FLEXURAL ANALYSIS OF FGM PLATE UNDER UNIFORM LINE LOAD BY USING MESHLESS METHOD.

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ABSTRACT: In present paper, functionally graded material plate under the uniformly distributed line load is considered for flexural analysis. The governing differential equation of the plate is obtained using energy principle. Multiquadric radial basic function based meshless method is applied for discreatization of the equation. The coding is developed in MATLAB to find out the results. Effect of gradation index, span to thickness ratio on deflection moments and stresses are carried out under uniform distributed line load .The present result are converge, new and may be used for validation in future.

Keywords: FGM, plates, Meshless method, MQ RBF, bending, uniform distributed line load.

Introduction

Composite materials play an important role in composite structure. Among all the composite materials functionally graded material (FGM) plates are the most emerging due to their material property character in one or more directions. Studies involving the bending response of FGMs structures have received the main attention for many researches in last few years. FGMs the non homogenous materials, which are commonly made of metal and ceramic, possess two main properties strength and thermal resistance. Over the last few decades many researchers lighted on the flexural response of functionally graded materials (FGM) plates by applying different plate theories and many solving techniques. The radial based functions (RBFs) were first used by Hardy [4] for the interpolation of geographical scattered data and then Kansa [6] applied for the solution of solutions to parabolic, hyperbolic and elliptic partial differential equations. (PDEs). Bo et al. [1] introduced the elasticity solutions for the static analysis of FG plates for different boundary conditions. Fasshauer, G. E [2] carried partial differential equations by collocation (RBFs) and multiresolution methods. Vaghefi et al. [7] introduced a three-dimensional static solution for thick FG plates by utilising a meshless Petrov– Galerkin method. Ferreira [3] carried the analysis of laminated composite plates using multiquadric radial basis function. In the present paper, the flexural response of functionally graded plates using multiquadric radial basis function is presented. Functionally graded plates with simply supported boundary conditions and uniformaly distributed line load subjected to uniform transverse pressure are analyzed.

Mathematical formulation

A rectangular shape plate of edge length a, b along x, y axes respectively and thickness h is the thickness along z axis whose mid plane is coinciding with x-y plane of the coordinate system is considered. The diagram of rectangular shaped functionally graded material (FGM) plate in rectangular coordinate system is shown in Figure 1.

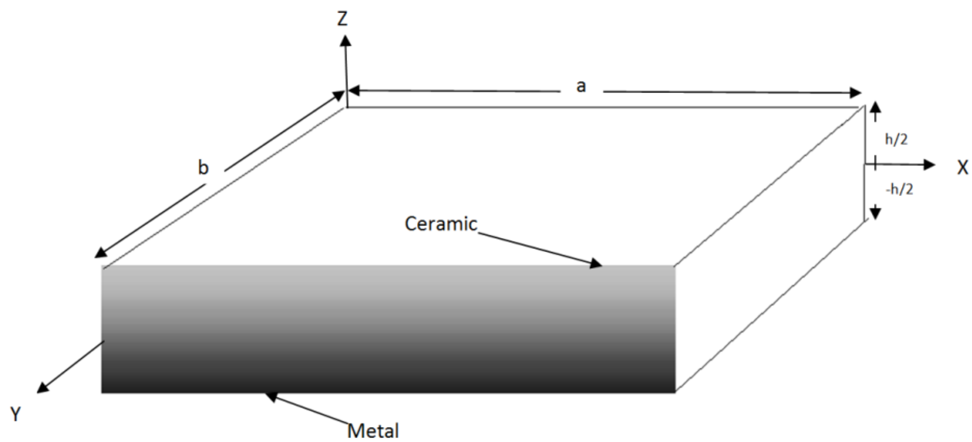

Fig 1. Geometry of rectangular FGM plate in rectangular coordinate system

The homogenization technique considered in this work is the law of mixtures, which provides the following elastic properties at each material layer. The top surface of the plate is ceramic rich and the bottom surface is metal rich.

$$V_c(z) = \left(\frac{2z+h}{2h}\right)^n \quad (1)$$

Where ‘n’ is exponent governing the material properties along the thickness direction known as volume fraction exponent or grading index,

