Optimized Components Ratio Model Development for Hollow Sandcrete Block Production Using Mixed Aggregate

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Abstract *: The The paper investigates the model development and optimization of the components ratio for the optimal compressive strength of 60/40 sand/quarry dust hollow sandcrete block. Quarry dust is a massive waste in quarry sites and can be combined with sand at studied ratios. The study applies the Scheffe's optimization approach to obtain a mathematically experimental model of the form* $f(x_{i1},x_{i2},x_{i3},x_{i4})$*, where* x_i *are proportions of the concrete components, viz: cement, sand, quarry dust and water. Scheffe's experimental design techniques are followed to mould various hollow block samples measuring 450mm x 225mm x 150mm and tested for 28 days strength. The task involved experimentation and design, applying the second order polynomial characterization process of the simplex lattice method. The model adequacy is checked using the control factors. Finally a software is prepared to handle the design computation process to take the desired property of the mix, and generate the optimal mix ratios.*

Keywords: *Simplex-lattice, Sandcrete, Pseudo-component, optimization, factor space.*

I. Introduction

The construction of structures is a regular operation which heavily involves sandcrete blocks for load bearing or non-load bearing walls. The cost/stability of this material has been a major issue in the world of construction where cost is a major index [1].

As it is, concrete is the main material of construction, and the ease or cost of its production accounts for the level of success in the area of environmental upgrading as in the construction of new roads, buildings, dams, water structures and the renovation of such structures. To produce the concrete several primary components such as cement, sand, gravel and some admixtures are to be present in varying quantities and qualities. Unfortunately, the occurrence and availability of these components vary very randomly with location and hence the attendant problems of either excessive or scarce quantities of the different materials occurring in different areas. Where the scarcity of one component prevails exceedingly, the cost of the concrete production increases geometrically. Such problems obviate the need to seek alternative materials for partial or full replacement of the scarce component when it is possible to do so without losing the quality of the concrete.

1.1 Optimization Concept

Planning is required for the success of every human activity. The target of planning is the maximization of the desired outcome of the venture. In order to maximize gains or outputs it is often necessary to keep inputs or investments at a minimum at the production level. The process involved in this planning activity of minimization and maximization is referred to as optimization, [2]. In the science of optimization, the desired property or quantity to be optimized is referred to as the objective function. The raw materials or quantities whose amount of combinations will produce this objective function are referred to as variables.

The variations of these variables produce different combinations and have different outputs. Often the space of variability of the variables is not universal as some conditions limit them. These conditions are called constraints. For example, money is a factor of production and is known to be limited in supply. The constraint at any time is the amount of money available to the entrepreneur at the time of investment. Hence or otherwise, an optimization process is one that seeks for the maximum or minimum value and at the same time satisfying a number of other imposed requirements [3]. The function is called the objective function and the specified requirements are known as the constraints of the problem.

1.2 Concrete Mix optimization

The task of concrete mix optimization implies selecting the most suitable concrete aggregates from the data base [4]. Several methods have been applied. Examples are by [5], [6]. [7] proposed an approach which adopts the equilibrium mineral assemblage concept of geochemical thermodynamics as a basis for establishing mix proportions. [8] reports that optimization of mix designs require detailed knowledge of concrete properties. Low water-cement ratios lead to increased strength but will negatively lead to an accelerated and higher shrinkage. Apart from the larger deformations, the acceleration of dehydration and strength gain will cause cracking at early ages.

1.3 Modeling

Different physical systems may correspond to the same mathematical model so that they can be solved by the same methods. This is an impressive demonstration of the unifying power of Mathematics [9]. Modeling means setting up mathematical models/formulations of physical or other systems. Many factors of different effects occur in nature in the world simultaneously dependently or independently. When they interplay they could inter-affect one another differently at equal, direct, combined or partially combined rates variationally, to generate varied natural constants in the form of coefficients and/or exponents. The challenging problem is to understand and asses these distinctive constants by which the interplaying factors underscore some unique natural phenomenon towards which their natures tend, in a single, double or multi phase system.

For such assessment a model could be constructed for a proper observation of response from the interaction of the factors through controlled experimentation followed by schematic design where such simplex lattice approach of the type of [10] optimization theory could be employed.

II. Literature Review

In the past, ardent researchers have done works in the behavior of concrete under the influence of its components. With given proportions of aggregates the compressive strength of concrete depends primarily upon age, cement content, and the cement-water ratio [11]. Of all the desirable properties of hardened concrete such as the tensile, compressive, flexural, bond, shear strengths, etc., the compressive strength is the most convenient to measure and is used as the criterion for the overall quality of the hardened concrete [3].

Simplex is a structural representation (shape) of lines or planes joining assumed positions or points of the constituent materials (atoms) of a mixture, and they are equidistant from each other [12]. When studying the properties of a q-component mixture, which are dependent on the component ratio only the factor space is a regular (q-1)–simplex [13]. Simplex lattice designs are saturated, that is, the proportions used for each factor have m + 1 equally spaced levels from 0 to 1 (x_i = 0, 1/m, 2/m, … 1), and all possible combinations are derived from such values of the component concentrations, that is, all possible mixtures, with these proportions are used.

