

FINITE DIFFERENCE ANALYSIS OF LAMINAR MIXED CONVECTION WITH VERTICAL CHANNEL IN DOWNFLOW

B. Rama Bhupal Reddy*

ABSTRACT

In this paper, finite difference analysis of Laminar mixed convection in vertical channel is studied. The distance between the plates is 'b' and g is the gravitational force. The buoyancy effect is taken into account according to the Boussinesq approximation. The flow problem is described by means of parabolic equations and the solutions are obtained by the use of an implicit finite difference technique coupled with a marching procedure. The velocity, the temperature and the pressure profiles are shown in graphs and their behaviour is discussed for different values of buoyancy parameter Gr/Re and Prandtl number Pr .

Keywords: *Mixed Convection, Vertical Channel*

*Associate Professor, Dept. of Mathematics, K.S.R.M.C.E., Kadapa

1. INTRODUCTION

Different orientation are heating conditions of the channel can induce different kinds of heated buoyant flows which enhance the heat transfer in different manners. For asymmetric heated channel, when the buoyancy parameter is not large, this small amount of the buoyant flow induced along the heated wall can either assist or oppose the main flow and cause either enhancement or reduction in the heat transfer. When the buoyancy parameter becomes large, the buoyant flow along the side wall becomes substantial. Depending upon the flow direction of the main stream, the buoyancy flow can cause different kinds of flow reversals which will alter the entire flow characteristic and enhance the heat transfer in different manners.

The laminar mixed convection in vertical or inclined ducts has been widely studied in the literature. Indeed, this research area has several technical applications, such as heat exchangers, cooling systems for electronic devices, solar collectors. Interesting results of the researches on this topic are collected in Refs. [1, 13]. Early exact solutions for the differential equations describing mixed convection in vertical channels and pipes were given by Ostrach [17] and Morton [16]. Ostrach [17] used a linearly changing wall temperature profile for vertical flow. He found instability and flow inversion for heating from below. Morton [16] predicted parallel flow for small Ra numbers in downward heated flow. He predicted that the flow would become unsteady and turbulent when the Raleigh number Ra reached 33. Quientiere [18] presented approximate analytical solutions for mixed convection between parallel plates. He related the heat transfer results to the inlet pressure defect and flowrate.

Rogers [19] and Yao [21-23] studied mixed convection by the use of stability analyses. Extensive work has been done to solve the flow field equation for mixed convection flows by numerical methods. The level of details of the results of these studies is highly dependent on the computing resources used. In the 1960s and 1970s general results were found for heat transfer effects for mixed convection flows by Lawrence [15], and Zeldin and Schmidt [24]. Beginning in the 1980s, numerical studies started being able to better resolve the velocity profiles of vertical pipe flows with mixed convection as shown in studies by Ingram [11, 12], Aung [2], Cebeci [6], Heggs [10], Wang [20] and Hebchi and Acharya [9]. Recent numerical studies by Evans [8] and Chang [7] have focused on the oscillatory flow reversal behavior that can occur in mixed convection flows.

Barletta [3] studied on the existence of parallel flow for mixed convection in an inclined duct. Barletta and Zanchini [4] studied mixed convection with variable viscosity in an inclined

channel with prescribed wall temperatures. Boulama and Galanis [5] studied analytical solution for fully developed mixed convection between parallel vertical plates with heat and mass transfer. Lavine [14] studied fully developed opposed mixed convection between inclined parallel plates.

2. NOMENCLATURE

b	Distance between the parallel plates
C_p	Specific heat of the fluid
g	Gravitational body force per unit mass (acceleration)
Gr	Grashoff number, $g\beta (T_1 - T_0) b^3 / \nu^2$
k	Thermal conductivity of fluid
p	Local pressure at any cross section of the vertical channel
p_0	Hydrostatic pressure
P	Dimensionless pressure inside the channel at any cross section, $(p - p_0) / \rho u_0^2$
Pr	Prandtl Number, $\mu C_p / k$
q''	Constant heat flux
Re	Reynolds number, $(b u_0) / \nu$
T	Dimensional temperature at any point in the channel
T_0	Ambient of fluid inlet temperature
T_w	Isothermal temperatures of circular heated wall
T_1, T_2	Isothermal temperatures of plate 1 and plate 2 of parallel plates
u	Axial velocity component
\bar{u}	Average axial velocity
u_0	Uniform entrance axial velocity
U	Dimensionless axial velocity at any point, u / u_0
v	Transverse velocity component
V	Dimensionless transverse velocity, v / u_0
x	Axial coordinate (measured from the channel entrance)
X	Dimensionless axial coordinate in Cartesian, $x / (b Re)$
y	Transverse coordinate of the vertical channel between parallel plates
Y	Dimensionless transverse coordinate, y / b
θ	Dimensionless temperature
ρ	Density of the fluid
ρ_0	Density of the fluid at the channel entrance