The volume fraction of the metal phase is obtained by

$$V_m(z) = 1 - V_c(z) \quad (2)$$

The material property gradation through the thickness of the plate is assumed to have the following form,

$$E(z) = \left[E_c - E_m\right]\left(\frac{2z+h}{2h}\right)^n + E_m \quad (3)$$

Here *E* denote the modulus of elasticity of FGM structure, while these parameters come with subscript m or c represent the material properties for pure metal and pure ceramic plate respectively., h is the thickness of the plate, E_m and E_c are the corresponding Young’s modulus of elasticity of metal and ceramic and z is the thickness coordinate.The displacement field at any point in the plate made up of uniform thickness is expressed as:

$$\begin{Bmatrix} U_x \\ U_y \\ U_z \end{Bmatrix} = \begin{Bmatrix} u_x(x,y) - z\dfrac{\partial u_z(x,y)}{\partial x} + \sin(\dfrac{\pi z}{h})\psi_x(x,y) \\ u_y(x,y) - z\dfrac{\partial u_z(x,y)}{\partial y} + \sin(\dfrac{\pi z}{h})\psi_y(x,y) \\ u_z(x,y) \end{Bmatrix} \quad (4)$$

The constitutive stress-strain relations for any FGM plate are expressed as:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{zx} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 & 0 & 0 \\ 0 & 0 & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & 0 \\ 0 & 0 & 0 & 0 & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

Where, the parameters Q_{ij} are the stiffness coefficients and are expressed in terms of elastics constants as:

$$\bar{Q}_{11} = \bar{Q}_{22} = \frac{E}{\left(1-\nu^2\right)}, \quad \bar{Q}_{12} = \frac{\nu E}{\left(1-\nu^2\right)}, \bar{Q}_{44} = \bar{Q}_{55} = \bar{Q}_{66} = G$$

The governing differential equations of plate are obtained using energy equation, in mathematical form it is expressed as:

$$\int_{t_1}^{t_2} \delta\left(U+V\right)dt = 0 \quad (5)$$

Where,, U = Strain energy

V = workdone due to transverse load

The strain energy of the plate due to internal stress resultants is expressed as:

$$U = \frac{1}{2}\int_{Volume} \left(\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{xy}\gamma_{xy} + \sigma_{yz}\gamma_{yz} + \sigma_{xz}\gamma_{xz}\right) dxdydz \quad (6)$$

$$V = \int_{Area} u_z q_z dxdy \quad (7)$$

The governing differential equations of plate are obtained using Hamilton's principle and expressed as :

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} = 0$$

$$\frac{\partial^2 \mathrm{M}_{xx}}{\partial \mathrm{x}^2} + \frac{\partial^2 \mathrm{M}_{yy}}{\partial \mathrm{y}^2} + 2\frac{\partial^2 \mathrm{M}_{xy}}{\partial \mathrm{x}\partial \mathrm{y}} + \mathrm{q}_z = 0$$

$$\frac{\partial M_{xx}^f}{\partial x} + \frac{\partial M_{xy}^f}{\partial y} - Q_x^f = 0 \qquad (8)$$

$$\frac{\partial M_{xy}^f}{\partial x} + \frac{\partial M_{yy}^f}{\partial y} - Q_y^f = 0$$

The force and moment resultants in the plate and plate stiffness coefficients are expressed as:

$$\mathrm{N_{ij}},\, \mathrm{M_{ij}},\, M_{ij}^f = \int_{-h/2}^{+h/2} (\sigma_{ij},\, z\sigma_{ij},\, z.\left[\sin(\frac{\pi z}{h})\right]\sigma_{ij})\,dz \qquad (9)$$

$$\mathrm{Q_x^f},\, \mathrm{Q_y^f} = \int_{-h/2}^{+h/2} \left(\sigma_{xz},\, \sigma_{yz}\right)\left(\frac{\partial\left[\sin(\frac{\pi z}{h})\right]}{\partial z}\right)dz \qquad (10)$$

$$A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij} = \int_{-h/2}^{h/2}\left\{Q(z)\times\left(1, z, z^2, z.\left[\sin(\frac{\pi z}{h})\right], z\times z.\left[\sin(\frac{\pi z}{h})\right], z.\left[\sin(\frac{\pi z}{h})\right]^2\right)\right\}dz \qquad (11)$$

i, j = 1, 2, 6

$$A_{ij} = \int_{-h/2}^{h/2}\left\{Q(z)\times\left(\frac{\partial\left[\frac{z}{4} - \frac{5}{3h^2}z^3\right]}{\partial z}\right)^2\right\}dz$$