Sandcrete blocks are masonry units used in all types of masonry constructions such as interior and exterior load bearing walls, fire walls party walls, curtain walls, panel walls, partition, backings for other masonry, facing materials, fire proofing over structured steel members, piers, pilasters columns, retaining walls, chimneys, fireplaces, concrete floors, patio paving units, kerbs and fences. The block is defined by ASIM as hollow block when the cavity area exceeds 25% of the gross cross-sectional area, otherwise it belongs to the solid category [14].

III. Theory Background

This is a theory where a polynomial expression of any degrees, is used to characterize a simplex lattice mixture components. In the theory only a single phase mixture is covered. The theory lends path to a unifying equation model capable of taking varying mix component variables to fix equal mixture properties. The optimization that follows selects the optimal ratio from the component ratios list that is automatedly generated. This theory is the adaptation to this work of formulation of response function for compressive strength of sandcrete block.

3.1 Simplex Lattice

Mathematically, a simplex lattice is a space of constituent variables of X_1, X_2, X_3,……, and X_i which obey these laws:

$X_i < 0$
$X \neq$ negative
$0 \leq x_i \leq 1$ (3.1)
$\sum x_i = 1$
$i=1$
That is, a lattice is an abstract space.

To achieve the desired strength of concrete, one of the essential factors lies on the adequate proportioning of ingredients needed to make the concrete. By some adaptations to the theory of [10], a model is developed by which, if the compressive strength desired is specified, possible combinations of needed ingredients to achieve the compressive strength can easily be predicted by the aid of computer, and if proportions are specified the compressive strength can easily be predicted.

3.2 Simplex Lattice Method

In designing experiment to attack mixture problems involving component property diagrams the property studied is assumed to be a continuous function of certain arguments and with a sufficient accuracy it can be approximated with a polynomial [13]. When investigating multi- components systems the use of experimental design methodologies substantially reduces the volume of an experimental effort. Further, this obviates the need for a special representation of complex surface, as the wanted properties can be derived from equations while the possibility to graphically interpret the result is retained. As a rule the response surfaces in multi-component systems are very intricate. To describe such surfaces adequately, high degree polynomials are required, and hence a great many experimental trials. If a mixture has a total of q components and x_1 be the proportion of the i^{th} component in the mixture such that,

$x_i >= 0$ (i=1,2, ….q), ... (3.2)

then the sum of the component proportion is a whole unity i.e.

$X_1 + X_2 + X_3 + X_4 = 1$ or $\sum X_i - 1 = 0$ (3.3)

where i = 1, 2, …., q… Thus the factor space is a regular (q-1) dimensional simplex. In (q-1) dimensional simplex if q = 2, we have 2 points of connectivity. This gives a straight line simplex lattice. If q=3, we have a triangular simplex lattice and for q = 4, it is a tetrahedron simplex lattice, etc. Taking a whole factor space in the design we have a (q,m) simplex lattice whose properties are defined as follows:

i. The factor space has uniformly distributed points,

ii Simplex lattice designs are saturated [13]. That is, the proportions used for each factor have m + 1 equally spaced levels from 0 to 1 (xi = 0, 1/m, 2/m, … 1), and all possible combinations are derived from such values of the component concentrations, that is, all possible mixtures, with these proportions are used.

Hence, for the quadratic lattice (q,2), approximating the response surface with the second degree polynomials (m=2), the following levels of every factor must be used 0, ½ and 1; for the fourth order (m=4) polynomials, the levels are 0, 1/4, 2/4, 3/4 and 1, etc; [10] showed that the number of points in a (q,m) lattice is given by

$C_{q+m-1} = q(q+1)(q+2) \ldots (q+m-1)/m!$ (3.4)

3.2.1The (4,2) Lattice Model

The properties studied in the assumed polynomial are real-valued functions on the simplex and are termed responses. The mixture properties were described using polynomials assuming a polynomial function of degree m in the q-variable $X_1, X_2 ......, X_q$, subject to equation (3.1), and will be called a (q,m) polynomial having a general form:

$$\hat{Y}= b_0 + \sum_{1\le i<q} b_iX_i + \sum_{1\le i<j\le q} b_{ij}X_iX_{ij} + \dots + \sum_{1\le i<j<k\le q} b_{ijk} + \sum b_{i1i2\dots in}X_{i1}X_{i2}\dots X_{in} \quad (3.5)$$

$$\hat{Y} = b_0 +b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{24}X_2X_4 + b_{23}X_2X_3+ b_{34}X_3X_4 \quad (3.6)$$

where b is a constant coefficient.

The relationship obtainable from Eqn (3.6) is subjected to the normalization condition of Eqn. (3.3) for a sum of independent variables. For a ternary mixture, the reduced second degree polynomial can be obtained as follows:

From Eqn. (3.3)

$$X_1+X_2 +X_3 +X_4=1 \quad (3.7)$$

i.e

$$b_0 X_2 + b_0X_2+ b_0 X_3+ b_0X_4 = b_0 \quad (3.8)$$

Multiplying Eqn. (3.7) by X_1, X_2, X_3, X_4, in succession gives

$$X_1^2 = X_1 - X_1X_2 - X_1X_3 - X_1X_4$$
$$X_2^2 = X_2 - X_1X_2 - X_2X_3 - X_2X_4 \quad (3.9)$$
$$X_3^2 = X_3 - X_1X_3 - X_2X_3 - X_3X_4$$
$$X_4^2 = X_4 - X_1X_4 - X_2X_4 - X_3X_4$$

