STRONG (WEAK) DOMINATION AND INDEPENDENT STRONG (WEAK) DOMINATION IN BIPOLAR FUZZY GRAPHS

V. Mohanaselvi[1]
Assistant Professor,
PG &Research Department of Mathematics,
Nehru Memorial College, Puthanampatti-621007,
Tamilnadu, India

S. Sivamani[2]
Assistant Professor,
Department of Mathematics,
CARE Group of Institutions, Trichy- 621009,
Tamilnadu ,India

Abstract

Let u and v be any two vertices of bipolar fuzzy graphs G. Then u strongly dominates v (v weakly dominates u) if (i) effective edge between u and v and (ii) dN (u) ≥ dN (v). A subset S of V is called a strong (weak) dominating set in bipolar fuzzy graphs G for every v $\in$V-S there exist u $\in$S such that u strongly dominates v (v weakly dominates u) and it is denoted by SBFD-set (WBFD-set). A strong (weak) dominating set S of bipolar fuzzy graphs G is said to be minimal strong (weak) dominating set if no proper subset of S is a strong (weak) dominating set of G. The minimum cardinality among all minimal strong (weak) dominating set is called strong (weak) bipolar fuzzy domination number of G, and is denoted by $\gamma_s(\gamma_w)$. A SBFD-set (WBFD-set) S of bipolar fuzzy graphs G is said to be an independent strong (weak) dominating set of G, if it is independent. The minimum cardinality of an independent strong (weak) dominating set is called the independent strong (weak) bipolar fuzzy dominating number and it is denoted by $\gamma_{is}(\gamma_{iw})$. In this paper, the strong (weak) domination number and independent strong (weak) domination number in bipolar fuzzy graphs are introduced and its bounds are obtained. Also the strong (weak) dominating set and independent strong (weak) dominating set are characterised.

Keywords

Bipolar fuzzy graph (BFG), strong (weak) dominating set, strong (weak) domination number, independent strong (weak) dominating set, independent strong (weak) domination number

AMS (MOS) SUBJECT CODES: 05C69, 05C72

1. INTRODUCTION

In 1965, L.A. Zadeh [8] introduced the notion of fuzzy subset of a set as a method of presenting uncertainty.

In 1962, the study of dominating sets in graphs was begun by O. Ore and C. Berge, the domination number, independent domination number are introduced by E.J. Cockayne and S.T. Hedetniemi in 1977.

A. Rosenfeld [4] introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness in 1975, whose basic idea was introduced by Kauffman in 1973.

In 1988, A. Somasundram and S. Somasundram [6] discussed domination in fuzzy graph. They defined domination using effective edges in fuzzy graph. In 2004
A. Somasundaram presents the concepts of independent domination, total domination, and connected domination of fuzzy graphs. In 2010, C. Natarajan and S.K. Ayyaswamy [3] introduce the strong (weak) domination in fuzzy

graph. In 2013, N. Vinoth kumar and G. Geetha Ramani [7] introduce the strong (weak) domination in intuitionistic fuzzy graph.

In 1994, W.R. Zhang [9] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are extension of fuzzy sets whose range of membership degree is [−1, 1]. In 2011, Akram [1] introduced the concept of bipolar fuzzy graphs and defined different operations on it. Bipolar fuzzy graph theory is now growing and expanding its applications.

Recently in 2013, M. G. Karunambigai, S. Sivasankar, M. Akram, K. Palanivel [2], introduce the concept of domination in bipolar fuzzy graphs.

The purpose of this paper is to introduce the concepts of strong (weak) domination, independent strong (weak) domination in bipolar fuzzy graphs. Also, strong (weak) domination number, independent strong (weak) domination number are defined for several classes of bipolar fuzzy graphs and investigated some of their properties.

Throughout this paper, a graph G = (V, E) denote a bipolar fuzzy graph without self loop and parallel edges with order p and size q..

2. STRONG (WEAK) DOMINATION IN BIPOLAR FUZZY GRAPHS

2.1 DEFINITIONS

Definition 2.1: Let u and v be any two vertices of G. Then u strongly dominates v (v weakly dominates u) if (i) effective edge between u and v (ii) $d_N(u) \geq d_N(v)$.