- μ Dynamic viscosity of the fluid
 ν Kinematic viscosity of the fluid, μ/ρ
 β Volumetric coefficient of thermal expansion

3. FORMULATION OF THE PROBLEM

We consider a steady developed laminar mixed convection flow between two vertical channels. The vertical plates are separated by a distance 'b'. The Cartesian coordinate system is chosen such that the x-axis is in the vertical direction that is parallel to the flow direction and the gravitational force 'g' always acting downwards independent of flow direction. The y-axis is orthogonal to the channel walls, and the origin of the axes is such that the positions of the channel walls are $y = 0$ and $y = b$. One wall is maintained at constant heat flux and the other is at isothermal condition. The fluid properties are assumed to be constant except for the buoyancy terms of the momentum equation. The geometry is illustrated in Figure 1.

The governing equations for the steady viscous flow with the following assumptions are made:

- (i) The flow is steady, viscous, incompressible and developed.
- (ii) The flow is assumed to be two-dimensional steady, and the fluid properties are constant except for the variation of density in the buoyancy term of the momentum equation.
- (iii) Energy dissipation is neglected.

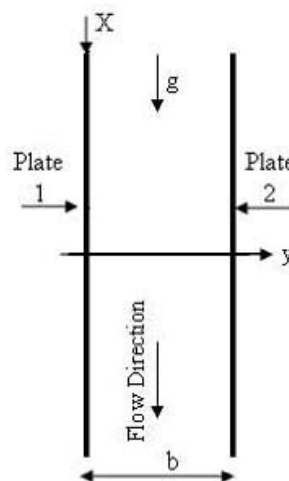


Figure 1: Schematic view of the system and coordinate axes corresponding to down-flow

After applying the above assumptions the boundary layer equations appropriate for this problem are

Continuity Equation

$$U \frac{\partial U}{\partial X} + V \frac{\partial V}{\partial Y} = 0 \quad (1)$$

X – Momentum Equation

$$U \frac{\partial U}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{dP}{dX} - \frac{Gr}{Re} \theta + \frac{\partial^2 U}{\partial Y^2} \quad (2)$$

Energy Equation

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} \quad (3)$$

The form of continuity equation can be written in integral form as

$$\int_0^1 U dY = 1 \quad (4)$$

The boundary conditions are

Entrance conditions

$$\text{At } X = 0, 0 < Y < 1 : U = 1, V = 0, \theta = 0, P = 0 \quad (5a)$$

No slip conditions

$$\left. \begin{array}{l} \text{At } X > 0, Y = 0 : U = 0, V = 0 \\ \text{At } X > 0, Y = 1 : U = 0, V = 0 \end{array} \right\} \quad (5b)$$

Thermal boundary conditions

$$\left. \begin{array}{l} \text{At } X > 0, Y = 0 : \left(\frac{\partial \theta}{\partial y} \right)_{Y=0} = -1 \\ \text{At } X > 0, Y = 1 : \theta = 0 \end{array} \right\} \quad (5c)$$

In the above, dimensionless parameters have been defined as:

$$\left. \begin{array}{l} U = u / u_0, V = vb / \nu, X = x / (b Re), Y = y / b \\ P = (p - p_0) / \rho u_0^2, Pr = \mu C_p / k, Re = (b u_0) / \nu \\ Gr = g\beta (T_1 - T_0) b^3 / \nu^2, \theta = (T - T_0) / (T_1 - T_0) \end{array} \right\} \quad (6)$$

The systems of non-linear equations (1) to (3) are solved by a numerical method based on finite difference approximations. An implicit difference technique is employed whereby the differential equations are transformed into a set of simultaneous linear algebraic equations.