Substituting Eqn. (3.8) into Eqn. (3.9), we obtain after necessary transformation that

$$\hat{Y} = (b_0 + b_1 + b_{11})X_1 + (b_0 + b_2 + b_{22})X_2 + (b_0 + b_3 + b_{33})X_3 + (b_0 + b_4 + b_{44})X_4 + (b_{12} - b_{11} - b_{22})X_1X_2 + (b_{13} - b_{11} - b33)X_1X_3 + (b_{14} - b_{11} - b_{44})X_1X_4 + (b_{23} - b_{22} - b_{33})X_2X_3 + (b_{24} - b_{22} - b_{44})X_2X_4 + (b_{34} - b_{33} - b_{44})X_3X_4 \quad (3.10)$$

If we denote

$\beta i = b_0 + b_i + b_{ii}$ and $\beta_{ij} = b_{ij} - b_{ii} - b_{jj}$,

then we arrive at the reduced second degree polynomial in 6 variables:

$$\hat{Y} = \beta_1X_1+ \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_{12}X_1X_2+ \beta_{13}X_1X_3 + \beta_{14}X_1X_4+ \beta_{23}X_2X_{23}+ \beta_{24}X_2X_4 + \beta 34X3X4 \quad (3.11)$$

Thus, the number of coefficients has reduced from 15 in Eqn 3.6 to 10 in Eqn 3.11. That is, the reduced second degree polynomial in q variables is

$$\hat{Y} = \Sigma\ \beta_iX_i +\Sigma\beta_{ij}X_iX_j \quad (3.12)$$

3.2.2 Construction of Experimental/Design Matrix

From the coordinates of points in the simplex lattice, we can obtain the design matrix. We recall that the principal coordinates of the lattice, only a component is 1 (Table 3.1) zero.

Table 3.1 Design matrix for (4,2) Lattice

N	X_1	X_2	X_3	X_4	Y_{exp}
1	1	0	0	0	Y_1
2	0	1	0	0	Y_2
3	0	0	1	0	Y_3
4	0	0	0	1	Y_4
5	½	1/2	0	0	Y_{12}
6	½	0	1/2	0	Y_{13}
7	1/2	0	0	1/2	Y_{14}
8	0	1/2	1/2	0	Y_{23}
9	0	1/2	0	1/2	Y_{24}
10	0	0	1/2	1/2	Y_{34}

Hence if we substitute in Eqn. (3.11), the coordinates of the first point (X1=1, X2=0, X3=0,and X4=0, Fig (3.1), we get that Y1= β1.

And doing so in succession for the other three points in the tetrahedron, we obtain
Y2= β2, Y3= β3, Y4= β4 (3.13)

The substitution of the coordinates of the fifth point yields
$Y_{12} = ½ X_1 + ½X_2 + ½X_1.1/2X_2$
$= ½ β_1 + ½ β_2 + 1/4 β_{12}$

But as $β_i$ = Yi then
$Y_{12} = ½ β_1 - ½ β_2 - 1/4 β_{12}$

Thus
$β_{12} = 4Y_{12} - 2Y_1 - 2Y_2$ (3.14)

And similarly,
$β_{13} = 4 Y_{13} - 2Y_1 - 2Y_2$
$β_{23} = 4 Y_{23} - 2Y_2 - 2Y_3$
etc.

Or generalizing,
$β_i = Y_i$ and $β_{ij} = 4Y_{ij} - 2Y_i - 2Y_j$ (3.15)

which are the coefficients of the reduced second degree polynomial for a q-component mixture, since the four points defining the coefficients $β_{ij}$ lie on the edge. The subscripts of the mixture property symbols indicate the relative content of each component X_i alone and the property of the mixture is denoted by Y_i.

3.2.3 Actual and Pseudo Components

The requirements of the simplex that
$\Sigma Xi = 1$
makes it impossible to use the normal mix ratios such as 1:3, 1:5, etc, at a given water/cement ratio. Hence a transformation of the actual components (ingredient proportions) to meet the above criterion is unavoidable. Such transformed ratios say X1(i), X2(i), and X3(i) and X4(i)for the ith experimental points are called pseudo components. Since X1, X2 and X3 are subject to $\Sigma Xi = 1$, the transformation of cement:sand:quarry dust :water at say 0.30 water/cement ratio cannot easily be computed because X1, X2, X3 and X4 are in pseudo expressions X1(i), X2(i), X3(i) and X4(i).For the ith experimental point, the transformation computations are to be done.
The arbitrary vertices chosen on the triangle are A1(1:6.25:3.75:0.32), A2(1:5.64:3.36:30) A3(1:4.88:2.92:0.29), and A4(1:6.26:3.74:0.37), based on experience and earlier research reports.