A subset S of V is called a strong (weak) dominating set in G for every v $\in$ V-S there exist u $\in$ S such that u strongly dominates v (v weakly dominates u) and it is denoted by SBFD-set (WBFD-set)

Definition 2.2: A strong (weak) dominating set S of G is said to be minimal strong (weak) dominating set if no proper subset of S is a strong (weak) dominating set of G. The minimum cardinality among all minimal strong (weak) dominating set is called strong (weak) bipolar fuzzy domination number of G, and is denoted by $\gamma_s(\gamma_w)$

Definition 2.3: A SBFD-set (WBFD-set) S of G is said to be an independent strong (weak) dominating set of G, if it is independent. The minimum cardinality of an independent strong (weak) dominating set is called the independent strong (weak) bipolar fuzzy dominating number and it is denoted by $\gamma_{is}(\gamma_{iw})$.

Definition 2.4: If the *minimum neighbourhood degree* of G is δ_N and the *maximum neighbourhood edge degree* of G is Δ_N, then $V_{\delta_N} = \{ v \in V : dN(v) = \delta_N \}$

$V_{\Delta_N} = \{ v \in V : dN(v) = \Delta_N \}$

Example: 2.5

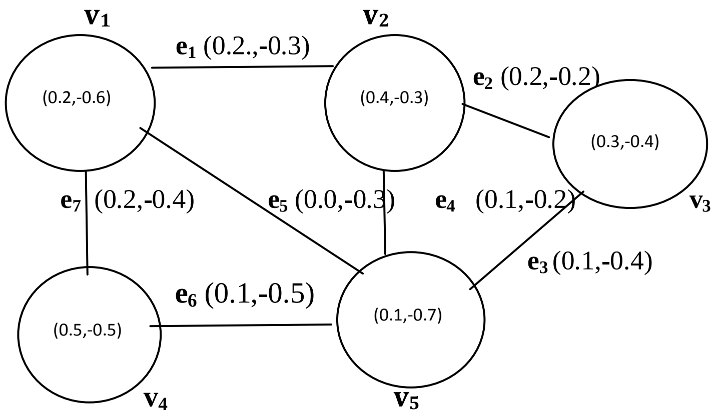

Figure: 1

For the figure: 1 the following results are obtained.

Order p = 2	SBFD-set is $\{v_1, v_5\}, \gamma_s = 0.5$
δ_E = 0.3, Δ_E = 0.65	WBFD-set is$\{v_2, v_3, v_4\}, \gamma_w = 1.5$
δ_N = 0.2, Δ_N = 0.95	Independent SBFD-set is $\{v_1, v_5\}$, $\gamma_{is} = 0.5$
Dominating set is $\{v_1, v_5\}$, γ = 0.5	Independent WBFD set is $\{v_2, v_3, v_4\}$, $\gamma_{iw} = 1.5$

Remark: 2.6 If S is a minimal SBFD set, then V-S need not be a WBFD set.

2.2. MAIN RESULTS

Theorem 2.7: A strong dominating set D of G is a minimal strong dominating set if and only if for each u ∈ D one of the following conditions holds.

(i) N (u) ∩ D = ϕ.

(ii)There is a vertex v ∈ V−D such that N (v) ∩ D = {u}.

Proof: Assume that D is a minimal strong dominating set of G. Then for every vertex u ∈ D, D − {u} is not a strong dominating set. Hence there exists v ∈ V− (D − {u}) which is not dominated by any vertex in D − {u}. If u= v, we get, N (u) ∩ D = ϕ. If u$\neq$v, v is not strongly dominated by D-{u}, but v is strongly dominated by D, then the vertex v is strongly dominated by u in D. Hence N (v) ∩ D = {u}.

Conversely, assume that D is a strong dominating set and for each vertex u ∈ D, one of the two conditions holds. Suppose D is not a minimal perfect dominating set, then there exists a vertex u ∈ D, D − {u} is a strong dominating set. Hence u is adjacent to at least one vertex in D−{u}, the condition (i) does not hold. If D−{u}is a dominating set then every vertex in V− D is adjacent to at least one vertex in

D −{u}, the second condition does not hold ,which is a contradiction to our assumption. So D is a minimal strong dominating set.

