Optimization of two stage supply chain Inventory models sales decision with advertising

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Abstract:

In a modern Business world technologies play an important role. Most of the companies use the technologies in their business and achieve their goal. One of the most important technologies in the business field is advertising. Through advertising many firms execute their different types of pricing strategies, to attract the buying behavior of the customers and to differentiate themselves from their competitors. With the help of advertising techniques vendor share their demand and inventory information with their customers to improve the coordination and reduce the lead times. Now we introduce the advertisement cost in the optimizing inventory and sales decisions in a two stage supply chain with imperfect production and backorders and maximize the optimization. Numerical example is provided to illustrate the advertisement cost in the expected total profit of the system.

Keywords: Integrated inventory, Pricing, Price sensitive demand, Lead time, Advertising

Introduction:

Inventory management is one of the most important key factors for any business manager to efficiently and successfully operate in the fiercely competitive modern global market. In today's competitive world companies require small lead times, low costs and high customer service levels to survive, because of this companies have become more customer focused. This result is that companies have been putting in significant effort to reduce their lead times, so that information sharing is very important for both buyers and suppliers in their business world. Information sharing may result in shorter lead times, reduced inventories and low operating cost which all contribute to higher supply chain profits (Cachon and Fisher 2000 , Yu et al 2001).

The main focus of companies in the 20th century was the customers. To retain the customers the vendor handles various techniques such as discount offer, different pricing strategies, concession in cost and so on. To make all these benefits are reaching to them, it has to be communicated to them at the right time in a right way. For achieving these companies will tackle the advertising techniques. With the help of advertisement companies promote their product, pricing strategies and service to the customers. These advertisements reach the customers via television, newspaper and direct mail, the latter in the form of sales circulars or coupons. Distribution of handbills is also one of the medium of advertising. The bills are issued to the people for them to get to know the motive of advertisement. Vendor may choose any medium to advertise their product. In this paper, we assumed that the vendor preferred the television for advertising their product. The aim of this paper is to study an integrated inventory model that considers operations and pricing decisions with Advertisement. The remainder of this paper is organized as follows, Section2 presents a review of related literature and section 3 defines assumptions and notations, Section4 formulate the mathematical model. In section 5 we develop a solution procedure for the model, Section 6 we provide some numerical example, and finally we conclude the paper in section7.

2. Literature review:

Researchers have studied how the lot sizing decisions of buyer and vendor interrelate. If the buyer for example determines an optimal order quantity based on his/her individual costs the resulting order quantity may be too small to be manufactured economically by the vendor. If both actors agreed to cooperate and to determine an optimal lot size based on the total cost of the system, then the position of the entire supply chain may be improved. In this line of thought Goyal (1976) was probably the first to derive an optimal lot size from the system's point of view which is calculated based on the sum of the buyer's and vendor's total cost functions. His work was encouraged by many researchers who extend Goyal's work to include multiple strategies (e.g Glock(2011),multiple actors on each stage (e.g Golck 2011Glock and kim (2014a,2014b) or productivity issues (e.g Glock 2012b). Porteus (1986) was one of the first to consider imperfect production in a lot size model. Salameh and Jaber (2000) considered lot sizing and inspection in an EOQ model where a random portion of units is defertive the model of Salameh and Jaber (2000) has received wide attention in the literature, which is documented in the review of khan et al(2011). Wee et al (2007) extended Salameh and Jaber's (2000) model into include shortages. Chang and Ho (2010) revisited the work of Wee et al (2007) and adopted the suggestion of Maddah and Jaber (2008) to use the renewal reward theorem to derive the expected profit per unit of time for their model. Ouyang et al (2006) presented other JELS models and assumed that defective rates are fuzzy. Whitin(1955) who was one of the first to consider price sensitive demand in an inventory model. Poretus(1985) obtained an explicit solution for this model. Viswanathan and Wang (2003) evaluated the effectiveness of quantity discounts as coordination mechanism in distribution channels with price sensitive demand. Sajadieh and Jokar (2009) provided integrated production inventory marketing model with linearly price dependent demand and maximized the joint total profit of vendor and buyer.

