Comparative study of two reliability models on a two -unit hot standby system with unannounced failures

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ABSTRACT

The present paper deals with the comparative study of two reliability models on a two-unit hot standby system wherein failures are unannounced. Initially, both the units are loaded but one of them is operative and the other is hot standby i.e. the total control of the machine is on the operative unit. Hot standby unit is hot in the sense that the latest information is transformed to it by the operative unit. The hot standby unit may also get failed. In the first model, the failure of a unit is not self announcing but it is revealed only when both the units get failed i.e. on the failure of the system whereas in the second model, the units are observed with regard to failures i.e. the failures are observable meaning thereby the failure of unit is detected as soon as it fails. Inspection is carried out for the failed unit to see as to whether it is repairable or irreparable and then it is repaired or replaced accordingly. Various measures of the system effectiveness are obtained by making use of semi-Markov processes and regenerative point technique. Study through graphs is also made.

Introduction

A large number of research papers in the field of reliability developed various reliability models for standby systems including Tuteja and Taneja (1992), Tuteja et al. (2001), Parashar and Taneja (2007), Zuhair and Rizwan (2007), Goyal et.al (2009), Rizwan et al. (2010), Mathew et al. (2011), Kumar and Kapoor (2012), Zhang et al. (2012), Singh and Taneja [2013]. So, a lot of work has been done in the field of reliability by large number of researchers considering two-unit systems similar or dissimilar. In most of the studies, it has been assumed that failures are self announcing. However, there may be situations where, in hot standby systems, we may not come to know whether the operative unit has got failed or not till the complete failure of both the units.

Taking the aspect that failures may not be self announcing, the present paper deals with the comparative study of two reliability models on a two-unit hot standby system wherein failures are unannounced. Initially, both the units are loaded but one of them is operative and the other is hot standby i.e. the total control of the machine is on the operative unit. Hot standby unit is hot in the sense that the latest information is transformed to it by the operative unit. The hot standby unit may also get failed. Two models have been discussed. In the first model, the failure of a unit is not self announcing but it is revealed only when both the units get failed i.e. on the failure of the system whereas in the second model, the units are observed with regard to failures i.e. the failures are observable meaning thereby the failure of unit is detected as soon as it fails. Inspection is carried out for the failed unit to see as to whether it is repairable or irreparable and then it is repaired or replaced accordingly. Various measures of the system effectiveness are obtained by making use of semi-Markov processes and regenerative point technique. Study through graphs is also made.

Notations

λ	Constant failure rate of operative unit
α	Constant failure rate of hot stand by unit.
p_1	probability that unit is repairable.
p_2	probability that unit is irreparable
$g(t)$, $G(t)$	p.d.f. and c.d.f. of repair time of ordinary repairman
$g_e(t)$, $G_e(t)$	p.d.f. and c.d.f. of repair time of expert repairman
$h_1(t)$, $H_1(t)$	p.d.f. and c.d.f. of time to inspection for detecting reparability of a failed unit
$h_e(t)$, $H_e(t)$	p.d.f. and c.d.f. of inspection time of expert repairman
$h_2(t)$, $H_2(t)$	p.d.f. and c.d.f. of replacement time

Symbols for the states of the system are

o	operative unit
hs	hot standby unit
F_{ui}	Failed unit under inspection by ordinary repairman
F_{uei}	Failed unit under inspection by expert repairman
F_{ur}	Failed unit under repair by ordinary repairman
F_{re}	Failed unit under repair by expert repairman
F_{rep}	Failed unit under replacement
F_{UI}/F_{UIe}	Inspection of the failed unit is continuing by the ordinary/expert from the previous state
F_{Ur}/F_{Re}	Repair of the failed unit continuing by the ordinary/expert from the previous state
F_{REP}	Replacement of the failed unit continuing by the ordinary repairman from the previous state

Model 1

The idea of inspection and replacement for a two-unit hot standby system is used in model 1, where failure of unit is not self announcing but it is revealed only when both the units get failed i.e. on the failure of the system. Initially both the units are loaded but one of them is operative and the other is hot standby the total control of the machines is on the operative unit. Hot standby unit is hot in the sense that the latest information is transformed to it by the operative unit. The hot standby unit may also get failed in standby unit. The ordinary repairman replaces the unit on finding it irreparable.

