
**STATISTICAL ANALYSIS OF RELIABILITY MODEL CONSISTING OF
TWO UNIT REDUNDANT SYSTEM WITH FAILURE RATES**

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ABSTRACT:

Failure rate is the frequency with which a system or component fails, expressed for example in failures per hour. It is often denoted by λ and is important in reliability theory. In practice, the reciprocal rate MTBF is more commonly expressed and used for high quality components or systems. In this paper the system consisting of two identical units in which one is operative and the other is considered as cold standby to analyze statistically. Under certain assumptions the analytical expressions for the mean time system failure is obtained.

1. INTRODUCTION:-

Reliability is both a desirable as well as necessary factor in the present day technology for achieving healthy economic progress of a nation. Now a days, reliability is not only a subject of study for scientists and academicians but also a serious concern to the engineers, manufacturer, economists and government leaders as well. Reliability considerations make more effective use of resources and it results in an increase in productivity and decrease in wastage of money, material and manpower. In the innovation of new and improved technological systems, reliability is acquiring special importance as one among the many important system measures such as performance, cost, etc.

Various researchers including [1,2,3,4] working in the field of reliability theory have studied stand by system with the assumption that the failure rates remain fixed throughout the life of the system. But, in real practical situations there exist such systems in which the expected down time can be minimized by operating the online unit at a tolerable reduced efficiency when the spare unit is not available. Also if the system is in down state, then it will be better to increase the repair and inspection rates of the failed unit.

Keeping the above situations in view, in this, paper it is analysed that two units cold standby system with changeable failure, repair and inspection rates. The system model under study is described as follows.

2. ASSUMPTIONS:-

- (i) The system consists of two identical units, initially one unit is operative and the other is kept as cold standby.
- (ii) Upon failure of an operative unit, the cold standby unit becomes operative instantaneously.

A single repair facility with 'FCFS' service discipline is used for repair, inspection and post repair.

After the repair, the unit goes for inspection for deciding whether the repair is perfect or not. If the repair of a unit is found to be perfect then the repaired unit becomes operative or cold standby otherwise it goes for post repair.

The failure rate of an operative unit decreases automatically when spare unit is not available.

The repair and inspection rate of the failed unit increases when the system is in down state.

Failure rate of operative unit is constant.

The rates of repair, inspection and post repair are constant in both up and down states identifying the suitable regenerative points following measures of system effectiveness are obtained:-

Transient and steady state transition probabilities.

Mean sojourn times in regenerative states.

Distribution of time to system failure and its mean (MTSF).

Point wise availability and steady availability of the system.

Expected busy period of the repairman in time interval (o,t).

Expected number of visits by the repairman in (o,t).