4. NUMERICAL SOLUTION

The solution of the governing equations for developing flow is discussed in this section. Considering the finite difference grid net work of figure 2, equations (2) and (3) are replaced by the following difference equations which were also used in [5]

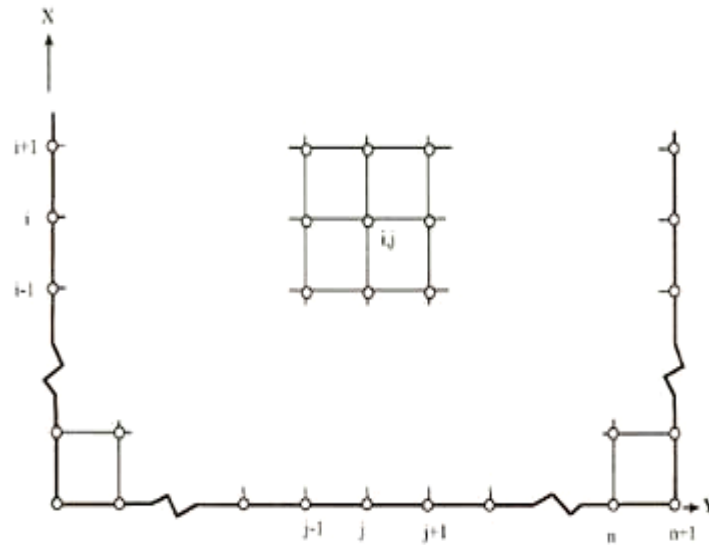


Figure 2: Mesh Network for Difference Representations

$$\begin{aligned}
 & U(i, j) \frac{U(i+1, j) - U(i, j)}{\Delta X} + V(i, j) \frac{U(i+1, j+1) - U(i+1, j-1)}{2\Delta Y} \\
 & = \frac{U(i+1, j+1) - 2U(i+1, j) + U(i+1, j-1)}{(\Delta Y)^2} - \frac{P(i+1) - P(i)}{\Delta X} - \frac{Gr}{Re} \theta(i+1, j)
 \end{aligned} \quad (7)$$

$$\begin{aligned}
 & U(i, j) \frac{\theta(i+1, j) - \theta(i, j)}{\Delta X} + V(i, j) \frac{\theta(i+1, j+1) - \theta(i+1, j-1)}{2\Delta Y} \\
 & = \frac{1}{Pr} \frac{\theta(i+1, j+1) - 2\theta(i+1, j) + \theta(i+1, j-1)}{(\Delta Y)^2}
 \end{aligned} \quad (8)$$

Numerical representation of the Integral Continuity Equation

The integral continuity equation can be represented by the employing a trapezoidal rule of numerical integration and is as follows:

$$\left[\sum_{j=1}^n U(i+1, j) + 0.5 (U(i+1, 0) + U(i+1, n+1)) \right] \Delta Y = 1$$

However, from the no slip boundary conditions

$$U(i+1, 0) = U(i+1, n+1) = 0$$

Therefore, the integral equation reduces to:

$$\left[\sum_{j=1}^n U(i+1, j) \right] \Delta Y = 1 \tag{9}$$

A set of finite-difference equations written about each mesh point in a column for the equation (7) as shown:

$$\beta_1 U(i+1,1) + \gamma_1 U(i+1,2) + \xi P(i+1) - \frac{Gr}{Re} \theta(i+1,1) = \phi_1$$

$$\alpha_2 U(i+1,1) + \beta_2 U(i+1,2) + \gamma_2 U(i+1,3) + \xi P(i+1) - \frac{Gr}{Re} \theta(i+1,2) = \phi_2$$

$$\alpha_n U(i+1, n-1) + \beta_n U(i+1, n) + \xi P(i+1) - \frac{Gr}{Re} \theta(i+1, n) = \phi_n$$

where

$$\alpha_k = \frac{1}{(\Delta Y)^2} + \frac{V(i, j)}{2\Delta Y}, \quad \beta_k = - \left[\frac{2}{(\Delta Y)^2} + \frac{U(i, j)}{\Delta X} \right]$$

$$\gamma_k = \frac{1}{(\Delta Y)^2} - \frac{V(i, j)}{2\Delta Y}, \quad \xi = \frac{-1}{\Delta X}, \quad \phi_k = - \left[\frac{P(i) + U^2(i, j)}{\Delta X} \right]$$

for k = 1, 2... n

A set of finite-difference equations written about each mesh point in a column for the equation (8) as shown:

$$\bar{\beta}_0 \theta(i+1,0) + (\bar{\alpha}_0 + \bar{\gamma}_0) \theta(i+1,1) + \dots = \bar{\phi}_0 - 2\Delta y \bar{\alpha}_0$$

$$\bar{\alpha}_n \theta(i+1,0) + \bar{\beta}_1 \theta(i+1,1) + \bar{\gamma}_1 \theta(i+1,2) + \dots = \bar{\phi}_1$$

$$\bar{\alpha}_n \theta(i+1, n-1) + \bar{\beta}_n \theta(i+1, n) = \bar{\phi}_n$$

where

$$\bar{\alpha}_k = \frac{1}{Pr(\Delta Y)^2} + \frac{V(i, j)}{2\Delta Y}, \quad \bar{\beta}_k = - \left[\frac{2}{Pr(\Delta Y)^2} + \frac{U(i, j)}{\Delta X} \right]$$

$$\bar{\gamma}_k = \frac{1}{Pr(\Delta Y)^2} - \frac{V(i, j)}{2\Delta Y}, \quad \bar{\phi}_k = - \frac{U(i, j) \theta(i, j)}{\Delta X}$$

for k = 0, 1, 2... n

Equation (1) can be written as

$$V(i+1, j) = V(i+1, j-1) - \frac{\Delta Y}{2\Delta x} (U(i+1, j) + U(i+1, j-1) - U(i, j) - U(i, j-1)), j = 1, 2, \dots, n \quad (10)$$

The numerical solution of the equations is obtained by first selecting the parameter that are involved such as Gr/Re and Pr. Then by means of a marching procedure the variables U, V, θ and P for each row beginning at row $(i+1) = 2$ are obtained using the values at the previous row 'i'. Thus, by applying equations (7), (8) and (9) to the points 1, 2, ..., n on row i, $2n+2$ algebraic equations with the $2n+2$ unknowns $U(i+1,1), U(i+1,2), \dots, U(i+1,n), P(i+1), \theta(i+1,0), \theta(i+1,1), \theta(i+1,2), \dots, \theta(i+1, n)$ are obtained. This system of equations is then solved by Gauss – Jordan elimination method. Equations (10) are then used to calculate $V(i+1,1), V(i+1,2), \dots, V(i+1,n)$.