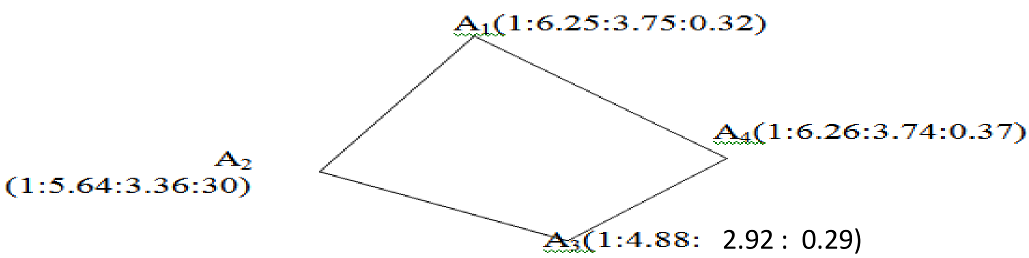

Fig 3.1 Tetrahedral Simplex

3.2.4 Transformation Matrix

If Z denotes the actual matrix of the ith experimental points, observing from Table 3.2 (points 1 to 4),
BZ = X =1 (3.16)
where B is the transformed matrix.
Therefore, $B = I.Z^{-1}$
Or $B=Z^{-1}$ (3.17)
For instance, for the chosen ratios A1, A2 , A3 and A4 (fig. 3.6),

$$Z = \begin{pmatrix} 1 & 6.25 & 3.75 & 0.32 \\ 1 & 5.64 & 3.36 & 0.30 \\ 1 & 4.88 & 2.92 & 0.29 \\ 1 & 6.26 & 3.74 & 0.37 \end{pmatrix} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3.18)$$

From Eqn 3.17,

$B = Z^{-1}$

$$Z^{-1} = \begin{pmatrix} -0.55 & 1.00 & 4.09 & -3.55 \\ -17.53 & 27.14 & -12.79 & 3.18 \\ 30.52 & -44.29 & 19.68 & -5.91 \end{pmatrix}$$

-10.39 -14.29 6.49 18.18

Hence,
$B\,Z^{-1} = Z.\,Z^{-1}$

$$= \begin{pmatrix} 1 & 6.25 & 3.75 & 0.32 \\ 1 & 5.64 & 3.36 & 0.30 \\ 1 & 4.88 & 2.92 & 0.29 \\ 1 & 6.26 & 3.74 & 0.37 \end{pmatrix} \begin{pmatrix} -0.55 & 1.00 & 4.09 & -3.55 \\ -17.53 & 27.14 & -12.79 & 3.18 \\ 30.52 & -44.29 & 19.68 & -5.91 \\ -10.39 & -14.29 & 6.49 & 18.18 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus, for actual component Z, the pseudo component X is given by

$$X \begin{pmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \\ X_4^{(i)} \end{pmatrix} = B \begin{pmatrix} -0.55 & 1.00 & 4.09 & -3.55 \\ -17.53 & 27.14 & -12.79 & 3.18 \\ 30.52 & -44.29 & 19.68 & -5.91 \\ -10.39 & -14.29 & 6.49 & 18.18 \end{pmatrix} Z \begin{pmatrix} Z_1^{(i)} \\ Z_2^{(i)} \\ Z_3^{(i)} \\ Z_4^{(i)} \end{pmatrix}$$

which gives the $X_i(i=1,2,3,4)$ values in Table 3.2.
The inverse transformation from pure component to actual component is expressed as

$$AX = Z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3.19)$$

where A = inverse matrix

$$A = Z\,X^{-1}.$$

From Eqn (3.16), X = BZ, therefore,

$$A = Z.\,(BZ)^{-1}$$
$$A = Z.Z^{-1}B^{-1}$$
$$A = IB^{-1}$$
$$= B^{-1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3.20)$$

This implies that for any pseudo component X, the actual component is given by

$$Z \begin{pmatrix} Z_1^{(i)} \\ Z_2^{(i)} \\ Z_3^{(i)} \\ Z_4^{(i} \end{pmatrix} = B \begin{pmatrix} 1 & 6.25 & 3.75 & 0.32 \\ 1 & 5.64 & 3.36 & 0.30 \\ 1 & 4.88 & 2.92 & 0.29 \\ 1 & 6.26 & 3.74 & 0.37 \end{pmatrix} X \begin{pmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \\ X_4^{(i)} \end{pmatrix} \quad . \quad . \quad . \quad . \quad . \qquad (3.21)$$

Eqn 3.21 is used to determine the actual components from points 5 to 10 , and the control values from points 11 to 13 (Table 3.2).

Table 3.2 Values for Experiment

N	X_1	X_2	X_3	X_4	RESPONSE	Z_1	Z_2	Z_3	Z_4
1	1	0	0	0	Y_1	1	6.25	3.75	0.32
2	0	1	0	0	Y_2	1	5.64	3.36	0.3
3	0	0	1	0	Y_3	1	4.88	2.92	0.29
4	0	0	0	1	Y_4	1	6.26	3.74	0.37
5	½	1/2	0	0	Y_{12}	1	5.945	3.555	0.31
6	½	0	1/2	0	Y_{13}	1	5.565	3.335	0.305
7	½	0	0	1/2	Y_{14}	1	6.255	3.745	0.345
8	0	1/2	1/2	0	Y_{23}	1	5.26	3.14	0.295
9	0	1/2	0	1/2	Y_{24}	1	5.95	3.55	0.335
10	0	0	1/2	1/2	Y_{34}	1	5.57	3.33	0.33
Control points									
11	0.25	0.25	0.25	0.25	Y_{1234}	1.00	5.76	3.44	0.32
12	0.5	0.25	0.25	0	Y_{1123}	1.00	5.76	3.45	0.31
13	0.25	0.5	0	0.25	Y_{1224}	1.00	5.95	3.55	0.32

3.2.5 Use of Values in Experiment

During the laboratory experiment, the actual components were used to measure out the appropriate proportions of the ingredients. Hence, cement, sand, aggregate dust and water were used for casting the samples. The values obtained are presented in Tables in section 5.