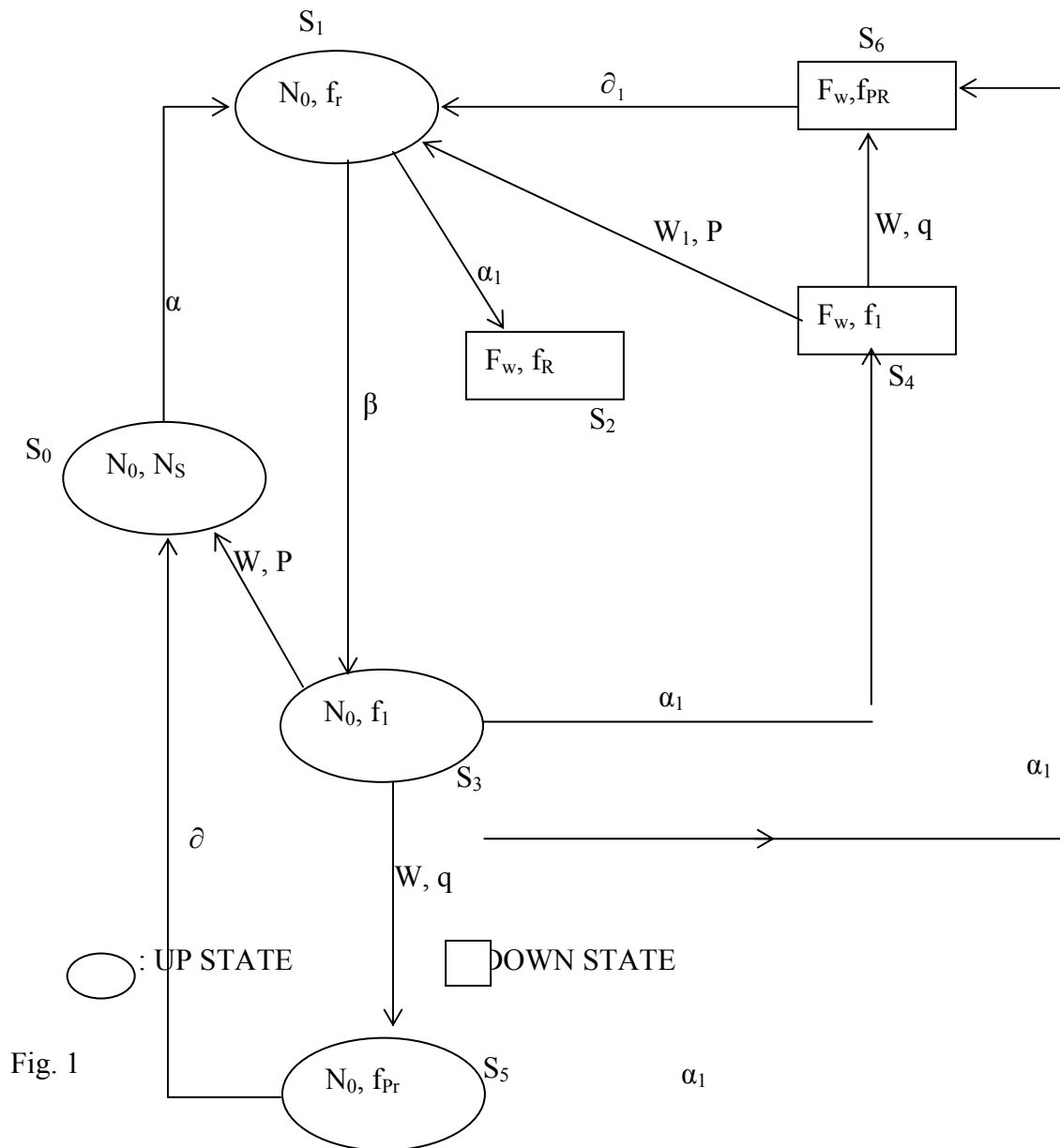


Fig. 1

3. NOTATION AND STATES OF THE SYSTEM

- $\alpha, /(<\alpha)$ - Failure rate of an operative unit when the standby unit is available/not available.
- β - Repair rate of failed unit in up state.
- $\beta, (>\beta)$ - Repair rate of failed unit in down state.
- δ - Rate of post repair in up state.
- $\delta, (>\delta)$ - Rate of post repair in down state.
- W - Rate of inspection in up state.
- $W(>W)$ - Rate of inspection in down state.
- P - Probability that the repaired unit is perfect after the repair ($=1-q$).
- N_0 - Unit is normal mode and operative.
- N_S - Unit is normal mode and cold standby.
- F_w - Failed unit waiting for repair.
- F_P - Failed unit under repair.
- F_R - Repair of failed unit is continued from earlier state.
- F_I - Failed unit under inspection after completing the repair.
- F_P - Failed unit under post repair.
- F_{PR} - Post repair of failed unit is continued from earlier state.

Using the above notations, the possible states of the system are;

Up states; $S_0: (N_0, N_S); S_1:(N_0, F_R); S_3:(N_0, F_R); S_5:(N_0, F_{PR})$

Down states; $S_2: (F_w, F_R); S_4:(F_w, F_I); S_6:(F_w, F_{PR})$

The possible transitions between the states are shown in Fig.1

4. PROBABILITIES: (TRANSITION AND STEADY STATE)

The epochs of entry into state S_0 - S_6 are regenerative points and E is the set of these states.

Let $T_0 (\equiv 0), T_1, T_2, \dots$ be the epochs of entry in to any state $S_i \in E$ and x_n be the state visited at epoch in T_{n+} i.e. just after the transition at T_n . Then $\{x_n, T_n\}$ is markov-renewal process with state space E and

$$Q_1(t) = P[x_{n+1} = S_j, T_{n+1} - T_n \leq t | x_n = S_i] \quad (1)$$

is the semi-marker Kernel over E . The transition probability matrix (t.p.m.) of the embedded markov chain is

$$P \equiv (P_{ij}) = [Q_i, (\infty)]$$

Considering the simple probabilistic arguments, the non zero elements of $[Q_i, (t)]$ are:-