The state transition diagram for the model is shown as in Fig. 1.

The following measures of the system effectiveness are obtained by making use of semi-Markov processes and regenerative point technique:

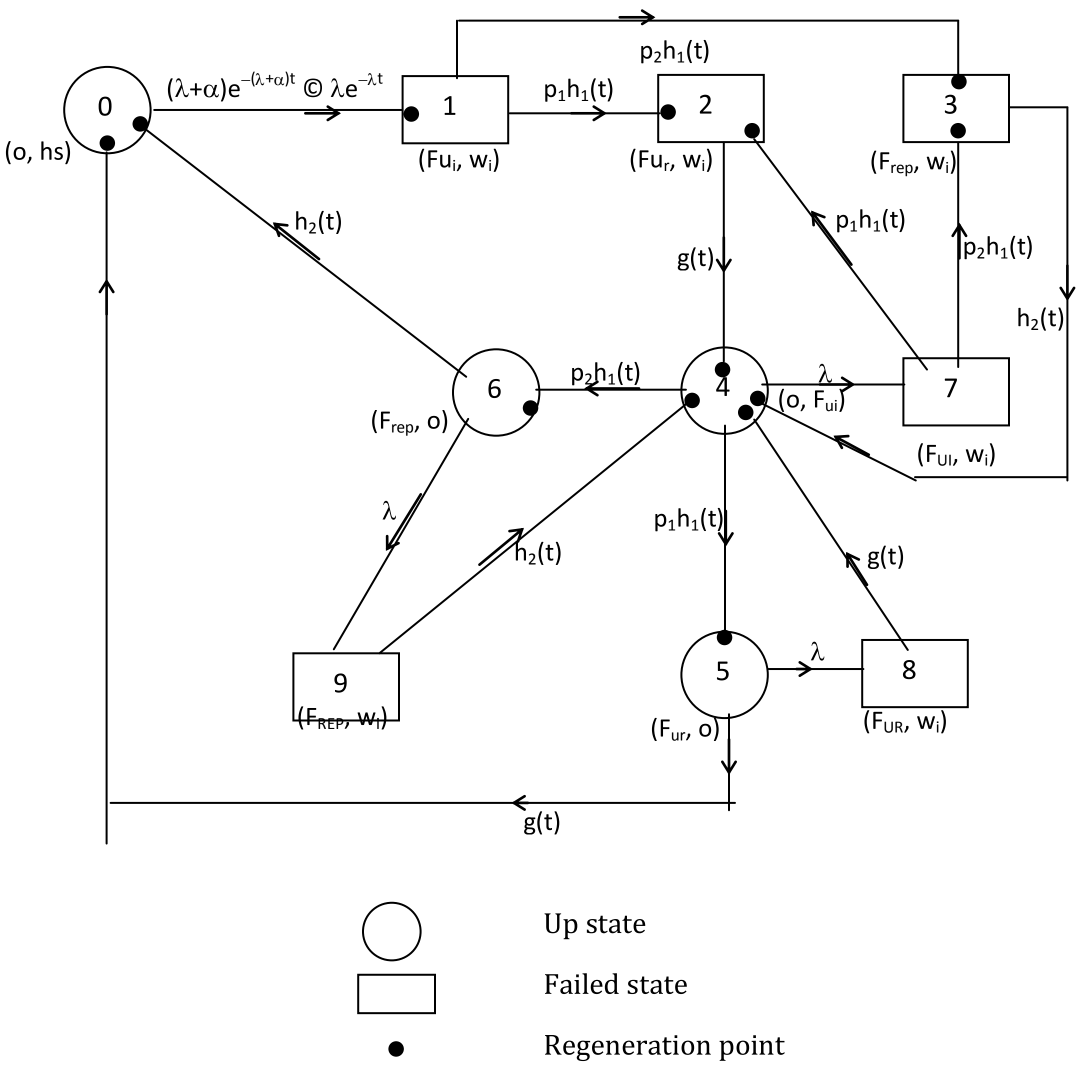


Fig. 1

Mean time to system failure (MTSF), $T_0 = \lim_{s \to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N}{D}$