(12)

i, j = 4, 5

$$where, \quad Q(z) = \left(\left(\left[Q_{ij}^c - Q_{ij}^m\right]\right)\left(\frac{2z+h}{2h}\right)^n + Q_{ij}^m\right)$$

The boundary conditions for an arbitrary edge with simply supported conditions are as follows:

$$x = 0, a : u_y = 0; \psi_y = 0; u_z = 0; M_{xx} = 0; N_{xx} = 0$$
$$y = 0, b : u_x = 0; \psi_x = 0; u_z = 0; M_{yy} = 0; N_{yy} = 0$$

1. SOLUTION METHODOLOGY

The governing differential equations (8) are expressed in terms of displacement functions. Radial basis function based formulation works on the principle of interpolation of scattered data over entire domain. A 2D rectangular domain having NB boundary nodes and ND interior nodes is shown in Figure-2.

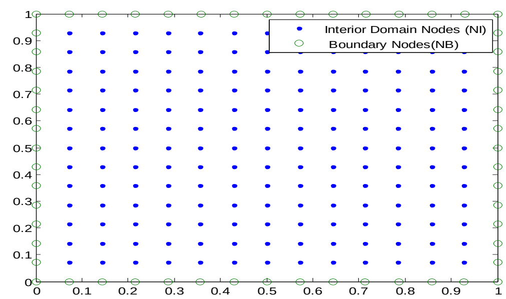

Fig 2. An arbitrary two dimensional domains

The variable $u_x, u_y, u_z, \psi_x, \psi_y$ can be interpolated in form of radial distance between nodes. The solution of the linear governing differential equations (8) is assumed in terms of multiquadric radial basis function for nodes 1:N, as;

$$u_x, u_y, u_z, \psi_x, \psi_y = \sum_{j=1}^{N} (\alpha_j^{u_x}, \alpha_j^{u_y}, \alpha_j^{u_z}, \alpha_j^{\psi_x}, \alpha_j^{\psi_y}) \; g\left(\left\| X - X_j \right\|, m, c\right)$$

Where, N is total numbers of nodes which is equal to summation of boundary nodes NB and domain interior nodes ND. $g\left(\left\| X - X_j \right\|, m, c\right)$ is multiquadric radial basis function expressed as $g = \left(r^2 + c^2\right)^m$, $(\alpha_j^{u_x}, \alpha_j^{u_y}, \alpha_j^{u_z}, \alpha_j^{\psi_x}, \alpha_j^{\psi_y})$ are unknown coefficients. $\left\| X - X_j \right\|$ is the radial distance between two nodes. Where, $r = \left\| X - X_j \right\| = \sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2}$ and m ,c are shape parameter. The value of 'm' and 'c' taken here is 0.5 and 1.3/(N)0.25.

2. COMPUTATION AND DISCUSSION OF RESULTS:

The study here has been focused on the flexural response of simply supported square functionally graded plates under line transverse loads. A RBF based meshless code in MATLAB 2013 is developed. Several examples have been analyzed and the computed results are compared. Based on convergence study, a 15×15 node is used throughout the study. The material properties of FGMs have been taken as follows:

Ceramic $E_c = 151 GPa$, $\nu_c = 0.3$ Aluminum (Al) $E_m = 70 GPa$, $\nu_m = 0.3$

In order to show the accuracy and efficiency of the present solution methodology, detailed convergence studies for simply supported FGM plate (a/h=20) is carried out . The convergences of the deflection are shown in Fig. 3. It can be seen that convergence achieved is within 1 % at 15×15 nodes.

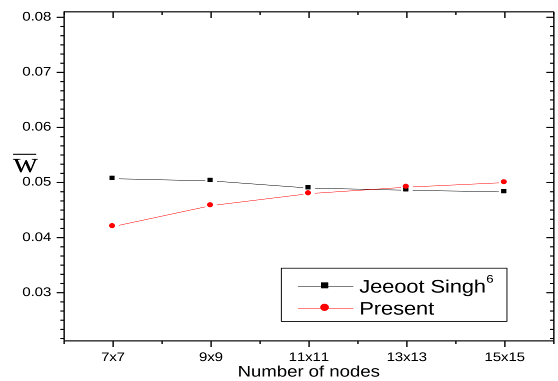

Fig. 3 Convergence study for deflection $\overline{w}$ of a simply supported FGM plate (a/h = 20, 'n'=2)