$$\begin{aligned}
 Q_{01}(t) &= 1 - e^{-\alpha t} \\
 Q_{12}(t) &= \alpha_1 \{1 - e^{-(\alpha_1 + \beta)t}\} / (\alpha_1 + \beta) \\
 Q_{13}(t) &= \beta \{1 - e^{-(\alpha_1 + \beta)t}\} / (\alpha_1 + \beta) \\
 Q_{24}(t) &= 1 - e^{-\beta t} \\
 Q_{30}(t) &= Pw \{1 - e^{-(w + \alpha_1)t}\} / (w + \alpha_1) \\
 Q_{34}(t) &= \alpha_1 \{1 - e^{-(w + \alpha_1)t}\} / (w + \alpha_1) \\
 Q_{35}(t) &= qw \{1 - e^{-(w + \alpha_1)t}\} / (w + \alpha_1) \\
 Q_{41}(t) &= P \{1 - e^{-w_1 t}\} \\
 Q_{46}(t) &= q \{1 - e^{-w_1 t}\} \\
 Q_{50}(t) &= \delta \{1 - e^{-(\delta + \alpha_1)t}\} / (\delta + \alpha_1) \\
 Q_{56}(t) &= \alpha_1 \{1 - e^{-(\delta + \alpha_1)t}\} / (\delta + \alpha_1) \\
 Q_{61}(t) &= 1 - e^{-\delta t}
 \end{aligned} \tag{2}$$

From the above transition probabilities, we can obtain the steady state probabilities, P_i 's by taking the limit as $t \rightarrow \infty$. Thus,

$$\begin{aligned}
 P_{01} = 1 & & ; & & P_{12} = \alpha_1 / (\beta + \alpha_1) \\
 P_{13} = \beta / (\beta + \alpha_1) & & ; & & P_{24} = 1 \\
 P_{30} = Pw / (w + \alpha_1) & & ; & & P_{34} = \alpha_1 / (w + \alpha_1) \\
 P_{35} = qw / (w + \alpha_1) & & ; & & P_{41} = P \\
 P_{46} = q & & ; & & P_{50} = \delta / (\delta + \alpha_1) \\
 P_{56} = \alpha_1 / (\delta + \alpha_1) & & ; & & P_{61} = 1
 \end{aligned} \tag{3}$$

Relation (3) satisfy the following equations:-

$$\begin{aligned}
 P_{12} + P_{13} = 1 & & ; & & P_{30} + P_{34} + P_{35} = 1 \\
 P_{41} + P_{46} = 1 & & ; & & P_{50} + P_{56} = 1
 \end{aligned} \tag{4}$$

5. M S T₂

Mean Sojourn time in state $S_i \in E$ is defined as the time of stay in state S_i before transition to any other state if T denotes the sojourn time in state S_i , then the mean sojourn time is

$$\mu_i = E(T) = \int P[T > t] dt \quad (5)$$

Considering the above result we can obtain the following expressions:-

$$\begin{aligned} \mu_0 &= 1/\alpha & ; & & \mu_1 &= 1/(\beta + \alpha_1) \\ \mu_2 &= 1/\beta_1 & ; & & \mu_3 &= 1/(w + \alpha_1) \\ \mu_4 &= 1/W_1 & ; & & \mu_5 &= 1/(\delta + \alpha_1) \\ \mu_6 &= 1/\delta_1 \end{aligned} \quad (6)$$

b) The contribution to mean sojourn time when the system transits from state S_i to S_j before transiting from any regenerative state $S_i \in E$, when it is counted from the epoch of entrance into state $S_i \in E$.

Mathematically,

$$m_{ij} = \int tdq_i(t) = -\int Q_i(t) dt \quad (7)$$

Thus

$$\begin{aligned} m_{01} &= 1/\alpha & ; & & m_{12} &= \alpha_1/(\beta + \alpha_1)^2 \\ m_{13} &= \beta/(\beta + \alpha_1)^2 & ; & & m_{24} &= 1/\beta_1 \\ m_{30} &= Pw/(w + \alpha_1)^2 & ; & & m_{34} &= \alpha_1/(w + \alpha_1)^2 \\ m_{35} &= qw/(w + \alpha_1)^2 & ; & & m_{41} &= P/w_1 \\ m_{46} &= q/w_1 & ; & & m_{50} &= \delta/(\delta + \alpha_1)^2 \\ m_{56} &= \alpha_1/(\delta + \alpha_1)^2 & ; & & m_{61} &= 1/\delta_1 \end{aligned} \quad (8)$$

