

Integrating Change Point into Software Reliability Growth Model along with Testing – Effort Function

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Abstract

In this paper a software reliability growth model (SRGM) based on non-homogenous Poisson process (NHPP) is proposed. The principal idea is to provide a SRGM which incorporates both exponentiated Weibull (EW) testing-effort function and change point. In the earlier research, it is found that the probability of fault detection is not constant. It can be changed at some point of time which is called a change point. The change can take place due to some important factors like the skill of test teams, program size and software testability. Therefore, we incorporated change point along with EW testing-effort function in the proposed SRGM. We performed the data analysis on real data set. Also, the parameters of the proposed SRGM are estimated through the least square estimation (LSE) and maximum likelihood estimation (MLE). Furthermore, we have drawn the required graphs and curve to evaluate the performance of proposed SRGM. In addition, we compared the proposed SRGM with some other existing SRGMs.

Keywords:

Software Reliability Growth Models, Non-Homogeneous Poisson Process, Change Point, Testing-Effort Function.

1. Introduction

Software that plays a pivotal role in the modern life is broadly classified as operating system and application software. Software is created by human and, therefore, a high degree of reliability cannot be guaranteed to perform without fault. Thus software reliability is crucial feature of the computer systems and the breakdown in the system could result in the fiscal, possessions, and human lose. It is defined as the probability of failure-free software operation in a specified environment for a specified period of time (Lyu, 1996).

The demand of complex software systems has increased more rapidly than the ability to design, implement, test and maintain them. When the

requirements for and dependencies on computers increase, the possibility of crises from computer failures also increases. The impact of these failures ranges from inconvenience to economic damages to loss of life. Needless to say, the reliability of computer systems has become major concern for our society. Although, measuring or predicting software reliability is a difficult task, it is important for the assessment of the performance of the underlying software system (Lyu, 1996). In addition to software reliability measurement, SRGMs help to predict the fault detection coverage in the testing phase. The above motivation yields the concept of proposed SRGM with exponentiated Weibull testing-effort and change point. (Lyu, 1996; Ahmad et al., 2007, 2012)

The actual data analysis is performed on real data set. Also, the parameters are estimated by least square estimation and maximum likelihood estimation methods. (Ahmad et al., 2010).

In the remaining of this paper, there are four more sections. In Section 2, we provide a brief review of the SRGM with exponentiated Weibull testing-effort function. In Section 3, we proposed a SRGM with exponentiated testing-effort function with change point. Furthermore, the numerical and data analysis takes place in section 4. In addition, we estimate the parameters, comparison criteras and plot the required graphs. Finally, conclusions are provided in Section 5.

2. Exponentiated Weibull (EW) Testing-Effort Function

During software testing phase, much testing-effort is consumed. The consumed testing-effort indicates how the errors are detected effectively in the software and can be modeled by different distributions (Putnam, 1978; Musa et al., 1987; Musa, 1999; Yamada et al., 1986, 1993; Kapur et al., 1999). Actually, the software reliability is highly related to the amount of testing-effort expenditures spent on detecting and correcting faults. Recently, Bokhari and Ahmad (2007) and Ahmad et al. (2007, 2010) proposed EW testing-effort function to predict the behavior of failure and fault of a software product. They have shown that EW testing-effort function is suitable and more flexible testing resource for assessing the reliability of software products. Therefore, we propose the EW curve as a more flexible testing-effort function that has no peak phenomenon during the software development process (Bokhari et al., 2005; Huang et al., 1997, 2000, 2002; Huang, 2004; Tian et al., 1995; Quadri et al., 2006).

The cumulative testing-effort expenditure (Ahmad et al., 2007, 2010) consumed in time $(0, t]$ (Yamada et al., 1985, 1986, 1993; Bokhari et al., 2005) is:

$$W(t) = \alpha(1 - e^{-\beta t^m})^\theta, \quad \alpha > 0, \beta > 0, m > 0, \theta > 0, \quad (1)$$

and the current testing-effort consumed at testing time t is:

$$w(t) = W'(t) = \alpha \cdot \beta \cdot m \cdot \theta \cdot t^{m-1} \cdot e^{-\beta t^m} (1 - e^{-\beta t^m})^{\theta-1}, \quad (2)$$

where α, β, m and θ are constant parameters, α is the total amount of testing-effort expenditures; β is the scale parameter, and m and θ are shape parameters.