Table1 Effect of span to thickness ratio on deflection, stresses and Moments of a simply supported FGM Plate (n=2)

	a/h						
	5	10	20	30	40	50	100
$\overline{w}$	0.0620	0.0522	0.0500	0.0496	0.0495	0.0494	0.0493
$\overline{\sigma}_{xx}$	10.3148	4.8041	2.3592	1.5677	1.1745	0.9391	0.4692
$\overline{\sigma}_{yy}$	15.0758	6.6755	3.2235	2.1348	1.5974	1.2765	0.6373
$\overline{\sigma}_{xy} \times 10^4$	4.5930	2.2171	1.0922	0.7246	0.5423	0.4334	0.2164
$\overline{\sigma}_{xz}$	2.9088	0.6860	0.1732	0.1556	0.1982	0.2180	0.2446
M_{xx}	0.1232	0.0619	0.0311	0.0207	0.0156	0.0124	0.0062
M_{yy}	0.1652	0.0832	0.0417	0.0278	0.0209	0.0167	0.0084
M_{xy}	0.0633	0.0291	0.0142	0.0094	0.0071	0.0057	0.0029
M_{xx}^f	0.1232	0.0619	0.0311	0.0207	0.0156	0.0124	0.0062
M_{yy}^f	0.1652	0.0832	0.0417	0.0278	0.0209	0.0167	0.0084
M_{xy}^f	0.0633	0.0291	0.0142	0.0094	0.0071	0.0057	0.0029

Table2 Effect of gradation index 'n' on deflection, stresses and Moments of a simply supported FGM Plate(a/h=5)

	'n'								
	0	0.25	0.5	0.75	1	2	5	10	Metal
$\overline{w}$	0.0395	0.0453	0.0499	0.0533	0.0559	0.0620	0.0684	0.0727	0.0851
$\overline{\sigma}_{xx}$	7.5528	8.3124	8.8691	9.2767	9.5821	10.3148	11.4451	12.5812	16.2849
$\overline{\sigma}_{yy}$	10.9942	12.1044	12.9181	13.5167	13.9689	15.0758	16.7618	18.3721	23.7046
$\overline{\sigma}_{xy}$	3.3955	3.7317	3.9761	4.1530	4.2840	4.5930	5.0919	5.6272	7.3211
$\overline{\sigma}_{xz}$	2.9003	2.9279	2.9523	2.9578	2.9513	2.9088	2.7392	2.7980	2.9000
M_{xx}	0.0834	0.0959	0.1043	0.1100	0.1141	0.1232	0.1354	0.1468	0.1799
M_{yy}	0.1123	0.1289	0.1402	0.1478	0.1533	0.1652	0.1816	0.1972	0.2421
M_{xy}	0.0427	0.0489	0.0532	0.0561	0.0583	0.0633	0.0702	0.0761	0.0921
M_{xx}^{f}	0.0834	0.0959	0.1043	0.1100	0.1141	0.1232	0.1354	0.1468	0.1799
M_{yy}^{f}	0.1123	0.1289	0.1402	0.1478	0.1533	0.1652	0.1816	0.1972	0.2421
M_{xy}^{f}	0.0427	0.0489	0.0532	0.0561	0.0583	0.0633	0.0702	0.0761	0.0921

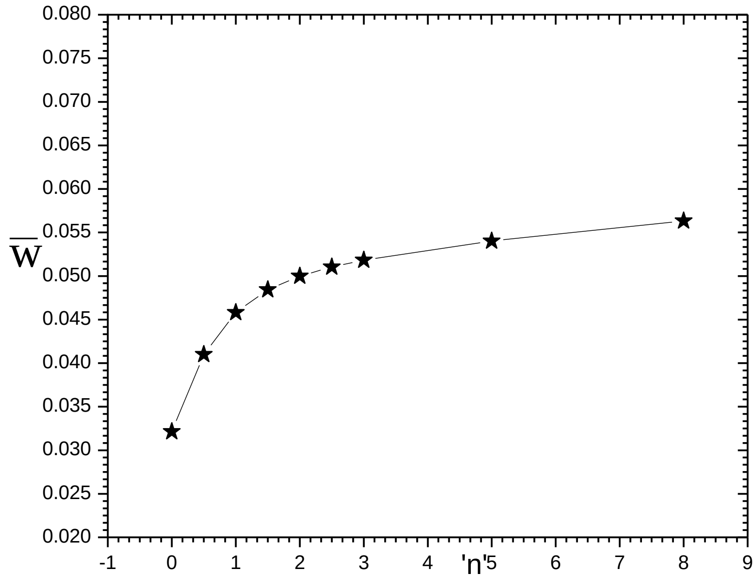

Fig 4. Effect of grading index 'n' on deflection of a square FGM plate (a/h=5)