Relations (8) satisfy the following experiences:-

$$m_{01} = 1/\alpha = \mu_0$$

$$m_{12} + m_{13} = 1/(\beta + \alpha_1) = \mu_1$$

$$m_{24} = 1/\beta_1 = \mu_2$$

$$m_{30} + m_{34} + m_{35} = 1/(w + \alpha_1) = \mu_3$$

$$\begin{aligned}
 m_{41} &= m_{46} = 1/w_1 = \mu_4 \\
 m_{50} + m_{56} &= 1/(\delta + \alpha_1) = \mu_5 \\
 m_{61} &= 1/\delta_1 = \mu_6
 \end{aligned} \tag{9}$$

6. MTSF

To investigate the distribution function $\pi_i(t) = P(T_i \leq t)$ of the time to system failure (T_i) with starting state S_1 , we regard all the failed states as absorbing in particular $\pi_0(t)$ can be evaluated as follows;

Starting from state S_0 , suppose the system enters the state S_1 during $(U, U + du)$,

$0 < U < t$ and then starting from S_1 it fails before completing of further time $(t-u)$. The probability of this contingency is

$$\pi_0(t) = \int_0^t \pi_1(t-U) dQ_o(t) = Q_{01}(t) \pi_1(t) \tag{10}$$

Similarly, to obtain $\pi_1(t)$, we observe that starting from state S_1 , the system can transit either to state S_2 or S_3 and as such $\pi_1(t)$ is the sum of the following two contingencies:-

The system transit to S_3 during $(t-u)$

The system fails before time t and reaches to state S_2

Thus

$$\pi_1(t) = Q_{12}(t) + Q_{13}(t) \pi_3(t) \tag{11}$$

In a similar fashion we can have:-

$$\pi_3(t) = Q_{34}(t) + Q_{30}(t) \pi_0(t) + Q_{35}(t) \pi_5(t)$$

$$\pi_5(t) = Q_{56}(t) + Q_{50}(t) \pi_0(t)$$

(12)

Taking Laplace transform of these relations, we obtained

$$\begin{aligned}
 \tilde{\pi}_0(s) &= \tilde{Q}_{01}(s) \cdot \tilde{\pi}_1(s) \\
 \tilde{\pi}_1(s) &= \tilde{Q}_{12}(s) + \tilde{Q}_{13}(s) \cdot \tilde{\pi}_3(s) \\
 \tilde{\pi}_3(s) &= \tilde{Q}_{34}(s) + \tilde{Q}_{30}(s) \cdot \tilde{\pi}_0(s) + \tilde{Q}_{35}(s) \cdot \tilde{\pi}_5(s) \\
 \tilde{\pi}_5(s) &= \tilde{Q}_{56}(s) + \tilde{Q}_{50}(s) \cdot \tilde{\pi}_0(s)
 \end{aligned} \tag{13}$$

Solving the above equations for $\pi_0(s)$ by omitting the argument "s" for brevity, we obtained:

$$\tilde{\pi}(s) = N_1(s) / D_1(s) \quad (14)$$

Where

$$N_1(s) = \tilde{Q}_{01} \left\{ \tilde{Q}_{12} + \tilde{Q}_{13} \tilde{Q}_{34} + \tilde{Q}_{13} \tilde{Q}_{35} \tilde{Q}_{56} \right\} \quad (15)$$

and

$$D_1(s) = 1 - \tilde{Q}_{01} \tilde{Q}_{13} \left(\tilde{Q}_{30} + \tilde{Q}_{35} \tilde{Q}_{50} \right) \quad (16)$$

Taking $S \rightarrow \infty$ in equation (65), we find

$$\tilde{\pi}_0(o) = 1, \text{ which implies that}$$

$\pi_0(t)$ is a proper c.d.f. Therefore, the mean time to system failure (MTSF) when it starts operation from S_0 is

$$\begin{aligned} E(t) &= \frac{d \tilde{\pi}_0(S)}{ds} \Big|_{s=0} \\ &= \frac{D_1'(o) - N_1(o)}{D_1(o)} \\ &= N_1 / D_1 \end{aligned} \quad (17)$$

Where

$$N_1 = \mu_0 + \mu_1 + P_{13} (\mu_3 + P_{35} \mu_5) \quad (18)$$

and

$$D_1 = 1 - P_{13} (P_{30} + P_{35} + P_{50}) \quad (19)$$

7. AVAILABILITY ANALYSIS

Define $M_1(t)$, as the probability that the system starting from up state $S_1 \in E$ remains up till time without passing through any regenerative state or returning to itself.

