$$

$$\phi_y(x, y, t) = \phi\phi_y(x, y)e^{i\omega t}$$

Where, N is total numbers of nodes which is equal to summation of boundary nodes NB and domain interior nodes ND. $g\left(\left\| X - X_j \right\|, m \right)$ is polynomial radial basis function expressed as $g = r^m$, $\delta = \alpha_j^w, \alpha_j^{\phi_x}, \alpha_j^{\phi_y}$ are unknown coefficients. $\left\| X - X_j \right\|$ is the radial distance between two nodes.

Where, $r = \left\| X - X_j \right\| = \sqrt{\left(x - x_j \right)^2 + \left(y - y_j \right)^2}$ and m is shape parameter. The value of 'm' taken here is 5. Polynomial radial basis function becomes singular, when r = 0 i.e. for zero distance. In order to eliminate the singularity, an infinitesimally small value is added into the r2 or zero distance. Mathematically it is explained as; $r^2 = r^2 + \mu^2$ when r = 0 or i = j; μ^2 is small numerical value of the order 10-10.

The discretized governing differential equation is expressed as eigenvalue for predicting the natural frequency.

$$\left([\mathrm{K}] - \omega^2 [\mathrm{M}] \right)\{\delta\} = \{0\}$$

Where [K] is the stiffness matrix and [M] refers to mass matrix, and the parameter refers to the frequency parameter.

NUMERICAL RESULTS AND DISCUSSIONS

A RBF based meshless code in MATLAB is developed following the analysis procedure as discussed above. In order to demonstrate the accuracy and applicability of present formulation, several examples have been analyzed and the computed results are compared with the published results. Based on convergence study, a 15×15 node is used throughout the study. The material properties of plate have been taken as follows:

E=1, ν=0.3. for isotropic plate.

E1/E2 = open; G12 = G13= 0.6×E2 ; G23=0.5×E2 ; ν12=0.25 and density ρ =1, for orthotropic plate.

The dimensionless natural frequency parameter is defined as:

$$\Omega = \omega\left(\frac{a^2}{\pi^2}\right)\left(\frac{\rho h}{D_{11}}\right)^{(1/2)}$$

Table 1: Frequency parameter of simply supported isotopic skew plate for first eight modes (a/h = 5)

Angle	Modes	Present	Wang[9]	Liew[13]
90	1	1.7748	1.7661	1.7661
	2	3.8811	3.8657	3.858
	3	3.8811	3.8657	3.858
	4	5.6244	5.5823	5.5737
	5	6.6204	7.3131	6.582
	6	6.6224	7.3131	6.582
	7	8.0338	8.5873	7.9485
	8	8.0338	8.5873	7.9485
75	1	1.903		
60	1	2.2513	2.1864	2.1719
45	1	2.9079	2.9755	2.9129
30	1	4.5425		

In order to show the accuracy and efficiency of the present solution methodology, detailed convergence studies for simply supported isotropic skew plate (a/h=5) is carried out. The plate edge length and thickness are denoted by ‘a’ and ‘h’, respectively. The convergences of the frequency parameters for all 8 modes are shown in Fig. 2&3. It can be seen that good convergence (within 1%) is achieved at 15×15 nodes. Table-1 shows that present results are in good agreement with results due to Wang [9] and Liew et al [13].

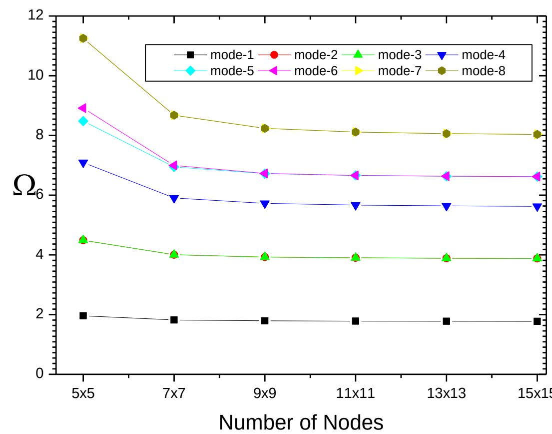

Figure 2: Convergence study for all 8 modes of simply supported isotropic square plate (a/h = 5)

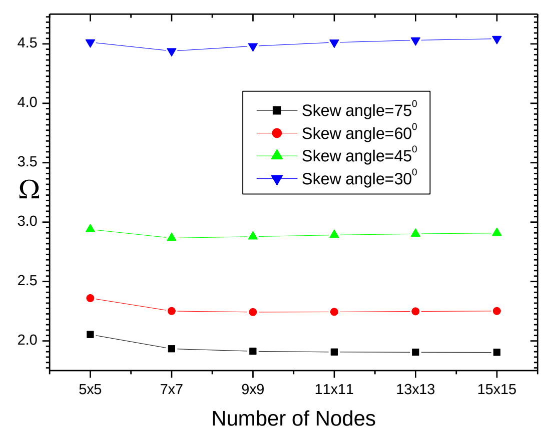

Figure 3: Convergence of frequency parameter for different skew angle of simply supported isotropic skew plate (a/h = 5)