Theorem 2.8: A weak dominating set D of G is a minimal weak dominating set if and only if for each u ∈ D one of the following conditions holds.
(i) N (u) ∩ D = ϕ.
(ii)There is a vertex v ∈ V−D such that N (v) ∩ D = {u}.
Proof is similar as in theorem 3.1.

Theorem 2.9: For any graph G,
(i) $\gamma \le \gamma_s \le p - \Delta_N$ (ii) $\gamma \le \gamma_w \le p - \delta_N$
Proof: Since every SBFD (WBFD) is a dominating set of G, we have $\gamma \le \gamma_s$, $\gamma \le \gamma_w$.
Let u ∈ V. If dN(u) = Δ_N , then V-N(u) is a SBFD set but not minimal. Therefore $\gamma_s \le$ |V − N (u)| = p − Δ_N Let v ∈ V. If dN(v) = δ_N , then V-N(v) is a WBFD set but not minimal. Therefore $\gamma_w \le$ |V − N (u)| = p − δ_N .

Corollary 2.10: For any graph G, $\gamma \le \gamma_s \le p - \Delta_N \le p - \Delta_E$
Proof: Since $\Delta_E \le \Delta_N$ and by theorem 3.3, we have $\gamma \le \gamma_s \le p - \Delta_N \le p - \Delta_E$

Corollary 2.11: For any graph G,
$\gamma \le \gamma_s \le p - \Delta_N \le p - \Delta_E \le p - \delta_E$
Proof: Since $\delta_E \le \Delta_E$ and by corollary 3.4, we have
$\gamma \le \gamma_s \le p - \Delta_N \le p - \Delta_E \le p - \delta_E$

Corollary 2.12: For any graph G
$\gamma \le \gamma_s \le p - \Delta_N \le p - \Delta_E \le p - \delta_E \le p - \delta_N$
Proof: Since $\delta_N \le \delta_E$ and by corollary 3.5, we have
$\gamma \le \gamma_s \le p - \Delta_N \le p - \Delta_E \le p - \delta_E \le p - \delta_N$

Lemma 2.13: If D is an independent WBFD set of G, then $D \cap V_{\delta N} \neq \phi$
Proof: Let v ∈ $V_{\delta N}$. Since D is independent WBFD, v ∈ D or there exist a vertex u ∈ D such that (u, v) is a strong edge in G for which dN(u) ≤ dN(v). If v ∈ D, then v ∈ $D \cap V_{\delta N}$
Hence $D \cap V_{\delta N} \neq \phi$. If v ∉ D, v ∈ $V_{\delta N}$ then
dN(v) = dN(u). Hence dN(u) = δ_N , ⇒ u ∈ $V_{\delta N}$.
Therefore u ∈ $D \cap V_{\delta N} \Rightarrow D \cap V_{\delta N} \neq \phi$

Lemma 2.14: If D is an independent SBFD set of G, then $D \cap V_{\Delta N} \neq \phi$
Proof is similar as in Lemma 2.13

Theorem 2.15: For any graph G, $\gamma_{iw} \leq p - \delta_N$

Proof: Let D be an independent WBFD- set of G.

Then by lemma 3.7 $D \cap V_{\Delta N} \neq \phi$

Let v$\in$ $D \cap V_{\delta N} \Rightarrow D \cap N(v) = \phi$.($\because$ D is independent).

$\Rightarrow$D $\subseteq V - N(v)$

$\Rightarrow$| D | $\leq |V - N(v)|$

$\Rightarrow \gamma_{iw} \leq p - dN(v)$

$\Rightarrow \gamma_{iw} \leq p - \delta_N$ ($\because v \in V_{\delta N}$).

Theorem 2.16: For any graph G, $\gamma_{is} \leq p - \Delta_N$

Proof is similar as in theorem 2.9.

Theorem 2.17: Let G be a BFG with $\gamma_{iw} = p - \delta_N$

and $v \in V_{\delta N}$. Then V-N (v) is independent.