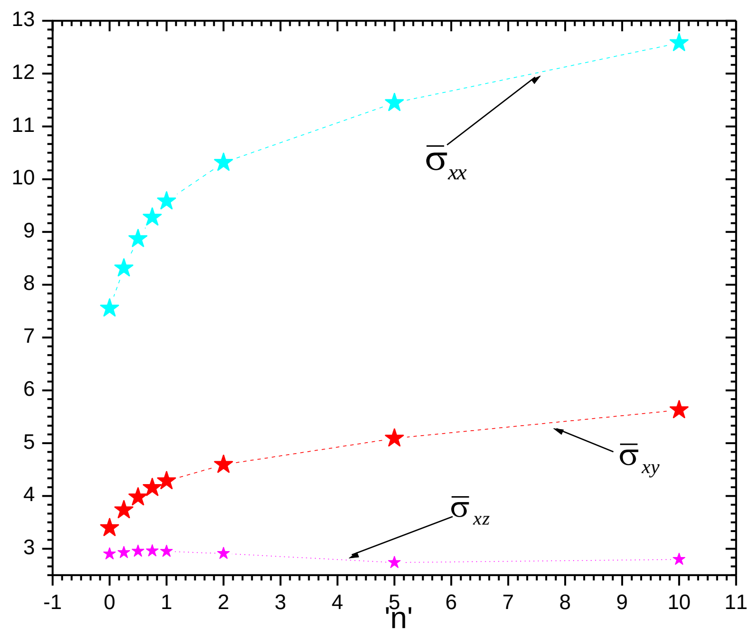

Fig 5. Effect of grading index 'n' on stresses of a square FGM plate (a/h=5)

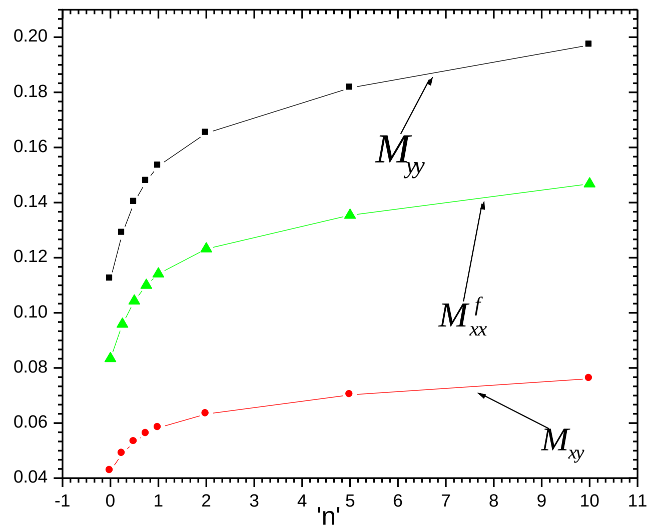

Fig 6 Effect of grading index 'n' on M_{yy} , M_{xy} and M_{xx}^{f} of a square FGM plate (a/h=5)

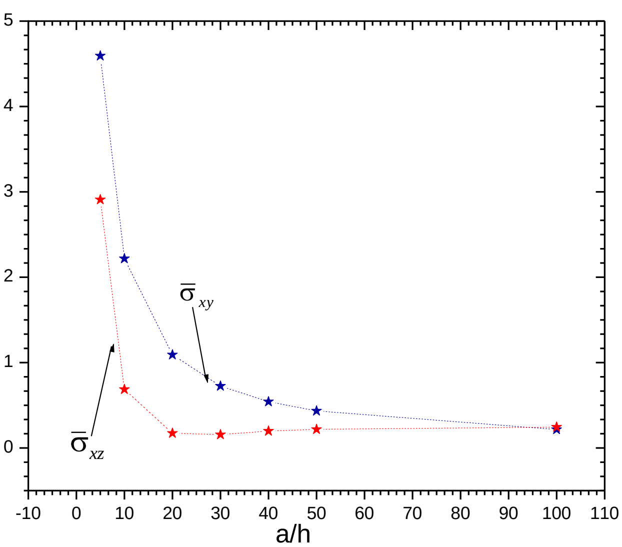

Fig 7 Effect of span to thickness ratio on stresses of a square FGM plate (‘n’=2)