Table 2: Frequency parameter of simply supported isotopic skew plate with skew angle

	Skew Angle				
a/h	90	75	60	45	30
10	1.936	2.1547	2.7096	3.7173	6.398
20	1.9718	2.2579	2.9442	4.161	7.5636
30	1.9631	2.2824	3.0225	4.3095	7.9439
40	1.9473	2.2919	3.068	4.395	8.1436
50	1.9299	2.297	3.102	4.4595	8.2843
100	1.8489	2.3125	3.221	4.7012	8.8034

Table 3: Frequency parameter of simply supported skew plate with skew angle (a/h = 100)

	Skew Angle				
E1/E2	90	75	60	45	30
3	1.3992	1.8838	2.5722	3.555	6.144
5	1.2442	1.6467	2.2501	3.0489	5.0246
10	1.1203	1.4205	1.9162	2.5252	3.8879
20	1.0587	1.2712	1.661	2.1309	3.0884
30	1.039	1.209	1.537	1.9385	2.7328
40	1.0293	1.1736	1.4699	1.8127	2.5174

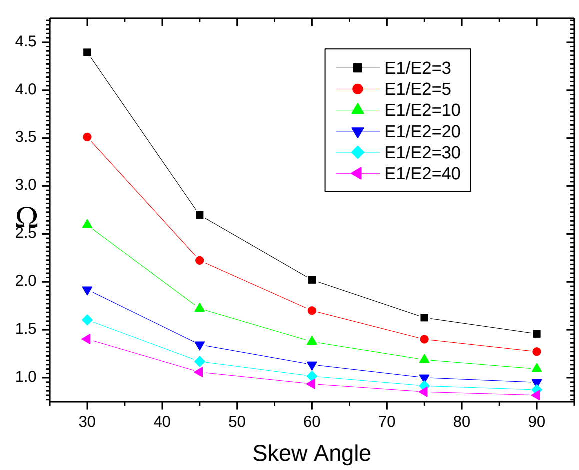

Figure 4: Effect of orthotropic ration on frequency parameter for different skew angle of simply supported skew plate (a/h = 10)

Table-2 and Table-3 shows the effect of span to thickness ratio and orthotropic ratio respectively with skew angle of a simply supported skew plate. Figure-4 presents that as skew angle decreases the frequency parameter increases and becomes more prominent for higher skew angles. It must be noted that skew angle 90 means rectangular plate.

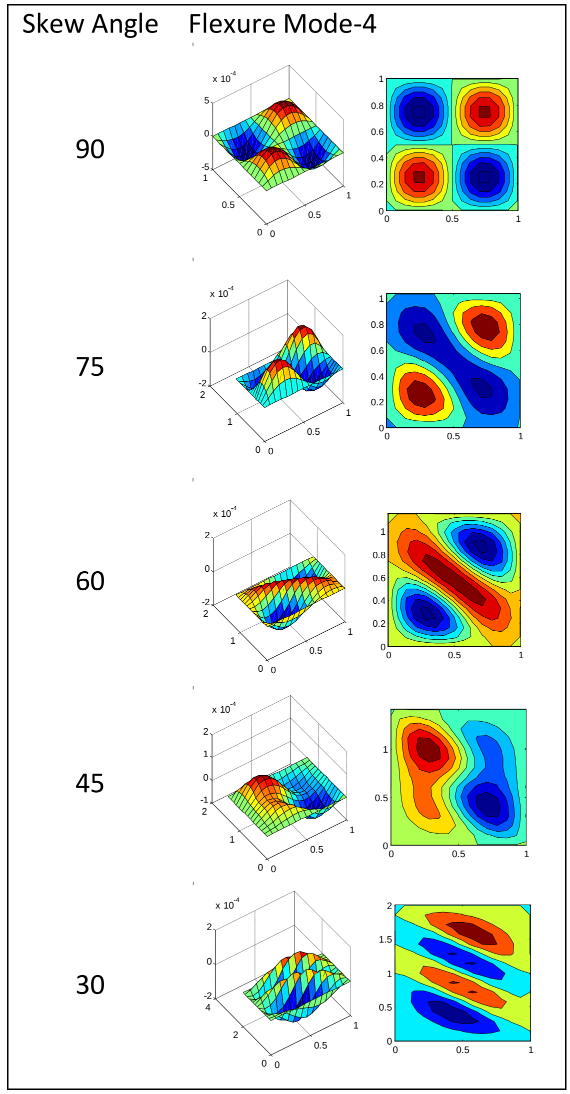

Figure 5: Flexure modes (mode-4) of simply supported skew plate with skew angle (a/h = 5).

CONCLUSIONS

Polynomial radial basis functions are used to predict the free vibration behavior of isotropic and orthotropic skew plates. Effect of orthotropic and span to thickness ratio on frequency parameter of simply supported skew plate is presented. Numerical results obtained are in good agreement with other published results. Some new results are also obtained.

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