Where $N = \alpha + 2\lambda$ and $D = \lambda(\lambda + \alpha)$

Availability analysis, $A_0 = N_1 / D_1$

$N_1 = \mu_0[p_{45}p_{50} + p_{46}p_{60}] + \mu_4 + \mu_5 p_{45} + \mu_6 p_{46}$

$D_1 = (p_{45}p_{50} + p_{46}p_{60})\mu_0 + (1 + p_{45}p_{50} + p_{46}p_{60})\mu_1 + \{p_{12}(p_{45}p_{50}$

$+ p_{46}p_{60}) + p_{42}^{(7)} + p_{45}\}\mu_2 + \{p_{13}(p_{45}p_{50} + p_{46}p_{60}) + p_{43}^{(7)} + p_{46}\}\mu_3$

Busy period analysis of ordinary repairman

(Repair Time Only) $B_0 = N_2 / D_1$

where $N_2 = \mu_2[(1 - p_{43}^{(7)} - p_{45}\, p_{54}^{(8)} - p_{46}\, p_{64}^{(9)})p_{12} + p_{42}^{(7)}\, p_{13}] + \mu_2 p_{45}$

Busy period analysis of ordinary repairman

(Inspection Time Only) $BI_0 = N_3 / D_1$

Where $N_3 = [p_{45}\, p_{50} + p_{46}\, p_{60} + 1]\mu_4$

Busy period analysis of ordinary repairman

(Replacement Time Only) $\mathbf{BR_0 = N_4 / D_1}$

Where $N_4 = [p_{46} + p_{43}^{(7)} + p_{13}(p_{45}p_{50} + p_{46}\, p_{60})]\mu_3$

Expected number of visits by ordinary repairman

$$V_0 = N_5 / D_1$$

Where $N_5 = p_{45}\, p_{50} + p_{46}\, p_{60}$

Expected number of replacements

$$R_0 = N_6 / D_1$$

Where $N_6 = p_{13}[p_{45}\, p_{50} + p_{46}\, p_{60} + p_{46}]$

Profit analysis

The expected total profit incurred to the system in steady-state is given by

$P_{41} = C_0A_0 - C_1B_0 - C_2BI_0 - C_3BR_0 - C_4V_0 - C_5R_0$

Where C_0 = revenue per unit up time of the system

C_1 = cost per unit time for which the repairman is busy under repair

C_2 = cost per unit time for which the repairman is busy under inspection

C_3 = cost per unit time for which the repairman is busy under replacement

C_4 = cost per visit of ordinary repairman

C_5 = cost per unit time for replacement

Particular case

For graphical interpretation the following particular case is considered :

$g(t) = \alpha_1 e^{-\alpha_1 t}$; $h_1(t) = \gamma_1 e^{-\gamma_1 t}$; $h_2(t) = \gamma e^{-\gamma t}$ $p_{01} = 1$; $p_{12} = p_1$;

$p_{13} = p_2$; $p_{24} = 1$; $p_{34} = 1$; $p_{45} = \dfrac{p_1\gamma_1}{\lambda+\gamma_1}$

$p_{46} = \dfrac{p_2\gamma_1}{\lambda+\gamma_1}$; $p_{47}^{(2)} = \dfrac{\lambda p_1}{\lambda+\gamma_1}$; $p_{47}^{(3)} = \dfrac{\lambda p_2}{\lambda+\gamma_1}$: $p_{50} = \dfrac{\alpha_1}{\lambda+\alpha_1}$;