3.3 Adequacy of Tests

This is carried out by testing the fit of a second degree polynomial [13]. After the coefficients of the regression equation has been derived, the statistical analysis is considered necessary, that is, the equation should be tested for goodness of fit, and the equation and surface values bound into the confidence intervals. In experimentation following simplex-lattice designs there are no degrees of freedom to test the equation for adequacy, so, the experiments are run at additional so-called control points.

The number of control points and their coordinates are conditioned by the problem formulation and experiment nature. Besides, the control points are sought so as to improve the model in case of inadequacy. The accuracy of response prediction is dissimilar at different points of the simplex. The variance of the predicted response, S_Y^2, is obtained from the error accumulation law. To illustrate this by the second degree polynomial for a ternary mixture, the following points are assumed:

X_i can be observed without errors [13].

the replication variance, S_Y^2, is similar at all design points, and

response values are the average of n_i and n_{ij} replicate observations at appropriate points of the simplex.

Then the variance $S_{\hat{Y}i}$ and $S_{\hat{Y}ij}$ will be

$$(S_{\hat{Y}}^2)_i=S_Y^2/n_i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3.22)$$

$$(S_{\hat{Y}}^2)_{ij}=S_Y^2/n_{ij}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3.23)$$

In the reduced polynomial,

$$\hat{Y} = \beta_1X_1+ \beta_2X_2 + \beta_3X_3 + \beta_4X_4 + \beta_{12}X_1X_2+ \beta_{13}X_1X_3 + \beta_{14}X_1X_4+ \beta_{23}X_2X_{23}+ \beta_{24}X_2X_4 + \beta_{34}X_3X_4 \quad . \quad . \quad . \qquad .(3.24)$$

If we replace coefficients by their expressions in terms of responses,

$\beta_i = Y_i$ and $\beta_{ij} = 4Y_{ij} - 2Y_i - 2Y_j$

$\hat{Y} = Y_1X_1 + Y_2X_2 + Y_3X_3 ++ Y_4X_4 + (4Y_{12} - 2Y_1 - 2Y_2)X_1X_2 + (4Y_{13} - 2Y_1 - 2Y_3)X_1X_3 + (4Y_{14} - 2Y_1 - 2Y_4)X_1X_4 + (4Y_{23} - 2Y_2 - 2Y_3)X_2X_3 + (4Y_{24} - 2Y_2 - 2Y_4)X_2X_4 + (4Y_{34} - 2Y_3 - 2Y_4)X_3X_4$

$= Y_1(X_1 - 2X_1X_2 - 2X_1X_3 - 2X_1X_4) + Y_2(X_2 - 2X_1X_2 - 2X_2X_3 - 2X_2X_4) + Y_3(X_3 - 2X_1X_3 + 2X_2X_3 + 2X_3X_4) + Y_4(X_4 - 2X_1X_4 + 2X_2X_4 + 2X_3X_4) + 4Y_{12}X_1X_2 + 4Y_{13}X_1X_3 + 4Y_{14}X_1X_4 + 4Y_{23}X_2X_3 + 4Y_{24}X_2X_4 + 4Y_{34}X_3X_4$ . .(3.25)

Using the condition $X_1+X_2+X_3+X_4=1$, we transform the coefficients at Y_i

$X_1 - 2X_1X_2 - 2X_1X_3 - 2X_1X_4 = X_1 - 2X_1(X_2 + X_3 + X_4)$

$= X_1 - 2X_1(1 - X_1) = X_1(2X_1 - 1)$ and so on. . (3.26)

Thus

$\hat{Y} = X_1(2X_1 - 1)Y_1 + X_2(2X_2 - 1)Y_2 + X_3(2X_3 - 1)Y_3 + X_4(2X_4 - 1)Y_4 + 4Y_{12}X_1X_2 + 4Y_{13}X_1X_3 + 4Y_{14}X_1X_4 + 4Y_{23}X_2X_3 + 4Y_{24}X_2X_4 + 4Y_{34}X_3X_4$ . . . . . (3.27)

Introducing the designation

$a_i = X_i(2X_1 - 1)$ and $a_{ij} = 4X_iX_j$ . . . . . (3.27)

and using Eqns (3.22) and (3.230) give the expression for the variance S_Y^2.

$S_{\hat{Y}}^2 = S_Y^2 \left(\sum_{1\le i\le q} a_{ii}/n_i + \sum_{1\le i<j\le q} a_{jj}/n_{ij}\right)$ . . . . .. (3.28)

If the number of replicate observations at all the points of the design are equal, i.e. $n_i=n_{ij}= n$, then all the relations for $S_{\hat{Y}}^2$ will take the form

$S_{\hat{Y}}^2 = S_Y^2\xi/n$ . . . . (3.29)

where, for the second degree polynomial,

$\xi = \sum_{1\le i\le q} a_i^2 + \sum_{1\le i<j\le q} a_{ij}^2$ . . (3.30)

As in Eqn (3.30), ξ is only dependent on the mixture composition. Given the replication Variance and the number of parallel observations n, the error for the predicted values of the response is readily calculated at any point of the composition-property diagram using an appropriate value of ξ taken from the curve.

Adequacy is tested at each control point, for which purpose the statistic is built:

$t = \Delta_Y/(S_{\hat{Y}}^2 + S_Y^2) = \Delta_Y n^{1/2} / (S_Y(1 + \xi))^{1/2}$ . . . .(3.31)

where $\Delta_Y = Y_{exp} - Y_{theory}$ . . . .(3.32)

and n = number of parallel observations at every point.

