INTRODUCTION TO GRAPH THEORY

Satish*

ABSTRACT

The field of mathematics plays vital role in various fields. One of the important areas in mathematics is graph theory which is used in structural models. This structural arrangements of various objects or technologies lead to new inventions and modifications in the existing environment for enhancement in those fields. This Paper describes the description of graph theory.

Keywords: Bipartite graph, Ad-hoc networks, Edges and Vertices.

*Extension lecturer, govt P.G College, Jind

INTRODUCTION:

A pair G = (V, E) with E ⊆ E(V) is called a graph (on V). The elements of V are the vertices of G, and those of E the edges of G. The vertex set of a graph G is denoted by VG and its edge set by EG. Therefore G = (VG, EG).

In literature, graphs are also called simple graphs; vertices are called nodes or points; edges are called lines or links. The list of alternatives is long (but still finite). A pair {u, v} is usually written simply as uv. Notice that then uv = vu. In order to simplify notations, we also write v ∈ G and e ∈ G instead of v ∈ VG and e ∈ EG.

DEFINITION: For a graph G, we denote νG = |VG| and εG = |EG| .

The number νG of the vertices is called the order of G, and εG is the size of G. For an edge e = uv ∈ G, the vertices u and v are its ends. Vertices u and v are adjacent or neighbours, if uv ∈ G. Two edges e1 = uv and e2 = uw having a common end, are adjacent with each other.

In mathematics **graph theory** is the study of *graphs*, which are mathematical structures used to model pairwise relations between objects. A "graph" in this context is made up of "vertices" or "nodes" and lines called *edges* that connect them. A graph may be *undirected*, meaning that there is no distinction between the two vertices associated with each edge, or its edges may be *directed* from one vertex to another; see graph (mathematics) for more detailed definitions and for other variations in the types of graph that are commonly considered. Graphs are one of the prime objects of study in discrete mathematics.

The graphs studied in graph theory should not be confused with the graphs of functions or other kinds of graphs.

Vertex (Node). A node *v* is a terminal point or an intersection point of a graph. It is the abstraction of a location such as a city, an administrative division, a road intersection or a transport terminal (stations, terminuses, harbors and airports).

Edge (Link). An edge *e* is a link between two nodes. The link (*i* , *j*) is of initial extremity*i* and of terminal extremity *j*. A link is the abstraction of a transport infrastructure supporting movements between nodes. It has a direction that is commonly represented as an arrow. When an arrow is not used, it is assumed the link is bi-directional.

Sub-Graph. A sub-graph is a subset of a graph *G* where *p* is the number of sub-graphs. For instance *G'* = (*v'*, *e'*) can be a distinct sub-graph of *G*. Unless the global transport system is considered in its whole, every transport network is in theory a sub-graph of another. For instance, the road transportation network of a city is a sub-graph of a regional transportation network, which is itself a sub-graph of a national transportation network.

HISTORY OF GRAPH THEORY:

The origin of graph theory started with the problem of Koinsber bridge, in 1735. This problem lead to the concept of Eulerian Graph. Euler studied the problem of Koinsberg bridge and constructed a structure to solve the problem called Eulerian graph. In 1840, A.F Mobius gave the idea of complete graph and bipartite graph and Kuratowski proved that they are planar by means of recreational problems. The concept of tree, (a connected graph without cycles[7]) was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. In 1852, Thomas Gutherie found the famous four color problem. Then in 1856, Thomas. P. Kirkman and William R.Hamilton studied cycles on polyhydra and invented the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. In 1913, H.Dudeney mentioned a puzzle problem. Eventhough the four color problem was invented it was solved only after a century by Kenneth Appel and Wolfgang Haken. This time is considered as the birth of Graph Theory.

Why study graph theory?

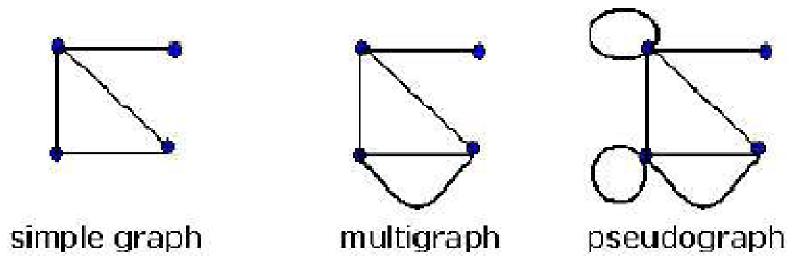
- Useful set of techniques for solving real-world problems – particularly for different kinds of optimisation.
- Graph theory is useful for analysing “things that are connected to other things”, which applies almost everywhere.
- Some difficult problems become easy when represented using a graph.
- There are lots of unsolved questions in graph theory: solve one and become rich and famous

TYPES OF GRAPH:

A graph that have nonempty set of vertices connected at most by one edge is called **simple graph**

When simple graphs are not efficient to model a citation, we consider **multigraphs**. They allow multiple edges between two vertices.