Proof: Let G be a BFG with $\gamma_{iw} = p - \delta_N$

and $v \in V_{\delta N}$. Suppose that V- N (v) is dependent. Applying the following algorithm we get an independent weak bipolar fuzzy dominating set of size at most $p - \delta_N - |v|$which is a contradiction to $\gamma_{iw} = p - \delta_N$

Hence V-N (v) is independent.

Algorithm:
S: = N (v)
D: = {v}
While S $\neq$ V
begin
Let u $\in$ {u $\in$ V-S: dN (u) is as small as possible
S: = S $\bigcup$ N (u)
D: = D $\bigcup$ {u}
end

Theorem 2.18: Let G be a BFG with $\gamma_{is} = p - \Delta_N$

and $v \in V_{\Delta N}$. Then V-N (v) is independent.

Proof is similar as in theorem 2.17.

Theorem 2.19: Let G be a BFG with $\gamma_{iw} = p - \delta_N$

iff V-N (v) is independent for every $v \in V_{\delta N}$.

Proof: Let $\gamma_{iw} = p - \delta_N$ and $v \in V_{\delta N}$.

Then V-N (v) is independent by theorem 2.17.

Conversely, Let V-N (v) is independent for every $v \in V_{\delta N}$.Let D be a minimum independent weak dominating bipolar fuzyy set. Hence $D \cap V_{\Delta N} \neq \phi$

Let v $\in D \cap V_{\delta N} \Rightarrow D \cap N(v) = \phi$.

Hence D = V-N (v) (since V-N (v) is independent), $\gamma_{iw} = p - \delta_N$

Theorem 2.20: Let G be a BFG with $\gamma_{is} = p - \Delta_N$
iff V-N (v) is independent for every $v \in V_{\Delta N}$.

Observations 2.21: For G, $\gamma_s \leq \gamma_w$ and $\gamma_{is} \leq \gamma_{iw}$

4. CONCLUSION

In this paper strong (weak) dominating set, independent strong (weak) dominating set, strong (weak) domination number and independent strong (weak) domination number are defined for bipolar fuzzy graphs. The necessary and sufficient conditions for the existence of strong (weak) dominating set are given. The upper bound of strong (weak) domination number and independent strong (weak) domination number are given in terms of its order and degree. The strong (weak) dominating sets are characterised when they are independent. Also we characterised independent strong (weak) dominating set when the upper bound of its strong (weak) domination number is sharp.

REFERENCES

[1] Akram, M., Bipolar fuzzy graphs, *Information Sciences*, Volume 181,(2011),5548-5564.

[2] Karunambigai, M.G., Akram, M., Palanivel,K., and Sivasankar, S., Domination in Bipolar Fuzzy Graph, Proceedings of the International Conference on Fuzzy Systems, FUZZ-IEEE- 2013, Hyderabad, India,(2013),1-6.

[3] Natarajan.C, and Ayyaswamy.S, Strong (weak) domination in fuzzy graphs, International Journal of Computational and Mathematical sciences, 2010

[4] Rosenfeld, A., Fuzzy graphs, Fuzzy Sets and their Applications to Cognitive and Decision Processes (Proc. U.S.-Japan Sem., Univ. Calif., Berkeley,Calif., 1974), Academic Press, New York, pp. 77-95, 1975.

[5] Mohanaselvi,V., S. Sivamani., Perfect domination in bipolar fuzzy graphs, Proceedings of the International Conference on Mathematical Methods and Computation,Trichy,India (2015),776-779,(Jamal Academic Research Journal: An Interdisciplinary ISSN 0973-0303).

[6] Somasundaram,A., and Somasundaram,S., Domination in fuzzy graphs-I, Pattern Recognition Letters, 19,(1988),787-791.

[7] Vinoth kumar,N. and Geetha Ramani,G.,Some domination parameters of the intuitionist fuzzy graph & its properties, International journal of mathematics & scientific computing, vol.1 (1) (2011), pp.5-6.

[8] Zadeh, L.A., Fuzzy Sets, Information and Control, Vol-8, 338-353, 1965.

[9] Zhang,W.R., Bipolar fuzzy sets and relations: a computational framework for cognitive modeling and multiagent decision analysis, *Proc. Of IEEE Conf.*, (1994).