The t-statistic has the student distribution, and it is compared with the tabulated value of $t_{\alpha/L}(V)$ at a level of significance α, where L = the number of control points, and V = the number for the degrees of freedom for the replication variance.

The null hypothesis is that the equation is adequate is accepted if $t_{cal} < t_{Table}$ for all the control points.

The confidence interval for the response value is

$\hat{Y} - \Delta \le y \le \hat{Y} + \Delta$ . . . . (3.33)

$\Delta = t_{\alpha/L,k} S_{\hat{Y}}$ . . . . (3.34)

where k is the number of polynomial coefficients determined.

Using Eqn (3.29) in Eqn (3.34)

$\Delta = t_{\alpha/k,v} S_Y(\xi/n)^{1/2}$ . . . . (3.35)

Where n is the number of parallel observations at every point

IV. Methodology

To be a good structural material, the material should be homogeneous and isotropic. The Portland cement, sandcrete or concrete are none of these, nevertheless they are popular construction materials (Wilby, 1963).The necessary materials required in the manufacture of the sandcrete in the study are cement, sand, quarry dust and water.

4.1 Materials

The sand material were collected at the Iyioku River sand basin in Enugu State and conformed to BS 882 and belongs to zone 2 of the ASHTO classification.

The water for use is pure drinking water which is free from any contamination i.e. nil Chloride content, pH =6.9, and Dissolved Solids < 2000ppm. Ordinary Portland cement is the hydraulic binder used in this project and sourced from the Dangote Cement Factory, and assumed to comply with the Standard Institute of Nigeria (NIS) 1974, and kept in an air-tight bag. The quarry dust was got from the Ishiagu quarry site and conformed to a maximum size of 2mm.

4.2 Preparation of Samples

The sourced materials for the experiment were transferred to the laboratory. The pseudo components of the mixes were designed following the background theory from where the actual variables were developed. The component materials were mixed at ambient temperature according to the specified proportions of the actual components generated in Table 3.2. In all, two hollow blocks of 450mm x225 x150mm for each of ten experimental points and three control points were cast for the compressive strength test, cured for 28 days after setting and hardening.

4.3 Strength Test

After 28 day of curing, the blocks were crushed to determine the sandcrete block strength, using the compressive testing machine to the requirements of BS 1881:Part 115 of 1986.

V. Result and Analysis

5.1 Determination of Replication Error And Variance of Response

To raise the experimental design equation models by the lattice theory approach, two replicate experimental observations were conducted for each of the ten design points. Below is the table which contain the results of two repetitions each of the 10 design points plus three control points of the (4,2) simplex lattice, and show the mean and variance values per test of the observed response, using the following mean and variance equations below:

$$\ddot{Y} = \sum_{i} (Y_r)/r \qquad (5.1)$$

where $\hat{Y}$ is the mean of the response values and r =1,2.

$$S_Y^2 = \sum^{n} [(Y_i - \ddot{Y}_i)^2]/(n-1) \qquad (5.2)$$

where n = 13.

Table 5.1 Result of the Replication Variance of the Compressive Strength Response for 450mm x225 x150mm Block

Experiment No (n)	Repetition	Response f_c (N/mm$^{2)}$	Response Symbol	$\sum Y_r$	$\ddot{Y}_r$	$\sum(Y_r - \ddot{Y}_r)^2$	S_i^2
1	1A 1B	1.42 1.54	Y_1	2.96	1.48	0.01	0.00
2	2A 2B	2.01 2.11	Y_2	4.12	2.06	0.01	0.00
3	3A 3B	1.90 1.61	Y_3	3.51	1.76	0.04	0.00
4	4A 4B	2.31 2.36	Y_4	4.67	2.34	0.00	0.00
5	5A 5B	2.20 1.76	Y_{12}	3.96	1.98	0.10	0.01
6	6A 6B	2.44 1.98	Y_{13}	4.42	2.21	0.11	0.01
7	7A 7B	1.42 1.54	Y_{14}	2.96	1.48	0.01	0.00
8	8A 8B	2.01 2.11	Y_{23}	4.12	2.06	0.01	0.00
9	9A 9B	1.90 1.61	Y_{24}	3.51	1.76	0.04	0.00
10	10A 10B	2.31 2.36	Y_{34}	4.67	2.34	0.00	0.00
			Control Points				
11	11A 11B	2.12 1.72	C_{1234}	3.84	1.92	0.08	0.01
12	12A 12B	1.44 1.98	C_{1123}	3.42	1.71	0.15	0.01
13	13A 130B	2.04 1.81	C_{1224}	3.85	1.93	0.03	0.00
						0.57	0.05

Replication Variance

$S_Y^2 = \sum S_i^2 = 0.05$

That's

Replication error $S_Y = (0.05)^{1/2} = 0.223$

Using the experimental values in Table 5.1 in Eqn (3.15), we get the coefficients of the reduced second degree polynomial as follows:

$\beta_1 = 1.48$, $\beta_2 = 2.06$, $\beta_3 = 1.76$, $\beta_4 = 2.34$, $\beta_{12} = 0.84$, $\beta_{13} = 2.37$, $\beta_{14} = -1.71$, $\beta_{23} = 0.61$
$\beta_{24} = -1.77$, $\beta_{34} = 1.16$

Thus, from Eqn (3.11),

$$\hat{Y}_c = 1.48X_1 + 2.06X_2 + 1.76X_3 + 2.34X_4 + 0.84X_1X_2 + 2.37X_1X_3 - 1.71X_1X_4 + 0.61X_2X_3 - 1.77X_2X_4 + 1.16X_3X_4 \quad (5.3)$$

Eqn 5.3 is the mathematical model of the compressive strength of hollow sandcrete block based on 28-day strength.