If that is not enough, we consider **pseudographs.** They allow edges connect a vertex to itself

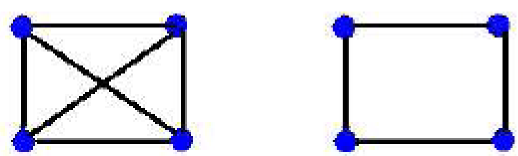


The set of edges is unordered. All such graphs are called **undirected graph.**

A **directed graph** consist of vertices and ordered pairs of edges. Note, multiple edges in the same direction are not allowed.

If multiple edges in the same direction are allowed, then a graph is called **directed multigraph**.

A graph in which every vertex has the same degree is called a **regular graph**. Here is an example of two regular graphs with four vertices that are of degree 2 and 3 correspondently



Connectivity

A path is a sequence of distinctive vertices connected by edges. Vertex v is reachable from u if there is a path from u to v. A graph is connected, if there is a path between any two vertices.

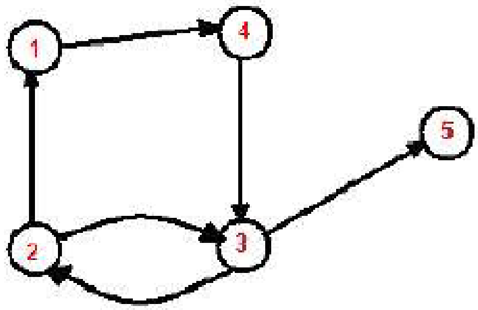
There are two standard ways to represent a graph:

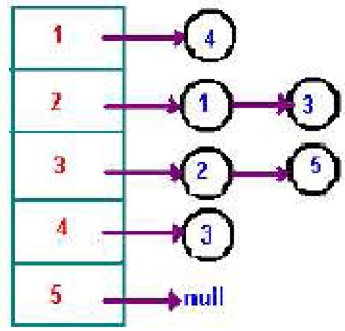
· as a collection of adjacency lists

· or as an adjacency matrix

An adjacency list representation is used for representation of the sparse graphs. An adjacency matrix representation may be preferred when the graph is dense.

The adjacency-list representation of a graph G consists of an array of linked lists, one for each vertex. Each such list contains all vertices adjacent to a chosen one. Here is an adjacency-list representation:





Vertices in an adjacency list are stored in an arbitrary order. A potential disadvantage of the adjacency-list representation is that there is no quicker way to determine if there is an edge between two given vertices.

Bipartite graphs

DEFINITION. A graph G is called bipartite, if VG has a partition to two subsets X and Y such that each edge uv ∈ G connects a vertex of X and a vertex of Y. In this case, (X, Y) is a bipartition of G, and G is (X, Y)-bipartite.

A bipartite graph G (as in the above) is complete (m, k)- bipartite, if |X| = m, |Y| = k, and uv ∈ G for all u ∈ X and v ∈ Y.

All complete (m, k)-bipartite graphs are isomorphic. Let Km, k denote such a graph.

A subset X ⊆ VG is stable, if G[X] is a discrete graph.

HOW TO DRAW GRAPH:

Graphs are represented graphically by drawing a dot or circle for every vertex, and drawing an arc between two vertices if they are connected by an edge. If the graph is directed, the direction is indicated by drawing an arrow.

A graph drawing should not be confused with the graph itself (the abstract, non-visual structure) as there are several ways to structure the graph drawing. All that matters is which vertices are connected to which others by how many edges and not the exact layout. In practice it is often difficult to decide if two drawings represent the same graph. Depending on the problem domain some layouts may be better suited and easier to understand than others.

The pioneering work of W. T. Tutte was very influential in the subject of graph drawing. Among other achievements, he introduced the use of linear algebraic methods to obtain graph drawings.

Graph drawing also can be said to encompass problems that deal with the crossing number and its various generalizations. The crossing number of a graph is the minimum number of intersections between edges that a drawing of the graph in the plane must contain. For a planar graph, the crossing number is zero by definition.

LINKS AND THEIR STRUCTURES

A transportation network enables flows of people, freight or information, which are occurring along its links. Graph theory must thus offer the possibility of representing movements as linkages, which can be considered over several aspects:

Connection. A set of two nodes as every node is linked to the other. Considers if a movement between two nodes is possible, whatever its direction. Knowing connections makes it possible to find if it is possible to reach a node from another node within a graph.

Path. A sequence of links that are traveled in the same direction. For a path to exist between two nodes, it must be possible to travel an uninterrupted sequence of links. Finding all the possible paths in a graph is a fundamental attribute in measuring accessibility and traffic flows.

Chain. A sequence of links having a connection in common with the other. Direction does not matter.

Length of a Link, Connection or Path. Refers to the label associated with a link, a connection or a path. This label can be distance, the amount of traffic, the capacity or any attribute of that link. The length of a path is the number of links (or connections) in this path.

Cycle. Refers to a chain where the initial and terminal node is the same and that does not use the same link more than once is a cycle.

Circuit. A path where the initial and terminal node corresponds. It is a cycle where all the links are traveled in the same direction. Circuits are very important in transportation because several distribution systems are using circuits to cover as much territory as possible in one direction (delivery route).

