# CONTACT CONFORMAL IN -SASAKIAN MANIFOLD

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In this paper, we have defined two quarter symmetric metric-F-T-connections in a Trans sasakian manifold, we have shown, by following the patterns of K. Yano (1976) and Mishra and Pandey (1980)

## 1. INTRODUCTION

Let $V_n$ (n= 2m+1) be an almost contact metric manifold equipped with a structure tensor F, of type (1.1) a contravariant vector T, a-l-form A and a metric tensor g, satisfying

(1.1)(a) $F^2X = -X + A(X)T$ (b) $FT = 0$ and

(1.2)(c) $g(T,X) = A(X)$ Also, a fundamental 2-form 'F in $V_n$ is defined as

(1.3) $`F(X,Y) = g(\bar{X},Y) = -g(X,\bar{Y}) = -`F(Y,X)$

Then we call the structure bundle {F,T,A,g} an almost contact metric structure.

An almost contact metric manifold $V_n$ is said to be normal if

(1.4)(a) $(d A)(X,Y) + 2`F(X,Y) = 0$ where,

(1.4)(b) $(d A)(X,Y) = (D_X A)(Y) - (D_Y A)(X)$

Here D is is the Riemannian Connection and 'd' is the exterior derivative in Vn

with regard to the metric tensor g.

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Now, an almost contact metric structure {F,T,A,g} on $V_n$ is called trans-sasakian structure [2], [3], if

(1.5) $(D_X F)(Y) = \alpha\left(g(X,Y)T - A(Y)X\right) + \beta\left\{`F(X,Y)T - A(Y)\overline{X}\right\}$

It can easily be seen that a trans-sasakian manifold is normal and in view of (1.5) one can easily obtain in $V_n$ the relations

(1.6)(a) $D_x T = -\alpha\overline{X} + \beta\left(X - A(X)T\right)$ and

(1.6)(b) $(D_X A)(Y) = -\alpha`F(X,Y) + \beta\, g\left(\overline{X},\overline{Y}\right)$

**REMARK**

(1) In the above and in what follows, X,Y,Z,......, etc. are tangent vector fields in $V_n$.

## 2. CONTACT CONFORMAL CONNECTION IN A TRANS-SASAKIAN MANIFOLD

Let us consider a conformal change of the metric tensor g which induces a new metric tensoer , given by

(2.1) $\tilde{g}(X,Y) = e^{2p}\, g(X,Y)$

with regard to this metric we take an affine connection B, which satisfies:-

(2.2) $(B_x\tilde{g})(Y,Z) = B_x\left\{e^{2p}\, g(Y,Z)\right\} = e^{2p}\, p(X)\, A(Y) A(Z),$

where p is a scalar point function in $V_n$ and

(2.3) being covariant derivative of the scalar p with respect to the metric tensor g, is a 1-form in $V_n$, where contravariant vector is P. Further, we assume that the torsion tensor of the connection B satisfies:-

(2.4) $S(X,Y) = -2`F(X,Y)U$

where U is certain contravariant vector field. In view of (2.2) and (2.4), we can easily obtain a relation between the connection B and the Riemannian Connection D[l], given by

(2.5) $B_y Z = D_y Z + \left\{Y - A(Y)T\right\} p(Z) + \left\{Z - A(Z)T\right\} p(Y)$

$-g\left(\overline{Y},\overline{Z}\right) p + u(Y)\overline{Z} + u(Z)\overline{Y} - `F(Y,Z)U,$

where, $u(X) \underline{\underline{def}}\; g(u,X)$

Now, we suppose B an F-connection so that

$$\left(B_yF\right)(Z)=0=\left(D_yF\right)(Z)+\{Y-A(Y)T\}\,p\left(\bar{Z}\right)-p(Z)\bar{Y}+p(Y)\bar{Z}-p(Y)\bar{Z}$$

$$+{}`F(Y,Z)P+g\left(\bar{Y},\bar{Z}\right)p+u(Y)\bar{\bar{Z}}-u(Y)\bar{\bar{Z}}+U\left(\bar{Z}\right)\bar{Y}+u(Z)$$

$$\{Y-A(Y)T\}-g\left(\bar{Y},\bar{Z}\right)U+{}`F(Y,Z)\bar{U}$$

Using (1.5), the above relation becomes

$$\alpha\{g(Y,Z)T-A(Z)Y\}+\beta\{{}`F(Y,Z)T-A(Z)\bar{Y}\}+p\left(\bar{Z}\right)\{Y-A(Y)T\}$$

$$-p(Z)\bar{Y}+{}`F(Y,Z)P+g\left(\bar{Y},\bar{Z}\right)\bar{P}+u\left(\bar{Z}\right)\bar{Y}+u(Z)\{Y-A(Y)T\}$$

$$-g\left(\bar{Y},\bar{Z}\right)U+{}`F(Y,Z)\bar{U}=0$$

Contracting the above equation with respect to Y, we have

$$-2m\alpha A(Z)+2mp\left(\bar{Z}\right)-p\left(\bar{Z}\right)+2mu(Z)-u(Z)+A(Z)-u(Z)$$

$$+A(u)A(Z)=0$$

or $\quad 2(m-1)\,p\left(\bar{Z}\right)+2(m-1)U(Z)-2A(Z)\{m\alpha-A(u)\}=0$

If we put

$A(U)=u(T)=\alpha,$ then

(2.6)(a) $\quad u(Z)=\alpha A(Z)-p\left(\bar{Z}\right) \quad$ or

(2.6)(b) $\quad U = \alpha T+\bar{P}$

here, we put $Q=\bar{P}$ so that $q(Z)=g(Q,Z)=-p\left(\bar{Z}\right)$ and $p(Q)=q(P)=0,$

then (2.6) becomes

(2.7)(a) $\quad u(Z)=\alpha A(Z)+q(Z) \quad$ and

(2.7)(b) $\quad U = \alpha T+Q$

Using (2.7) in (2.5), we have

(2.8) $\quad B_YZ=D_YZ+\left(Y-A(Y)T\right)p(Z)+\{Z-A(Z)T\}\,p(Y)$

$$-g\left(\bar{Y},\bar{Z}\right)P+\{\alpha A(Y)+q(Y)\}\,\bar{Z}+\{\alpha A(Z)+q(Z)\}\bar{Y}$$

$$-{}`F(Y,Z)\{\alpha T+Q\}$$

Further, we suppose that B is a T-connection,

Then

$$B_YT=0=D_YT+p(T)\{Y-A(Y)T\}+\alpha\bar{Y}$$

Using (1.6)(a) in this equation, we have

$$-\alpha\overline{Y}+\beta\{(Y-A)T\}+p\{Y-A(Y)T\}+\alpha\overline{Y}=0$$

which implies that

(2.9) $p(T)=A(P)=-\beta$

**Proposition (2.1):** In a Trans-sasakian manifold the affine connection B, which is an F-T- connection and whose torsion tensor satisfies (2.4), is given by (2,8) with the conditions $u(T)=a=A(U)$, $p(T)$=-b=$A(P)$

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