5.2 Test of Adequacy of the Compressive strength Model

Eqn 5.3, the equation model, will be tested for adequacy against the controlled experimental results.

We recall our statistical hypothesis as follows:

1. Null Hypothesis (H_0): There is no significant difference between the experimental values and the theoretical expected results of the compressive strength.
2. Alternative Hypothesis (H_1): There is a significant difference between the experimental values and the theoretical expected results of the compressive strength.

5.2.1 t-Test for the Compressive strength Model

If we substitute for X_i in Eqn 5.4 from Table 3.3, the theoretical predictions of the response ($\hat{Y}$) can be obtained. These values can be compared with the experimental results (Table 5.1). For the t-test (Table 5.2), a, ξ, t and Δ_y are evaluated using Eqns 3.31, 3.32, 3.35, 3.27 and 3.30 respectively.

Table 5.2 t-Test for the Test Control Points

N	CN	I	J	a_i	a_{ij}	a_i^2	a_{ij}^2	ξ	$\ddot{Y}$	$\hat{Y}$	Δ_y	t
		1	2	-0.125	0.250	0.016	0.063					
1	C_1	1	3	-0.125	0.250	0.016	0.063					
		1	4	-0.125	0.250	0.016	0.063		1.92			
		2	3	-0.125	0.250	0.016	0.063			2.00	0.82	4.85
		2	4	-0.125	0.250	0.016	0.063					
		3	4	-0.125	0.250	0.016	0.063					
						0.094	0.375	0.15				
		1	2	0.000	0.500	0.000	0.250					
2	C_2	1	3	0.000	0.500	0.000	0.250					
		1	4	0.000	0.000	0.000	0.000		1.71			
		2	3	0.000	0.250	0.000	0.063			2.13	0.42	2.31
		2	4	0.000	0.000	0.000	0.000					
		3	4	0.000	0.000	0.000	0.000					
						0.000	0.563	0.32				
		1	2	-0.125	0.500	0.016	0.250					
		1	3	-0.125	0.000	0.016	0.000					
		1	4	-0.125	0.250	0.016	0.063					
3	C_3	2	3	-0.125	0.000	0.016	0.000		1.93	1.76	0.17	0.93
		2	4	-0.125	0.500	0.016	0.250					
		3	4	-0.125	0.000	0.016	0.000					
						0.094	0.563	0.33				

Significance level α = 0.05,

i.e. $t_{\alpha/L}(V) = t_{0.05/3}(13)$, where L=number of control points.

From the Student's t-table, the tabulated value of $t_{\alpha/L}(V) = t_{0.05/3}(13)$ is found to be 2.450 which is greater than most of the calculated t-values in Table 5.2. Hence we can accept the Null Hypothesis.
From Eqn 3.35, with k=10 and $t_{\alpha/k,v} = t_{0.05/k}(13) = 3.01$,

Δ = 0.18 for C_{1234}, 0.26 for $C_{1124\,=0.26}$, and 0.27 for C_{1224},
which satisfies the confidence interval equation of
Eqn 3.33 when viewed against most response values in Table 5.2.

5.3 Computer Program

A computer program is developed for the model. In the program any desired compressive strength can be specified as an input and the computer processes and prints out possible combinations of mixes that match the property, to the following tolerance:

Compressive Strength - 0.001 N/mm^2,

Interestingly, should there be no matching combination, the computer informs the user of this. It also checks the maximum value obtainable with the model.

5.3.1 Choosing a Combination

It can be observed that the strength of 2.4125 N/sq mm yielded 6 combinations. To accept any particular proportions depends on the factors such as workability, cost and honeycombing of the resultant lateritic concrete.

VI. Conclusion and Recommendation

6.1 Conclusion

Henry Scheffe's simplex design was applied successfully to prove that the compressive strength of sandcrete block is a function of the proportion of the ingredients (cement, sand quarry dust and water), but not the quantities of the materials.

The maximum compressive strength obtainable with the compressive strength model is 2.4125 N/sq mm. See the computer run outs (Appendix 1) which show all the possible sandcrete mix options for the desired compressive strength property, and the choice of any of the mixes is the user's.

One can also draw the conclusion that the maximum values achievable, within the limits of experimental errors, is quite below that obtainable using only sand as aggregate. This is due to the predominantly high silt content of quarry dust.

It can be observed that the task of selecting a particular mix proportion out of many options is not easy, if workability and other demands of the resulting sandcrete have to be satisfied. This is an important area for further research work.

The research work is a great advancement in the search for the applicability of quarry dust sandcrete production in regions where sand is extremely scarce with the ubiquity of quarry dust.

6.2 Recommendations

From the foregoing study, the following could be recommended:

i) The model can be used for the optimization of the strength of concrete made from cement, quarry dust and water.

ii) Quarry dust aggregates cannot adequately substitute sharp sand aggregates for heavy construction.

iii) More research work need to be done in order to match the computer recommended mixes with the workability of the resulting concrete.

iii) The accuracy of the model can be improved by taking higher order polynomials of the simplex.