The concept of co-op advertising was developed by Berger (1972) which highlighted that mathematical modeling could yield improved managerial decisions and better performance of whole channel. Huang, Li(2002),Zhu and Chau (2002) developed independently a co-op advertising model for a one manufacturer, one retailer supply chain. Szmerekovsky and Zhang (2009) further extended the work of Huang et el (2002) and Huang and Li (2001) in developing a price discount model to coordinate advertising expanses of two parties. Shang et al (2011) developed the cooperative advertising model that involves one manufacturer and two retailers. Our review of the literatures showed that joint economic lot size models have frequently been studied in the past and that the production of defective items and pricing decision have been studied in the stream of research, however that imperfect production planned shortages and pricing with advertisement techniques have been studied in combination in the context of JELS model.

3. Assumptions and Notations:

This paper extends the work of M.A.Rad et al (2014). The following assumptions are made in developing the proposed models.

1. There is a single vendor and single buyer for a single product.
2. Shortages are allowed at the buyer if they are completely backordered.
3. The demand rate is a decreasing function of the selling price with D(p) = $\alpha p^{-\beta}$
   Where α > 0 is a scaling factor and β > 1 is the index of price elasticity.
4. Each shipment the buyer receives contains Y percent of defective items where y is uniformly distributed on [L,J] with $0 \le L \le J \le 1$
5. The unit purchase price paid by the buyer to the vendor. W is a decreasing function of the mean rate of defectives (i.e). w(y)=m-ky and its assumed to be higher than the unit production cost C.
6. Each shipment received by the buyer is subjected to 100% screening with a screening rate of x_b to separate good items from defective ones to avoid shortage occur during screening it is assumed that the screening rate x_b is higher than the demand rate D and that the number of good items in each shipment is at least equal to the demand that occur during the screening time.
7. The Expected number of good quality items in each shipment (1-y)Q is equal to the demand during the order cycle T. (i.e) (1-Y)Q=DT.
8. The capacity utilization ρ is defined as the ratio of the market demand the production rate R (i.e) ρ =D/R.
9. Lead time L is a constant. The lead time demand is normally distributed with mean DL and standard deviation $\sigma\sqrt{L}$
10. The reorder point r = expected demand during lead time + safely stock. (i.e) r = DL+k$\sigma\sqrt{L}$ where k is the safety stock factor.
11. The initialization cost of casting an advertisement is the amount of money spent by the vendor to get in connection with the advertising agency, which is assumed to be a constant.

The parameter used in this paper is defined as follows.

D (p)	: demand rate [units/year]
α	: scaling factor of the demand function
β	: index of price elasticity of the demand function
S	: vendor's setup cost [$/setup]
A	: buyer's ordering cost [$/setup]
h_v	: vendor's inventory holding cost[$/unit*year]
h_b	: buyer's inventory holding cost[$/unit*year]
π	: backordering cost[$/unit]
C	: unit production cost[$/unit]
CI_b	: buyer's screening cost[$/unit]
CI_v	: vendor's screening cost[$/unit]
C_w	: vendor's warranty cost for defective units [$/unit]
F	: fixed transportation cost per shipment [$/order]
v	: unit variable cost for handling or receiving an item[$/unit]
y	: rate of defectives, which is uniformly distributed on [L,J]
var	: variance of defectives
Y	: Mean rate of defectives
W(Y)	: unit purchase price [$/unit]
x_b	: screening rate of the buyer[unit/year]
x_v	: screening rate of the vendor[unit/year]
R	: production rate [unit/year]

ρ : ratio of the market demand rate to the production rate (i.e) ρ =D/R.

ρ_b : ratio of the market demand rate to the screening rate of the buyer, i.e $\rho_b = \frac{D}{x_b}$

ρ_v : ratio of the market demand rate to the screening rate of the vendor i.e, $\rho_v = \frac{D}{x_v}$

T : order cycle[year]
p : buyer's selling price[$/unit](decision variable)
Q : buyer's order quantity [units](decision variable)
b : buyer's backorder quantity[units](decision variable)
n : number of shipments(decision variable)
T : order cycle
L : lead time
A_1 : advertisement cost per unit time (i.e) $A_1(Q) = I + f_1 + v_1 + nt$

Where, I = initialization cost
f_1 = fixed cost per advertisement
v_1 = variable cost per advertisement
n_1 = number of times per advertisement taken
t = cast of telecasting the advertisement

EJTP : expected joint total profit [$/year]

3.Mathematical Model:

We derive the mathematical model under joint optimization.