Using the above definition, we have

$$\begin{aligned} M_o(t) &= e^{-\alpha_1 t} \\ M_1(t) &= e^{-(\beta + \alpha_1)t} \\ M_3(t) &= e^{-(w + \alpha_1)t} \\ M_5(t) &= e^{-(\delta + \alpha_1)t} \end{aligned} \quad (20)$$

It is observed that the epoch of entry into the S_0 - S_i are regenerative points and E is the set of these states.

Using the theory of regenerative process, the point-wise availability $A_0(t)$ is the sum of the following probabilities.

Starting from state S_0 , the system remains there till epoch t , the

probability of this contingency is equal to $M_0(t)$.

Starting from state S_0 , the system transits to states S_1 , during

$(U, U+dU)$, $U < t$ and then starting from state S_1 it is available at epoch t , the probability of this contingency is equal to

$$\int_0^t q_{01}(U)A_1(t-u)du = q_{01}(t)\ell A_1(t)$$

Hence

$$A_0(t) = M_0(t) + q_{01}(t)\ell A_1(t) \quad (21)$$

Similarly, $A_1(t)$ is the sum of the following two independent probabilities

Probability that the system initially up in state S_1 and it remains up at time t without transiting to any regenerative state E .

Probability that the system transits from initial state S_1 to any one of the regenerative state S_i , $i = 2, 3$ during $(u, u+du)$, $u < t$ and then starting from state S_1 , it is availability at epoch t

Thus

$$A_1(t) = M_1(t) + q_{12}(t)\ell A_2(t) + q_{13}(t)\ell A_3(t) \quad (22)$$

Similarly

$$\begin{aligned} A_2(t) &= q_{24}(t)\ell A_4(t) \\ A_3(t) &= M_3(t) + q_{30}(t)\ell A_0(t) + q_{34}(t)\ell A_4(t) + q_{35}(t)\ell A_5(t) \\ A_4(t) &= q_{41}(t)\ell A_1(t) + q_{46}(t)\ell A_6(t) \\ A_5(t) &= M_5(t) + q_{50}(t)\ell A_0(t) + q_{56}(t)\ell A_6(t) \\ A_6(t) &= q_{61}(t)\ell A_1(t) \end{aligned}$$

(23)

Taking Laplace transform of the above equations a system of resulting equations in $A_1^*(s)$ can be expressed in matrix form as

$$(A_0^*, A_1^*, A_2^*, A_3^*, A_4^*, A_5^*, A_6^*) = Q^{-1}(M_0^*, M_1^*, 0, M_3^*, 0, M_5^*, 0) \quad (24)$$

Where

$$Q = \begin{pmatrix} 1 & -q_{q1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -q_{12}^* & -q_{13}^* & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -q_{24}^* & 0 & 0 \\ -q_{30}^* & 0 & 0 & 1 & -q_{34}^* & -q_{35}^* & 0 \\ 0 & -q_{41}^* & 0 & 0 & 1 & 0 & -q_{46}^* \\ -q_{50}^* & 0 & 0 & 0 & 0 & 1 & -q_{56}^* \\ 0 & -q_{61}^* & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Computing the relevant cofactors of first column of Q, the solution of (82) gives the laplace transform of point-wise availability $A_0(t)$ as

$$A_0^*(s) = N_2(s) / D_2(s) \quad (25)$$

$$N_2(s) = M_0^* \begin{pmatrix} 1 - q_{12}^* q_{24}^* (q_{41}^* + q_{46}^* q_{61}^*) - q_{13}^* q_{34}^* (q_{41}^* + q_{46}^* q_{61}^*) \\ -q_{13}^* q_{35}^* q_{56}^* q_{61}^* + q_{01}^* M_1^* + q_{13}^* (M_3^* + q_{35}^* M_5^*) \end{pmatrix} \quad (26)$$

and

$$D_2(s) = 1 - q_{12}^* q_{24}^* (q_{41}^* + q_{46}^* q_{61}^*) - q_{13}^* \begin{pmatrix} q_{34}^* (q_{41}^* + q_{46}^* q_{61}^*) + q_{35}^* q_{56}^* q_{61}^* \\ + q_{01}^* (q_{30}^* + q_{35}^* q_{50}^*) \end{pmatrix} \quad (27)$$