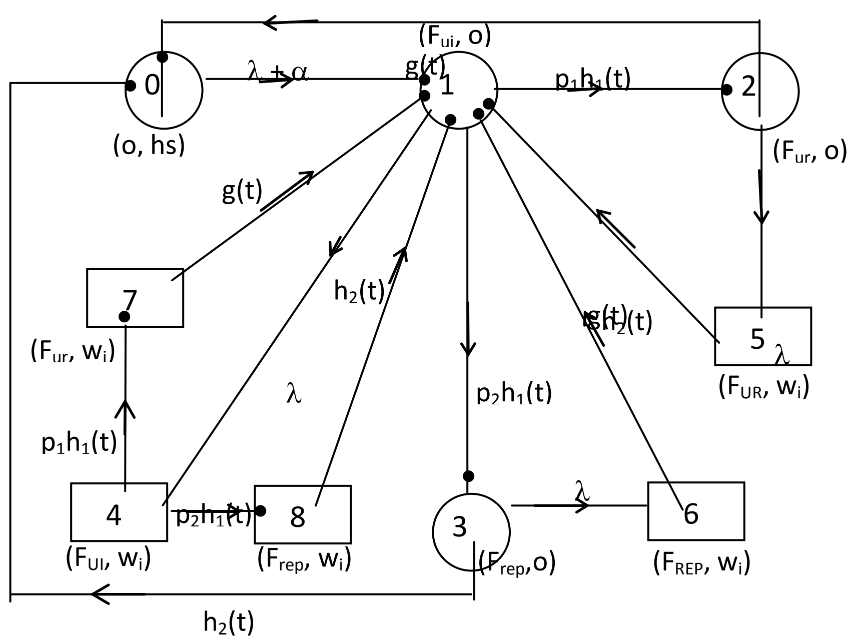
$p_{54}^{(8)} = \dfrac{\lambda}{\lambda+\alpha_1}$; $p_{60} = \dfrac{\gamma}{\lambda+\gamma}$: $p_{64}^{(9)} = \dfrac{\lambda}{\lambda+\gamma}$: $\mu_0 = \dfrac{\alpha+2\lambda}{\lambda(\lambda+\alpha)}$;

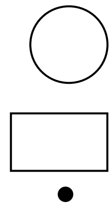
$\mu_1 = \dfrac{1}{\gamma_1}$, $\mu_3 = \dfrac{1}{\gamma}$, $\mu_4 = \dfrac{1}{\lambda+\gamma_1}$, $\mu_5 = \dfrac{1}{\lambda+\alpha_1}$, $\mu_6 = \dfrac{1}{\lambda+\gamma}$

we can find profit for this particular case.

Model 2

In this model, we analyse a two-unit hot standby system. It is assumed that failures are observable i.e. failure of a unit is detected as soon as it fails. On the failure of a unit, it goes under inspection which is carried out by an ordinary repairman to detect whether the unit is repairable or not. Two reliability models have been studied. In the first model, it is assumed that when the ordinary repairman declares that unit is not repairable, it is replaced with a new one. The unit is replaced if the ordinary repairman declares that unit is not repairable state transition diagram is shown as in Fig. 2





Up state

Failed state

Regeneration point

Fig. 2

Following measures of the system effectiveness are obtained by making use of semi-Markov processes and regenerative point technique.

Mean time to system failure $T_0 = N/D$

where $N = \mu_0 + \mu_1 + p_{12}\mu_2 + p_{13}\mu_3$

$D = 1-(p_{12}p_{20} + p_{13}p_{30})$

Availability analysis $A_0 = \dfrac{N_1}{D_1}$

where $N_1 = (p_{12}\, p_{20} + p_{13}p_{30})\mu_0 + \mu_1 + p_{12}\mu_2 + p_{13}\mu_3$

$D_1 = (p_{12}p_{20} + p_{13}p_{30})\mu_0 + k_1 + (p_{12} + p_{17}^{(4)})\mu_7 + (p_{13} + p_{18}^{(4)})\mu_8$