3.1Buyer's perspective:

The buyer's expected total profit per unit time can be expressed as,

$$ETP_B(p,Q,b) = (p - W(Y) - \frac{CI_b + v}{(1-Y)})\alpha p^{-\beta} - \frac{(A+F)\alpha p^{-\beta}}{Q(-Y1} - \frac{h_b Q}{2(1-Y)}\left\{E(1-y)^2 + 2Y\rho_b\right\} - h_b\sigma\sqrt{L}k - \frac{(\pi\sigma\sqrt{L}\psi(k) + h_b)b^2}{2Q(1-Y)} + bh_b \quad (1)$$

Where $\psi(k) = \phi(k) - k[1-\Phi(k)]$ and ϕ and Φ are standard normal probability density function and distribution function respectively.

3.2 Vendor's perspective

The vendor's expected total profit per unit time is formulated as follows,
$ETP_v(n)$

$$= (W(Y) - \frac{C + YC_w}{(1-Y)})\alpha p^{-\beta} - \frac{A_1\alpha p^{-\beta}}{Q(1-Y)} - \frac{S\alpha p^{-\beta}}{nQ(1-Y)} - \frac{h_v Q}{2}\left[\frac{(2-n)\rho}{(1-Y)} + n - 1\right] \quad (2)$$

3.3 Integrated approach:

Vendor and buyer make decision jointly to determine the best policy for the supply chain in this case the expected joint total profit per unit time is given by,

EJTP(p,Q,b,n) =

$$(p-\frac{CI_b+v+C+YC_w}{(1-Y)})\alpha p^{-\beta}-\frac{(\frac{S}{n}+A+F+A_1)\alpha p^{-\beta}}{Q(1-Y)}-\frac{h_bQ}{2(1-Y)}\left\{E(1-y)^2+2Y\rho_b\right\}$$

$$-\frac{h_vQ}{2}\left[\frac{(2-n)\rho}{(1-Y)}+n-1\right]-\frac{(\pi\sigma\sqrt{L}\psi(k)+h_b)b^2}{2Q(1-Y)}+bh_b-h_b\sigma\sqrt{L}k \quad (3)$$

In this case of coordination the objective is to maximize eq(3) and to find optimal values for the decision variable p,Q,b,and n.

EJTP (p,Q,b,n) concave in b for given values of p,Q and n the unique optimal value of b can be derived as follows.

$$b^*=\frac{h_b}{\pi\sigma\sqrt{L}\psi(k)+h_b}Q(1-Y) \quad (4)$$

By substituting eq(4) into eq(3) and by rearranging the resulting term we get.

$$\text{EJTP}(p,Q,n) = (p-\frac{CI_b+v+C+YC_w}{(1-Y)})\alpha p^{-\beta}-\frac{(\frac{S}{n}+A+F+A_1)\alpha p^{-\beta}}{Q(1-Y)}$$

$$-\frac{Q}{2}\left\{h_b[\frac{E(1-y)^2}{(1-Y)}+\frac{2Y\rho_b}{(1-Y)}-\frac{h_b(1-Y)}{\pi\sigma\sqrt{L}\psi(k)+h_b}]+h_v[\frac{(2-n)\rho}{(1-Y)}+n-1]\right\}$$

$$-h_b\sigma\sqrt{L}k \quad (5)$$

EJTP(p,Q,n) is a concave function in Q , since forgiven values of p and n the first order partial derivative of eq(5) with respect to Q is

$$Q^*=\sqrt{\frac{2\alpha p^{-\beta}(\frac{S}{n}+A+F+A_1)}{(1-Y)H(n)}} \quad (6)$$

This is the optimal value of Q for fixed values of p and n.