3. Software Reliability Growth Model (SRGM) with Exponentiated Weibull Testing-Effort Function and Change Point

There are some basic assumptions for a SRGM with EW testing-effort and change point (Yamada et al., 1985a, 1986; 1993; Kapur et al., 1999; Kuo et al., 2001; Huang and Kuo, 2002; Huang, 2004):

- The software system is subject to failures at random times caused by errors remaining in the system.
- Each time a failure occurs, the error that caused it is immediately removed and no new errors are introduced.
- Testing-effort expenditures are described by the EW curve.
- The mean number of errors detected in the time interval $(t, t + \Delta t)$ to the current
- Testing-effort expenditures are proportional to the mean number of remaining errors in the system.
- The error detection phenomenon in software testing is modeled by a non-homogeneous Poisson process (NHPP).
- The proportionality is a constant over time.

We describe the SRGM with exponentiated Weibull testing-effort function and change point as follows (Huang, 2004):

$$\frac{dm(t)}{dt} \times \frac{1}{w(t)} = r \times (a - m(t)) \quad (3)$$

$$\text{and } r(t) = \begin{cases} r_1, & 0 \leq t \leq \tau \\ r_2, & t > \tau \end{cases}$$

The solution for equation (3) under the marginal condition $m(t)=0$,

$$m(t) = a(1 - e^{rW(t)}) \quad (4)$$

Solving equation (3) under the boundary condition $m(0) = 0$ and $W(0) = 0$ and $r(t) = r_2$ for $0 \leq t < \tau$,

$$m(t) = a(1 - e^{-r_1 W(t)}) \quad (5)$$

Plug in the value of $W(t)$ from the equation(1), we get

$$m(t) = a \left(1 - e^{-r_1 \alpha (1 - e^{-\beta t m})^\theta} \right) \quad (6)$$

Again, solving equation (3) under the boundary condition $m(0) = 0$ and $W(0) = 0$ and $r(t) = r_2$ for $t < \tau$,

$$m(t) = a \times [1 - e^{-(r_1 W(\tau) - r_2 W(\tau) + r_2 W(t))}] \quad (7)$$

Plug in the value of $W(t)$ from the equation(1), we get

$$= a \times \left[1 - e^{-(r_1 W(\tau) - r_2 W(\tau) + r_2 \alpha (1 - e^{-\beta t m})^\theta)} \right] \quad (8)$$

This is a SRGM by considering the exponentiated Weibull testing-effort function and change point.

4. Numerical and Data Analysis

4.1. Description of Real Data Set

This section will evaluate the performance of SRGM with exponentiated Weibull testing-effort function and change point with the help of real data set. The data is from a study by Ohba (1984). The system was PL/1 database application software (Huang, 2004).

4.2. Models Compared

Since the model is new, that's why we need to compare its performance with other existing models. We will compare with Ahmad's model, Huang's model, Yamada's delayed S-shaped Model and Goel and Okumoto model.

4.3. Comparison Criteria

To judge the performance of proposed SRGM, we have three comparison criteria.

4.3.1. The Accuracy of Estimation

The Accuracy of Estimation (AE) is defined as:

$$\left| \frac{m_a - a}{m_a} \right| \quad (9)$$

where m_a is the actual cumulative number (Goel et al.,1979) of detected faults after the test, and a is the estimated number of initial faults.

4.3.2. Mean Square Errors

We use the mean of (Kapur et al.,1996) squared errors (MSE) for long term predictions since it provides better understood measure of the differences between actual and predictive values (Lyu et al.,1992) .

The MSE can be calculated as follows:

$$\frac{\sum_{i=1}^k [m(t_i) - m_i]^2}{k} \quad (10)$$

A smaller MSE indicates a smaller fitting error and better performance.

4.3.3. Relative Error

The capability of the model to predict failure behavior from present and past failure behavior is called predictive validity, which can be represented by computing the relative error (RE) for the data (Musa et al., 1987; Huang, 2004).

$$\text{Relative Error (RE)} = \frac{m(t_q) - q}{q} \quad (11)$$

4.4 Methods for Estimation of Model Parameters

The parameters of the SRGM are estimated by least square method and maximum likelihood method (Ahmad et. al, 2011).