Busy period analysis of ordinary repairman

(Repair Time Only) $B_0 = \dfrac{N_2}{D_1}$

where $N_2 = \{p_{12} + p_{17}^{(4)}\}\mu_7$

Busy period analysis of ordinary repairman

(Inspection Time Only) $BI_0 = \dfrac{N_3}{D_1}$

where $N_3 = \mu_1$

Busy period analysis of ordinary repairman

(Replacement Time Only) $BR_0 = \dfrac{N_4}{D_1}$

where $N_4 = (p_{13} + p_{18}^{(4)})\mu_8$

Expected number of visits by the repairman

$$V_0 = \frac{N_5}{D_1}$$

where $N_5 = 1 - p_{12}\, p_{21}^{(5)} - p_{13}\, p_{31}^{(6)} - p_{17}^{(4)} - p_{18}^{(4)}$

Expected number of replacements

$$R_0 = \frac{N_6}{D_1}$$

where $N_6 = p_{13} + p_{18}^{(4)}$

Profit analysis

The expected total profit incurred to the system in steady-state is given by

$P_{51} = C_0A_0 - C_1B_0 - C_2BI_0 - C_3BR_0 - C_4V_0 - C_5R_0$

where C_0 = revenue per unit up time of the system

C_1 = cost per unit time for which the repairman is busy under repair

C_2 = cost per unit time for which the repairman is busy under inspection

C_3 = cost per unit time for which the repairman is busy under replacement

C_4 = cost per visit of ordinary repairman

C_5 = cost per unit time for replacement

Particular case

For graphical interpretation the following particular case is considered:

$g(t) = \alpha_1 e^{-\alpha_1 t}$; $h_1(t) = \gamma_1 e^{-\gamma_1 t}$; $h_2(t) = \gamma e^{-\gamma t}$: $p_{01} = 1$; $p_{12} = \dfrac{p_1\gamma_1}{\gamma_1 + \lambda}$;

$p_{13} = \dfrac{p_2\gamma_1}{\gamma_1 + \lambda}$: $p_{14} = \dfrac{\lambda}{\gamma_1 + \lambda}$; $p_{17}^{(4)} = \dfrac{p_1\lambda}{\gamma_1 + \lambda}$; $p_{18}^{(4)} = \dfrac{p_2\lambda}{\gamma_1 + \lambda}$

$p_{20} = \dfrac{\alpha_1}{\alpha_1 + \lambda}$; $p_{21}^{(5)} = \dfrac{\lambda}{\alpha_1 + \lambda}$; $p_{25} = \dfrac{\lambda}{\alpha_1 + \lambda}$: $p_{30} = \dfrac{\gamma}{\gamma + \lambda}$;

$p_{31}^{(6)} = \dfrac{\lambda}{\gamma + \lambda}$; $p_{36} = \dfrac{\lambda}{\gamma + \lambda}$: $p_{71} = 1$; $p_{81} = 1$

$\mu_0 = \dfrac{1}{\lambda+\alpha}$; $\mu_1 = \dfrac{1}{\gamma_1+\lambda}$; $\mu_2 = \dfrac{1}{\alpha_1+\lambda}$: $\mu_3 = \dfrac{1}{\gamma+\lambda}$;

$\mu_7 = \dfrac{1}{\alpha_1}$; $\mu_8 = \dfrac{1}{\gamma}$: $K_1 = \dfrac{1}{\gamma_1}$

Using the above equations, we can find profit of the particular system.