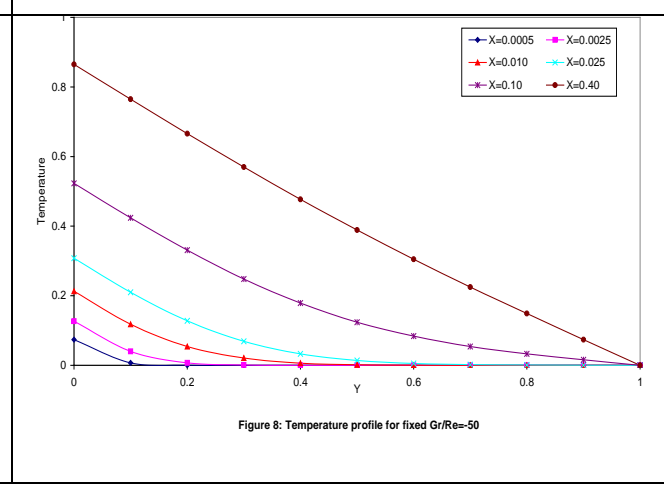
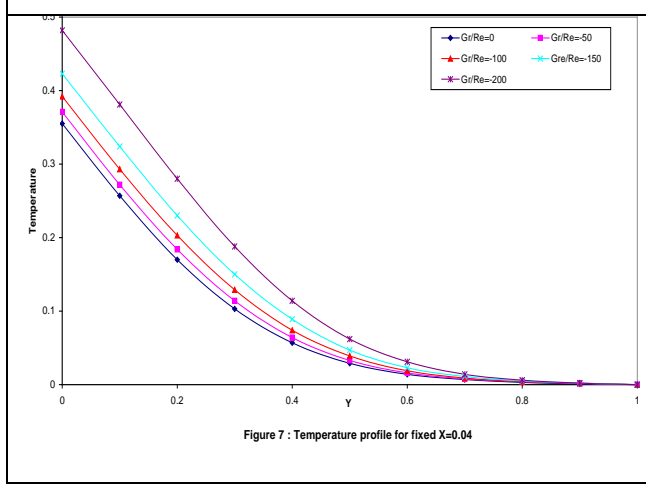
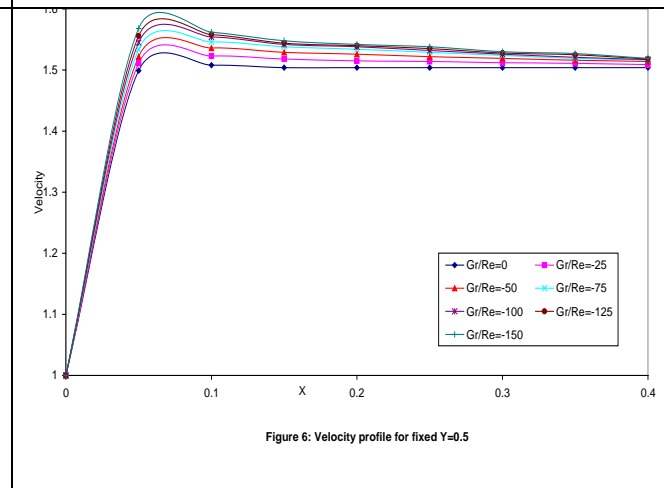
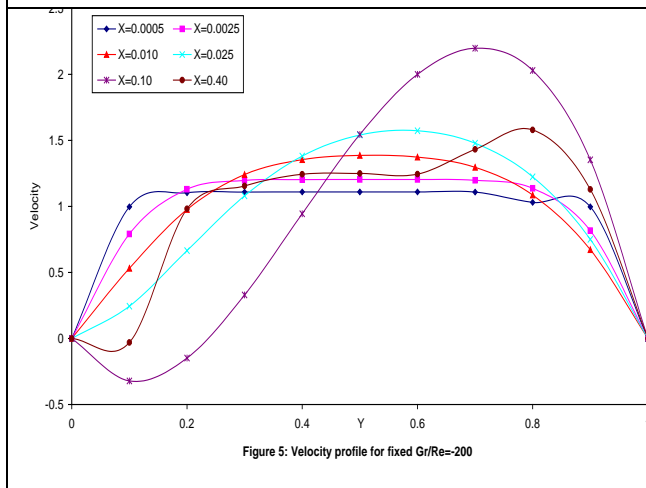
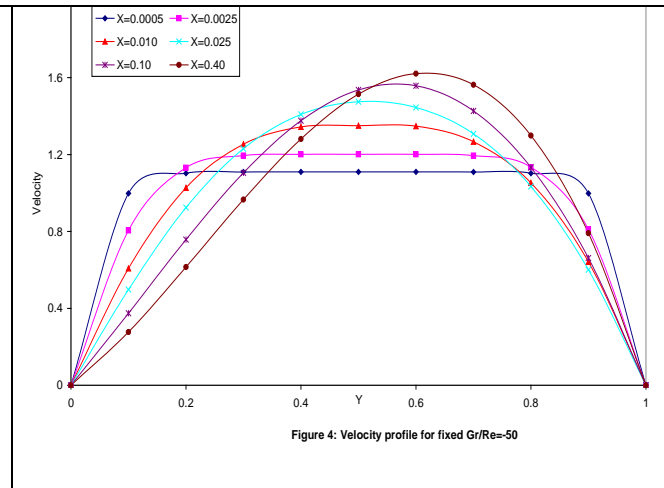
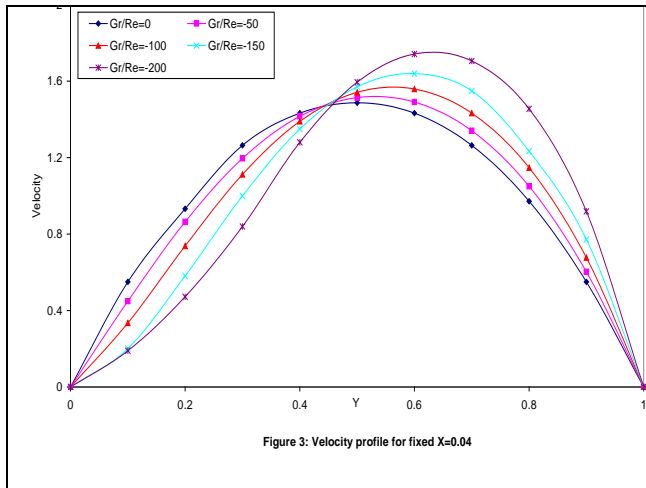
5. RESULTS AND DISCUSSION