4.4.1 Least Square Estimation (LSE)

The parameters α , β , θ and m in the exponentiated Weibull testing-effort function can be estimated by the method of LSE. These parameters are determined for n observed data pairs in the form (t_k, W_k) ($k = 1, 2, \dots, n$; $0 < t_1 < t_2 < \dots < t_n$), where W_k is the cumulative testing-effort consumed in time $(0, t_k]$. The least squares estimators $\hat{\alpha}, \hat{\beta}, \hat{\theta}$, and \hat{m} can be obtained by minimizing:

$$S(\alpha, \beta, \theta, m) = \sum_{k=1}^n (W_k - W(t_k))^2 \quad (12)$$

4.4.2 Maximum Likelihood Estimation (MLE)

Once the estimates of α , β , θ and m are known, the parameters of SRGM would be estimated through MLE method. The estimators for a , r_1 and r_2 are determined for the n observed data pairs in the form (t_k, y_k) ($k = 1, 2, \dots, n$; $0 < t_1 < t_2 < \dots < t_n$), where y_k is the cumulative number of software errors detected up to time t_k of $[0, t_k]$. Then the likelihood function for the unknown parameters a , r_1 and r_2 in the NHPP model (Equation 8) is given by:

$$= \prod_{k=1}^n \frac{L'(a, r_1, r_2)}{[m(t_k) - m(t_{k-1})]^{(y_k - y_{k-1})}} \cdot e^{-[m(t_k) - m(t_{k-1})]},$$

where $t_0 = 0$ and $y_0 = 0$,

The maximum likelihood estimates of SRGM parameters a , r_1 and r_2 can be obtained by solving the following three equations:

$$\frac{\partial L}{\partial a} = 0 \quad (14)$$

$$\frac{\partial L}{\partial r_1} = 0 \quad (15)$$

$$\frac{\partial L}{\partial r_2} = 0 \quad (16)$$

The above non-linear equations can be solved numerically.

4.5. Estimation of Parameters and Model Comparison

In this section we are going to estimate the parameters of testing-effort and change point with real data set. Also, we evaluate the different comparison criteria to check the performance of proposed SRGM. In order to estimate the parameters α , β , θ and m of exponentiated Weibull testing-effort function; we fit the actual testing-effort data into equation (2) and solve it by using the MLE. That is, we minimize the sum of squares given in equation (12) and the estimated parameters are obtained as:

$$\hat{\alpha} = 112.9117, \quad \hat{\beta} = 0.000031, \\ \hat{\theta} = 0.3968, \quad \text{and } \hat{m} = 2.8114$$

Fig 1 and 3 graphically illustrate the comparisons between the observed data and the estimated exponentiated Weibull testing-effort data. Here, the fitted curves are shown as a dotted line and the solid line represents actual software data (Ahmad et al., 2007). Using the estimated parameters α , β , θ and m , the other parameters a , r_1 and r_2 in equation (8) can be solved numerically by the MLE. These parameters are listed in Table I.

Table I: Comparison Table

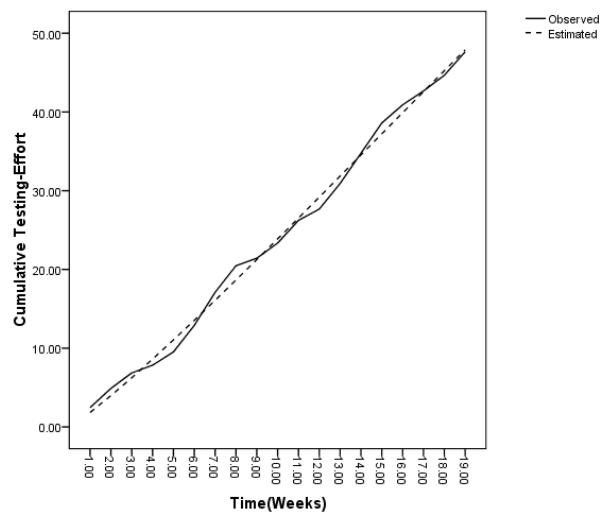
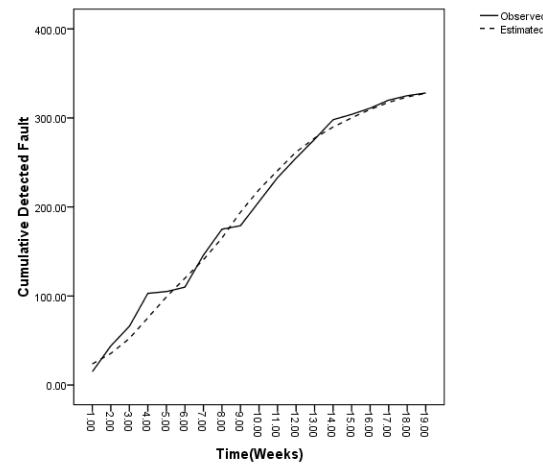
Model	A	r	τ	AE (%)	MSE
Proposed Model (Equation 8)	419.42	$r_1=0.03$ $r_2=0.02$	4	17.15	391.04
Proposed Model (Equation 8)	413.27	$r_1=0.03$ $r_2=0.03$	5	15.44	679.84
Proposed Model (Equation 8)	423.02	$r_1=0.03$ $r_2=0.03$	6	18.16	1142.64
Proposed Model (Equation 8)	453.83	$r_1=0.03$ $r_2=0.03$	7	26.76	1813.22
Proposed Model (Equation 8)	514.54	$r_1=0.02$ $r_2=0.02$	8	43.72	2692.74
Proposed Model (Equation 8)	627.68	$r_1=0.02$ $r_2=0.02$	9	75.33	3756.08
Proposed Model (Equation 8)	864.10	$r_1=0.01$ $r_2=0.01$	10	141.37	4967.90
Ahmad et al. Model	565.16	0.02	-	0.36	117.58
Chin – Yu Huang Model	394.07	0.42	-	10.06	118.29