Taking the limit as $S \rightarrow 0$ in the above relations (27), we obtained

$$\begin{aligned} D_2(0) &= 1 - P_{12} P_{24} (P_{41} + P_{46} P_{61}) - P_{13} (P_{34} P_{41} + P_{46} P_{61}) + P_{35} P_{56} P_{61} + P_{01} (P_{30} + P_{35} P_{50}) \\ &= 1 - P_{12} (P_{41} + P_{46}) - P_{13} \{P_{34} (P_{41} + P_{46}) + P_{35} P_{56} + P_{30} + P_{35} P_{50}\} \\ &= 1 - P_{12} - P_{13} (P_{34} + P_{35} + P_{36} + P_{30} + P_{35} P_{50}) \\ &= 1 - P_{12} - P_{13} \{P_{34} + P_{35} (P_{56} + P_{50}) + P_{30}\} \\ &= 1 - P_{12} - P_{13} (P_{34} + P_{35} + P_{30}) \\ &= 1 - P_{12} - P_{13} = 1 - 1 = 0 \end{aligned} \quad (28)$$

Thus the steady state availability when the system initially starts operation from s_0 is obtained as

$$A_0(\infty) = \lim_{s \rightarrow \infty} S.A_0^*(s) = N_2(0) / D_2'(0) = N_2 / D_2 \quad (29)$$

Where

$$N_2 = \mu_1 + P_{13} \left\{ (P_{30} + P_{35}P_{50})\mu_0 + \mu_3 + P_{35}\mu_5 \right\} \quad (30)$$

And

$$D_2 = P_{13} (P_{30} + P_{35}P_{50})\mu_0 + \mu_1 + P_{12}\mu_2 + P_{13}\mu_3 + (P_{12} + P_{13}P_{34})\mu_4 + P_{13}P_{35}\mu_5 + (P_{12}P_{46} + P_{13}P_{34}P_{46} + P_{13}P_{35}P_{56})\mu_6 \quad (31)$$

8. BUSY PERIOD ANALYSIS

$W_1(t)$ is defined as the probability that the repairman is busy initially in state S_1 and remains busy at epoch t without transiting to any other regenerative state. Therefore,

$$W_1(t) = e^{-(\beta+\alpha_1)t} \quad (32)$$

Similarly,

$$\begin{aligned} W_2(t) &= e^{-\beta_1 t} & ; & & W_3(t) &= e^{-(w+\alpha_1)t} \\ W_4(t) &= e^{-w_1 t} & ; & & W_5(t) &= e^{-(\delta+\alpha_1)t} \\ W_6(t) &= e^{-\delta_1 t} & & & & \end{aligned} \quad (33)$$

Also $B_1(t)$ is defined as the probability that the repairman is busy at epoch t starting from state $S_1 \in E$ thus by probabilistic arguments, we have

$$\begin{aligned} B_0(t) &= q_{01}(t)\ell B_1(t) \\ B_1(t) &= W_1(t) + q_{12}(t)\ell B_2(t) + q_{13}(t)\ell B_3(t) \\ B_2(t) &= W_2(t) + q_{24}(t)\ell B_4(t) \\ B_3(t) &= W_3(t) + q_{30}(t)\ell B_0(t) + q_{34}(t) + q_{35}(t)\ell B_5(t) \\ B_4(t) &= W_4(t) + q_{41}(t)\ell B_1(t) + q_{46}(t)\ell B_6(t) \\ B_5(t) &= W_5(t) + q_{50}(t)\ell B_0(t) + q_{56}(t)\ell B_6(t) \\ B_6(t) &= W_6(t) + q_{62}(t)\ell B_1(t) \end{aligned} \quad (34)$$

Taking laplace transforms of these equation, we obtained a set of linear equations in $B_2^*(s)$ whose solution (omitting the argument “s” for brevity) may be expressed in matrix form as:-

$$\left(B_0^*, B_1^*, B_2^*, B_3^*, B_4^*, B_5^*, B_6^* \right) = Q^{-1} \left(o, w_0^*, w_1^*, w_2^*, w_3^*, w_4^*, w_5^*, w_6^* \right) \quad (35)$$