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APPENDIX 1

```
'QBASIC BASIC PROGRAM THAT OPTIMIZES THE PROPORTIONS OF SANDCRETE MIXES
   'USING THE SCHEFFE'S MODEL FOR CONCRETE COMPRESSIVE STRENGTH
   CLS
   C1$ = "(ONUAMAH.HP) RESULT OUTPUT ": C2$ = "A COMPUTER PROGRAM "
   C3$ = "ON THE OPTIMIZATION OF A 4-COMPONENT SANDCRETE MIX"
   PRINT C2$ + C1$ + C3$
   PRINT
   'VARIABLES USED ARE
   'X1, X2, X3,X4, Z1, Z2, Z3,Z4, Z$,YT, YTMAX, DS
   'INITIALISE I AND YTMAX
   I = 0: YTMAX = 0
    FOR MX1 = 0 TO 1 STEP .01
     FOR MX2 = 0 TO 1 - MX1 STEP .01
      FOR MX3 = 0 TO 1 - MX1 - MX2 STEP .01
        MX4 = 1 - MX1 - MX2 - MX3
        YTM = 1.48 * MX1 + 2.06 * MX2 + 1.76 * MX3 + 2.34 * MX4 + .84 * MX1 * MX2 + 2.37 * MX1 * MX3 -
1.71 * MX1 * MX4 + .61 * MX2 * MX3 - 1.77 * MX2 * MX4 + 1.16 * MX3 * MX4
        IF YTM >= YTMAX THEN YTMAX = YTM
      NEXT MX3
     NEXT MX2
    NEXT MX1
   INPUT "ENTER DESIRED STRENGTH, DS = "; DS
   'PRINT OUTPUT HEADING
   PRINT
   PRINT TAB(1); "No"; TAB(10); "X1"; TAB(18); "X2"; TAB(26); "X3"; TAB(32); "X4"; TAB(40);
"YTHEORY"; TAB(48); "Z1"; TAB(56); "Z2"; TAB(62); "Z3"; TAB(70); "Z4"
   PRINT
   'COMPUTE THEORETICAL STRENGTH, YT
    FOR X1 = 0 TO 1 STEP .01
     FOR X2 = 0 TO 1 - X1 STEP .01
      FOR X3 = 0 TO 1 - X1 - X2 STEP .01
       X4 = 1 - X1 - X2 - X3
       YT = 1.48 * X1 + 2.06 * X2 + 1.76 * X3 + 2.34 * X4 + .84 * X1 * X2 + 2.37 * X1 * X3 - 1.71 * X1 * X4 + .61
* X2 * X3 - 1.77 * X2 * X4 + 1.16 * X3 * X4
       IF ABS(YT - DS) <= .001 THEN
       'PRINT MIX PROPORTION RESULTS
       Z1 = X1 + X2 + X3 + X4: Z2 = 6.25 * X1 + 5.64 * X2 + 4.88 * X3 + 6.26 * Z4: Z3 = 3.75 * X1 + 3.36 * X2 +
2.92 * X3 + 3.74 * X4: Z4 = .32 * X1 + .3 * X2 + .29 * X3 + .37 * X4
       I = I + 1
       PRINT TAB(1); I; USING "##.###"; TAB(7); X1; TAB(15); X2; TAB(23); X3; TAB(32); X4; TAB(40);
YT; TAB(48); Z1; TAB(56); Z2; TAB(62); Z3; TAB(70); Z4
       PRINT
       PRINT
       IF (X1 = 1) THEN 550
      ELSE
       IF (X1 < 1) THEN GOTO 150
      END IF
```

```
150     NEXT X3
     NEXT X2
    NEXT X1
   IF I > 0 THEN 550
   PRINT
   PRINT "SORRY, THE DESIRED STRENGTH IS OUT OF RANGE OF MODEL"
   GOTO 600
550 PRINT TAB(5); "THE MAXIMUM VALUE PREDICTABLE BY THE MODEL IS "; YTMAX; "N / Sq; mm; "
600 END
```

```
 ENTER DESIRED STRENGTH, DS = ? 1.4
 No     X1      X2      X3       X4    YTHEORY   Z1       Z2      Z3       Z4
  1    0.630  0.000  0.000   0.370  1.400      1.000  3.937  3.746   0.339
  2    0.650  0.010  0.000   0.340  1.400      1.000  6.238   3.743  0.337
  3    0.690  0.020  0.000   0.290  1.400      1.000  6.534   3.739  0.334
  4    0.700  0.020  0.000   0.280  1.399      1.000  6.579   3.739  0.334
  5    0.760  0.020  0.000   0.220  1.400      1.000  6.951   3.740  0.331
  6    0.830  0.010  0.000   0.160  1.400      1.000  7.313   3.744  0.328
    THE MAXIMUM VALUE PREDICTABLE BY THE MODEL IS  2.4125 N / Sq mm
 Press any key to continueA COMPUTER PROGRAM (ONUAMAH.HP) RESULT OUTPUT ON THE
 OPTIMIZATION OF A 4-COMPONE
 NT SANDCRETE MIX
 ENTER DESIRED STRENGTH, DS = ? 2.9
 No     X1     X2     X3     X4    YTHEORY   Z1     Z2    Z3     Z4
 SORRY, THE DESIRED STRENGTH IS OUT OF RANGE OF MODEL
 Press any key to continue
```