To simplify notation we define,

$$H(n)=\left\{h_b[\frac{E(1-y)^2}{(1-Y)}+\frac{2Y\rho_b}{(1-Y)}-\frac{h_b(1-Y)}{\pi\sigma\sqrt{L}\psi(k)+h_b}]+h_v\left[\frac{(2-n)\rho}{(1-Y)}+n-1\right]\right\} \quad (7)$$

Substituting eq (6) into eq(5) we get the following expected joint total profit function which is a function of the two variables p and n.

$$EJTP(p,n) = (p - \frac{CI_b + v + C + YC_w}{(1-Y)})\alpha p^{-\beta} - \sqrt{\frac{2\alpha p^{-\beta}(\frac{S}{n} + A + F + A_1)H(n)}{(1-Y)}} - h_b\sigma\sqrt{L}k \quad (8)$$

For given values of p, maximizing EJTP(p,n) in n is equivalent to minimizing the following expression,

$$EJTP' = \frac{2\alpha p^{-\beta}(\frac{S}{n} + A + F + A_1)H(n)}{(1-Y)} \quad (9)$$

We first assume that n is a continuous variable. As EJTP' is convex in n the following equation for n can be obtained by solving $\frac{\partial EJTP'}{\partial n} = 0$

$$n = \sqrt{\frac{S\left\{h_b\left[\frac{E(1-y)^2}{(1-Y)} + \frac{2Y\rho_b}{(1-Y)} - \frac{h_b(1-Y)}{\pi\sigma\sqrt{L}\psi(k) + h_b}\right] + h_v\left[1 - \frac{\rho}{(1-Y)}\right]\right\}}{h_v\left(\left(A + F + A_1\right)\left(1 - \frac{\rho}{(1-Y)}\right)\right)}} \quad (10)$$

Let $n^- = \lfloor n \rfloor$, where $\lfloor n \rfloor$ represents the nearest integer smaller than or equal to.

The optimal value for the selling price p for a fixed value of n can be obtained by taking the first order partial derivative of EJTP(p,n) in eq(8) with respect to p and setting it equal to zero and solving for p numerically. For example the fsolve procedure of MATLAB could be used to solve this equation as was done in this paper,

$$\frac{\partial EJTP(p,n)}{\partial p} = \alpha p^{-\beta} - \alpha\beta p^{-\beta-1}\left(p - \frac{CI_b + v + C + YC_w}{(1-Y)}\right)$$

$$+ \frac{\beta\sqrt{\frac{\alpha p^{-\beta}\left(\frac{S}{n} + A + F + A_1\right)H(n)}{(1-Y)}}}{p\sqrt{2}} = 0 \quad (11)$$

4. Solution Procedures:

The following algorithm is used to calculate the optimal solution for the model developed in above section

Step1 : set $n = n^-$ by solving eq (10)
Step 2 : determine $p^{(n)}$ by solving eq(11)
Step 3 : compute the value of $Q^{(n)}$ using eq(6)
Step 4: calculate the values of $b^{(n)}$ using eq(4)
Step 5: compute EJTP ($p^{(n)}, Q^{(n)}, b^{(n)}, n$) using (3)
Step 6: let n=n+1, repeat steps 2 to 5 to find EJTP ($p^{(n)}, Q^{(n)}, b^{(n)}, n$).
Step 7: If EJTP ($p^{(n)}, Q^{(n)}, b^{(n)}, n$) $\geq$ EJTP ($p^{(n-1)}, Q^{(n-1)}, b^{(n-1)}, n-1$) go to step (6) otherwise, the optimal solution is (p^*, Q^*, b^*, n^*)= EJTP ($p^{(n-1)}, Q^{(n-1)}, b^{(n-1)}, n-1$)

5. Numerical Examples:

To illustrate the behavior of our model the following parameters are used. A=$100/order, F=$100/order, S=$1200/setup C=$2.5/units, v=$1/units, CI_b=$0.1/unit, π=$1.5/(unit/year), I=1000,$f_1$=250,$v_1$=250,$n_1$=7,t=500, h_b=$0.86/(unit/year), h_v=0.25/(unit/year), ρ=0.8, ρ_b=0.3, σ=5, $\alpha = 300000$ and β =1.25. Therefore, D (p)= $300000p^{-1.25}$ and W(Y)=9-20Y.Cw=$11/units. The percentage of defective items ,y is a continuous random variable which is uniformly distributed on (0,0.04) as in Salameh and Jaber (2000)

The optimal values of p,q,b,n and the expected total system profit for the jointly optimized model are p*=20.3. n*=2,Q*=9000,b*=758.52.EJTP*=10482.

6. Conclusion:

In this we study the optimizing inventory model in a two level supply chain with Advertisement cost. According to the numerical result, we can able to understand that the technology is very important to the vendor to execute their product in front of their customers. Introducing the advertisement cost is used to reduce the number of shipments for the vendors.

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