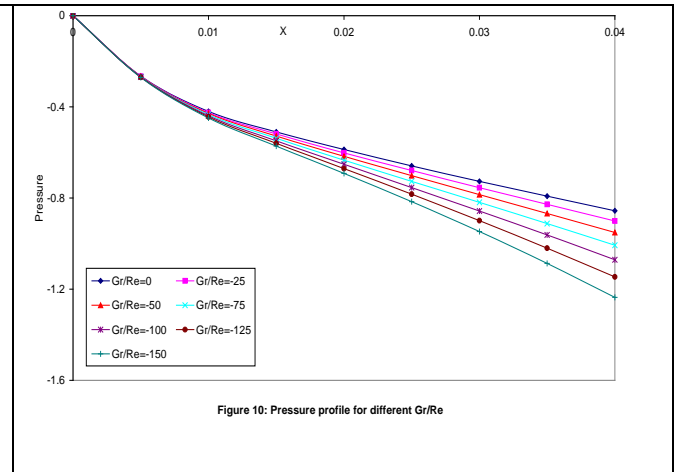
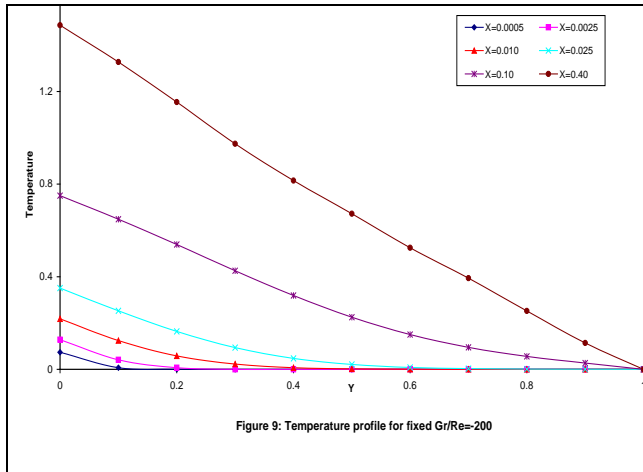
The numerical solution of the equations is obtained by first selecting the parameters that are involved such as Gr/Re and Pr, for fixed Pr = 0.7. The developments of velocity profiles along the axial distance for laminar mixed convection in vertical channels are shown in the figures 3 to 6. These figures show that the fluid decelerates near the two walls of the channel (due to the formation of the two boundary layers on the walls) and accelerates in the core region as a result of the continuity principle. However, the velocity profile recovers and attains its asymptotic fully developed parabolic profile.

Figures 7 to 9 show how the temperature profiles are affected by constant heating of the wall along the axial direction between the vertical parallel plates. Figure 7 and 8 is plotted for fixed Gr/Re = -50 and Gr/Re = -200 respectively for different X values (one wall is maintained at constant heat flux and the other is at isothermal condition). These figures show that the development of their invariant analytical fully developed profiles. Higher temperature near the heated wall result in buoyancy aiding effect for upward flow while the cooling effects of the cold wall results in opposite buoyancy effects and for large asymmetrical heating conditions flow reversal might take place at the cold wall.

Figure 10 depict the variation of pressure along the axial direction for different buoyancy parameters Gr/Re. These figures show how the two hydrodynamic parameters are developing downstream of the channel at different heating rates represented buoyancy parameter Gr/Re. These negative values of the pressure will continue increasing due to the corresponding negative pressure gradient for the cases of pure forced convection (Gr/Re = 0) and the buoyancy-opposed flow cases with negative values of Gr/Re. For pure forced convection and relatively low negative values of Gr/Re for buoyancy-opposed flows, the pressure gradient

develops from a very high negative value at the entrance and continues negative with a decreasing negativity till it reaches asymptotically to its fully developed negative value. The direct result of the act of these two forces for buoyancy opposed flow is to create and develop more negative pressure in the flow direction which is evidently presented for cases of $Gr/Re \leq 0$. It is worth mentioning here that large values of the opposing buoyancy parameters lead to flow reversal and consequentially it leads faster to flow instability.





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