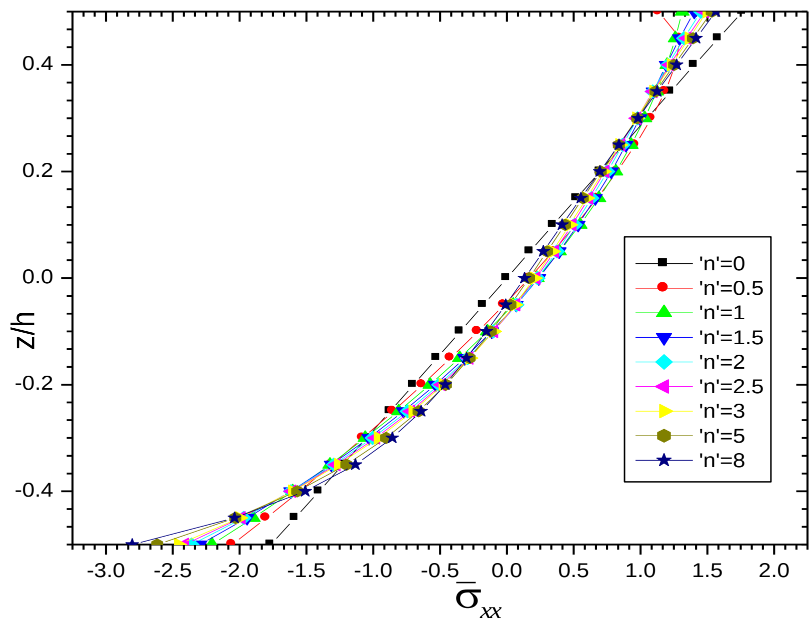

Fig 8 Effect of grading index 'n' on normalized stress $\bar{\sigma}_{xx}$ of simply supported square FGM plate along the thickness

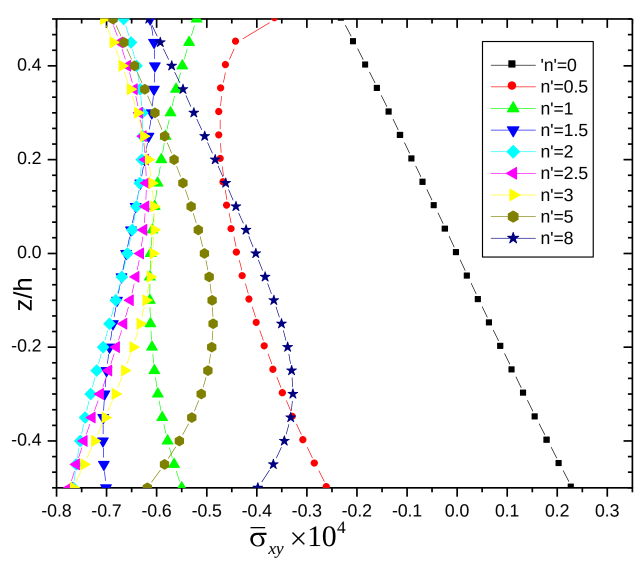

Fig 9: Effect of grading index 'n' on normalized stress $\bar{\sigma}_{xy}$ of simply supported square FGM plate along the thickness

The result obtained for deflection, stresses and moments due to different span ratio for simply supported FGM plates with gradation index 2 is shown in table 1 and table 2 shows the effect of gradation index 'n' for a thick simply supported FGM plates (a/h=5) . It is observed from Fig 4, Fig 5 and Fig 6 that the effect of grading index is more prominent when the value of n is less than 2 for deflection stresses and moments respectively. Fig 7 shows the variation in deflection become almost negligible as plates become thinner (i.e a/h > 30). However it is more prominent for thick plate. Fig 8 and Fig 9 represent the through thickness variation of stresses for different values of gradation index 'n'.

3. CONCLUSION

Bending response of functionally graded material plate (FGM) is presented using shear deformation theory. The effect of span to thickness ratio decreases for a/h ≥ 30. The effect of gradation index 'n' is prominent for lesser values of 'n' and decreases as 'n' increases. The present results can be used for validation purpose. Present solution mythology is good for obtaining the result and the concentrated load. The same can be extended for other types of concentrated load like sinusoidal varying line load, point load, patch load etc.

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