Chin – Yu Huang Model	398.11	$r_1=0.05$ $r_2=0.04$	4	11.20	69.43
Chin – Yu Huang Model	398.114	$r_1=0.04$ $r_2=0.04$	5	11.20	69.43
Chin – Yu Huang Model	398.11	$r_1=0.04$ $r_2=0.04$	6	11.20	69.43
Chin – Yu Huang Model	398.01	$r_1=0.04$ $r_2=0.04$	7	11.17	69.41
Chin – Yu Huang Model	398.01	$r_1=0.04$ $r_2=0.04$	8	11.17	69.41
Chin – Yu Huang Model	398.01	$r_1=0.04$ $r_2=0.04$	9	11.17	69.41
Yamada Delayed S-Shaped Model	374.05	0.19	-	14.48	168.67
Goel and Okumoto Model	513.14	0.053	-	43.34	222.09

Table II : Relative Error

Percentage of Data Used	Relative Error (RE)
32	0.20628005
37	0.052395232
42	- 0.006590506
47	0.079640971
53	0.028479617
58	- 0.014245439
63	- 0.032523129
68	- 0.047251208

74	-0.06558535
79	- 0.035464668
84	- 0.011988327
89	0.001996853
95	0.025681202
100	0.053109103

Graphs**Fig 1 Observed / Estimated Cumulative Testing-effort vs. Time****Fig 2 Observed / Estimated Cumulative Detected Fault vs. Time**

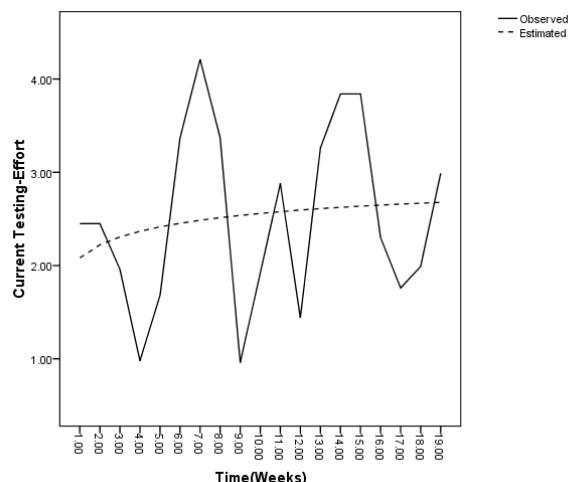


Fig 3 Observed / Estimated Current Testing-effort vs. Time

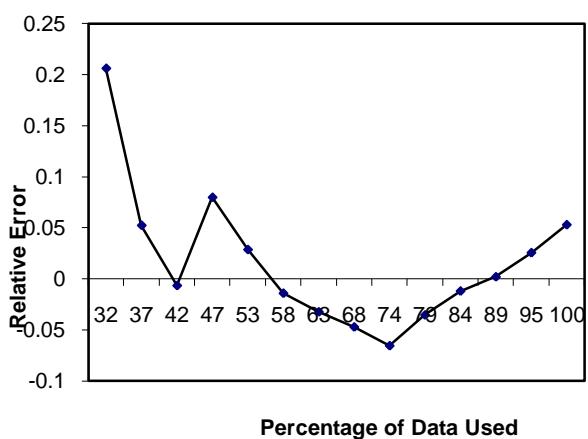


Fig 4 Relative Error Curve for the Proposed Model

5. Conclusions

In this paper we proposed a SRGM with exponentiated Weibull testing-effort function and change point. The number of initial faults, the fault detection rate, the accuracy of estimation, the mean square errors and the relative error are estimated. The proposed SRGM is compared with other existing SRGMs using different criteria. It is shown that proposed SRGM has better prediction capability as compare to other existing SRGMs. Also, the relative error of the proposed model approaches to zero faster as compared with the other

models. The results obtained show better fit and wider applicability of the proposed model on real data set. The graphs are also yield the considerable fitting of data.

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