Clique. A clique is a maximal complete subgraph where all vertices are connected.

Cluster. Also called community, it refers to a group of nodes having denser relations with each other than with the rest of the network. A wide range of methods are used to reveal clusters in a network, notably they are based on modularity measures (intra- versus inter-cluster variance).

Ego network. For a given node, the ego network corresponds to a sub-graph where only its adjacent neighbors and their mutual links are included.

Nodal region. A nodal region refers to a subgroup (tree) of nodes polarized by an independent node (which largest flow link connects a smaller node) and a number of subordinate nodes (which largest flow link connects a larger node). Single or multiple linkage analysis methods are used to reveal such regions by removing secondary links between nodes while keeping only the heaviest links.

Dual graph. A method in space syntax that considers edges as nodes and nodes as edges. In urban street networks, large avenues made of several segments become single nodes while intersections with other avenues or streets become links (edges). This method is particularly useful to reveal hierarchical structures in a planar network.

PROPERTIES:

Symmetry and Asymmetry: A graph is symmetrical if each pair of nodes linked in one direction is also linked in the other. By convention, a line without an arrow represents a link where it is possible to move in both directions. However, both directions have to be defined in the graph. Most transport systems are symmetrical but asymmetry can often occur as it is the case for maritime (pendulum) and air services. Asymmetry is rare on road transportation networks, unless one-way streets are considered.

Assortativity and Disassortativity: Assortative networks are those characterized by relations among similar nodes, while disassortative networks are found when structurally different nodes are often connected. Transport (or technological) networks are often disassortative when they are non-planar, due to the higher probability for the network to be centralized into a few large hubs.

Completeness: A graph is complete if two nodes are linked in at least one direction. A complete graph has no sub-graph and all its nodes are interconnected.

Connectivity: A complete graph is described as connected if for all its distinct pairs of nodes there is a linking chain. Direction does not have importance for a graph to be connected, but may be a factor for the *level* of connectivity. If $p>1$ the graph is not connected because it has more than one sub-graph (or component). There are various levels of connectivity, depending on the degree at which each pair of nodes is connected.

Complementarity :Two sub graphs are complementary if their union results in a complete graph. Multimodal transportation networks are complementary as each sub-graph (modal network) benefits from the connectivity of other sub-graphs.

Root: A node *r* where every other node is the extremity of a path coming from *r* is a root. Direction has an importance. A root is generally the starting point of a distribution system, such as a factory or a warehouse.

Trees: A connected graph without a cycle is a tree. A tree has the same number of links than nodes plus one. ($e = v-1$). If a link is removed, the graph ceases to be connected. If a new link between two nodes is provided, a cycle is created. A branch of root *r* is a tree where no links are connecting any node more than once. River basins are typical examples of tree-like networks based on multiple sources connecting towards a single estuary. This structure strongly influences river transport systems.

APPLICATIONS OF GRAPH THEORY:

Graph theoretical concepts are widely used to study and model various applications, in different areas. They include, study of molecules, construction of bonds in chemistry and the study of atoms. Similarly, graph theory is used in sociology for example to measure actors prestige or to explore diffusion mechanisms. Graph theory is used in biology and conservation efforts where a vertex represents regions where certain species exist and the edges represent migration path or movement between the regions. This information is important when looking at breeding patterns or tracking the spread of disease, parasites and to study the impact of migration that affect other species. Graph theoretical concepts are widely used in Operations Research. For example, the traveling salesman problem, the shortest spanning tree in a weighted graph, obtaining an optimal match of jobs and men and locating the shortest path between two vertices in a graph. It is also used in modeling transport networks, activity networks and theory of games. The network activity is used to solve large number of combinatorial problems. The most popular and successful applications of networks in OR is the planning and scheduling of large complicated projects. The best well known problems are PERT(Project Evaluation Review Technique) and CPM (Critical Path Method). Next, Game theory is applied to the problems in engineering, economics and war science to find optimal way to perform certain tasks in competitive environments. To represent the method of finite game a digraph is used. Here, the vertices represent the positions and the edges represent the moves.

SUMMARY:

The main aim of this paper is to present the importance of graph theoretical ideas in various areas of compute applications for researches that they can use graph theoretical concepts for the research. An overview is presented especially to project the idea of graph theory.

REFERENCES:

1. Gian Luca Marcialis, Fabio Roli, Alessandra Serrau, "Graph Based and Structural Methods for Fingerprint Classification, Springer verlag, Berlin Heidelberg 2007
2. Bollobás, B. *Graph Theory: An Introductory Course.* New York: Springer-Verlag, 1979.
3. Chartrand, G. *Introductory Graph Theory.* New York: Dover, 1985.
4. Gould, R. (Ed.). *Graph Theory.* Menlo Park, CA: Benjamin-Cummings, 1988.
5. Gross, J. T. and Yellen, J. *Graph Theory and Its Applications.* Boca Raton, FL: CRC Press, p. 13, 1999.
6. Harary, F. *Graph Theory.* Reading, MA: Addison-Wesley, 1994.
7. Trudeau, R. J. *Introduction to Graph Theory.* New York: Dover, 1994.