Where

$$Q = \begin{bmatrix} 1 & -Q_{01}^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -Q_{12}^* & -Q_{13} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -Q_{24}^* & 0 & 0 \\ -Q_{30}^* & 0 & 0 & 1 & -Q_{34}^* & -Q_{35}^* & 0 \\ 0 & -Q_{41}^* & 0 & 0 & 1 & 0 & -Q_{46}^* \\ -Q_{50}^* & 0 & 0 & 0 & 0 & 1 & -Q_{56}^* \\ 0 & -Q_{61}^* & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Computing the relevant cofactors of first column of Q the solution of (35) give the laplace transform of $B_0(t)$ as $B_0^*(s) = N_3(s) / D_2(s)$ (36)

Where $D_2(S)$ is same as (85) and

$$N_3(S) = q_{01}^* [W_1^* + q_{12}^* W_2^* + q_{24}^* (N_4^* + q_{46}^* W_6^*) + q_{13}^* (W_3^* + q_3^* (W_4^* + q_{46}^* W_6^*)) + q_{35}^* (W_5^* + q_{35}^* W_6^*)] \quad (37)$$

Thus the steady state probability that the system having started from S_0 is under repair is given by

$$B_0 = \lim_{t \rightarrow \infty} B_0(t) = \lim_{s \rightarrow 0} B_0^*(s) = N_3 / D_2 \quad (38)$$

Where D_2 is same as in (31) and

$$N_3 = \mu_1 + P_{12} \mu_2 + (P_{12} + P_{13} \times P_{34}) (\mu_4 + P_{46} \mu_6) + P_{13} (\mu_3 + P_{35} (\mu_5 + P_{56} \mu_6)) \quad (39)$$

CONCLUSION:

In this concluding section on the basis of above results , we have obtained the expected number of visits by the repairman so that the cost analysis can be done.

Let $V_1(t)$ denote the expected number of visits by the repairman in $(0,t)$ given that the system initially starts from regenerative state S_1 . then the following recursive relations can be obtained:

$$\begin{aligned}
V_0(t) &= Q_{01}(t)[1 + v_1(t)] \\
V_1(t) &= Q_{12}(t)v_2(t) + Q_{13}(t)v_3(t) \\
V_2(t) &= Q_{24}(t)v_4(t) \\
V_3(t) &= Q_{30}(t)v_0(t) + Q_{34}(t)v_4(t) + Q_{35}(t)v_5(t) \\
V_4(t) &= Q_{41}(t)v_1(t) + Q_{46}(t)v_6(t) \\
V_5(t) &= Q_{50}(t)v_0(t) + Q_{56}(t)v_6(t) \\
V_6(t) &= Q_{61}(t)v_1(t)
\end{aligned}
\tag{40}$$

Taking Laplace-Stieltjes transform of these relations and solving for $V_0(s)$ (by omitting the argument "s" for brevity) we obtained

$$\tilde{V}_0(s) = N_4(s) | D_3(s) \tag{41}$$

Where

$$N_4(s) = \tilde{Q}_{01} \left\{ 1 - \tilde{Q}_{12} \tilde{Q}_{24} \left(\tilde{Q}_{41} + \tilde{Q}_{46} \tilde{Q}_{61} \right) - \tilde{Q}_{13} \tilde{Q}_{34} \left(\tilde{Q}_{41} + \tilde{Q}_{46} \tilde{Q}_{61} \right) - \tilde{Q}_{13} \tilde{Q}_{35} \tilde{Q}_{56} \tilde{Q}_{61} \right\} \tag{42}$$

And

$$D_3(s) = 1 - \tilde{Q}_{12} \tilde{Q}_{24} \left(\tilde{Q}_{41} + \tilde{Q}_{46} \tilde{Q}_{61} \right) - \tilde{Q}_{13} \left\{ \begin{array}{l} \tilde{Q}_{41} + \tilde{Q}_{46} \tilde{Q}_{61} + \tilde{Q}_{35} \tilde{Q}_{56} \tilde{Q}_{61} \\ + \tilde{Q}_{01} \left(\tilde{Q}_{30} + \tilde{Q}_{35} \tilde{Q}_{50} \right) \end{array} \right\} \tag{43}$$

In steady state the no. of visits per unit time is given by

$$V_0 = \lim_{t \rightarrow \infty} [V_0(t) | t] = \lim_{s \rightarrow 0} s V_0(s) = N_4 | D_2 \tag{44}$$

Where D_2 is same as in (31) and

$$N_4 = P_{13} (P_{30} + P_{35} + P_{50}) \tag{45}$$

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