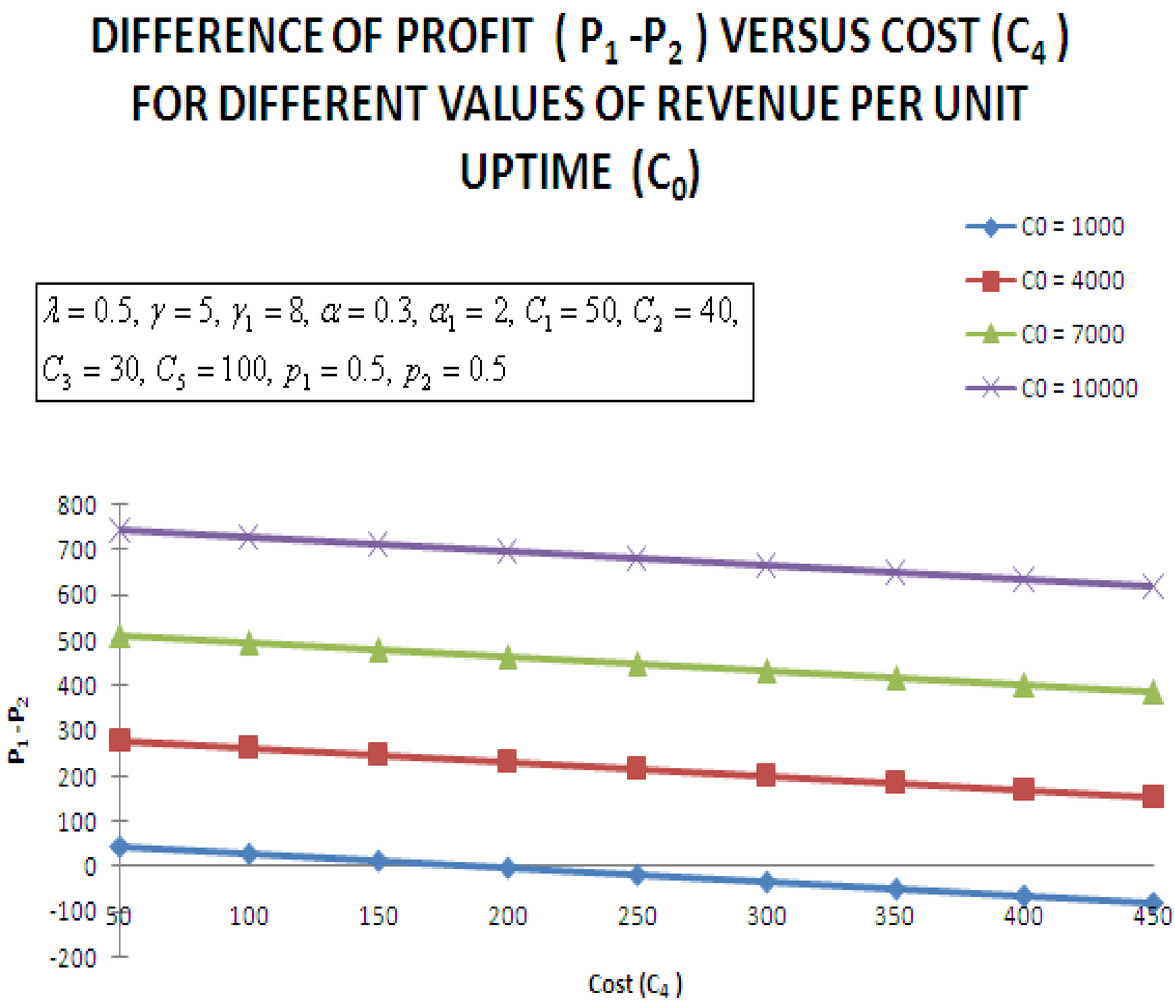


Fig. 3

Comparison Between Model 1 and Model 2

Fig. 3 shows the behavior of the difference between $P_1 – P_2$ with respect to cost (C_4) for different values of C_0 where P_1 and P_2 are the profits for Model 1 and Model 2 respectively. The following conclusions are drawn:

If C_0 = 1000, then P_1–P_2 > or = or < 0 according as C_4 < or = or > 162. So, Model 1 is better or worse than Model 2

if C_4 < or > 162. Both the Models are equally good if C_4 = 161.6.

For C_0 = 4000, C_0 = 7000 or C_0 = 10000; the difference becomes negative at a very high cost (C_4). Therefore, Model 2 is better than Model 1 till C_4 becomes very high

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