STRUCTURAL DYNAMIC ANALYSIS OF CANTILEVER BEAM STRUCTURE

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ABSTRACT

Structural dynamic analysis is required to understand the vibration related behavior of structures including their eigenvalues, modeshapes and frequency response functions. In this paper, structural dynamic analysis of a cantilever beam structure has been performed. Spatial finite element model of the beam structure is formulated and then analyzed further to produce modal model and response model of the structure. The work is helpful in understanding the dynamic behavior such as extreme versus intermediate positions of modeshapes and point versus transient frequency response functions of the cantilever beam structure.

***Keywords**: Structural dynamic analysis, Finite element method.*

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1. INTRODUCTION

Better dynamic testing and analysis tools are becoming the need of the day with the ever increasing demands for better performance and the use of lighter materials in modern day machines and structures. With the modern high performance engines, one can achieve very high speeds in no time, which results in increased vibration and noise problems. Further in the automotive, aircraft and spaceship industries, there is an ever existing demand of attaining better fuel economy; which can be met to a good extent by using thin products as well as with the use of light weight materials such as aluminium and plastics composites instead of the conventionally used heavy weight materials such as steels. Thin and light weight products have lot more tendencies to vibrate than their thick and heavy weight counterparts. Excessive vibrations can even result in pre-mature failure of products, whether it is the suspension of an automobile, wing of an aircraft, the printed-circuit-board (PCB) installed in a spaceship, blades of an air-cooler, or the compact-disc (CD) of a computer etc. On the other hand, consumers of today's world desire for non-vibrating and silent functioning of such products. Thus it becomes very important for engineers to understand the vibration behavior of structures through their dynamic analysis [1]. Dynamic analysis aims at understanding, evaluating and modifying the structural dynamic behavior which involves many terms such as natural frequencies, eigenvalues, eigenvectors, damping ratios, Frequency Response Functions (FRFs) etc.

2. DYNAMIC ANALYSIS OF BEAM STRUCTURE

Dynamic analysis involves the formation of model of the system using Finite Element (FE) method [2]. In this method, a complex continuous region of a structure is discretized into simple geometric shapes called finite elements such as an axial element, torque element, beam bending element, thin plate bending element, thick plate bending element, etc. Boundary points of such finite elements are called as nodes. The results of structural dynamic analysis can be used for finite element model updating [3]. Dynamic analysis of structures is also the backbone of dynamic design of structures [4, 5]. In this paper the cantilever beam structure, as shown in Fig. 1, has been taken as a case study because of its simplicity and also the ability to represent a variety of mechanical products such as wing of an aircraft, rotor blade of a helicopter, blade of a ceiling fan, needle of a clock, shelve of a civil structure, solar panel of a satellite etc. Dimensions of the cantilever beam structure of MS material are 910 x 50 x 5 mm, having a mass density of 7800 kg/m^3. These dimensions have also been reported by Arora et al. but for a fixed-fixed beam structure [6]. The FE model of the beam under considerations is developed using 60 beam type elements, each FE having two nodes, thereby resulting in total 61 nodes. At each node, two degrees of freedom are measured, out of which one is the displacement in y-direction and the other is the rotation about z-axis. Both the degrees of freedom of node number 1 are fully constrained. For dynamic analysis purpose, each element is expressed in the form of elemental mass, stiffness and damping matrices. Eq. (1) and (2) represent the elemental mass and stiffness matrices for a beam element.

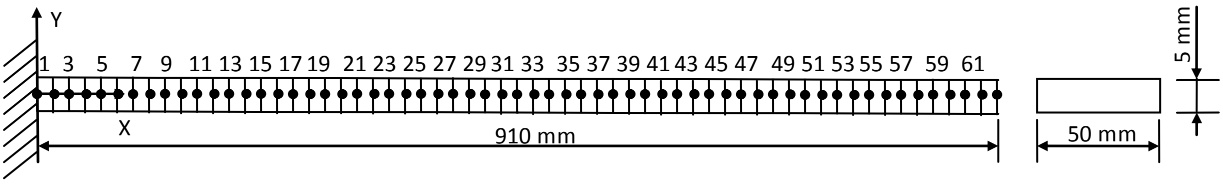
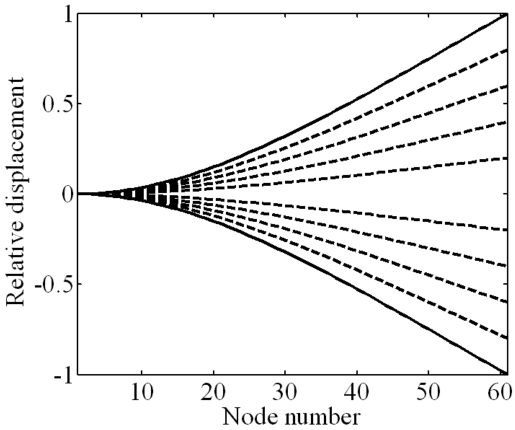


Fig. 1: FE model of a cantilever beam structure

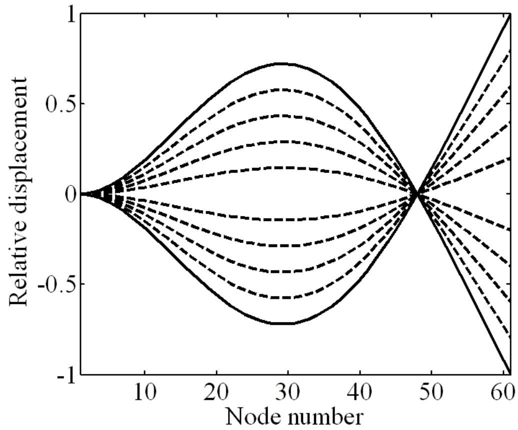
$$[m_e] = \frac{\rho A a}{105}\begin{bmatrix} 78 & 22a & 27 & -13 \\ 22a & 8a^2 & 13a & -6a^2 \\ 27 & 13a & 78 & -22a \\ -13a & -6a^2 & -22a & 8a^2 \end{bmatrix} \quad (1)$$

$$[k_e] = \frac{EI}{2a^3}\begin{bmatrix} 3 & 3a & -3 & 3a \\ 3a & 4a^2 & -3a & 2a^2 \\ -3 & -3a & 3 & -3a \\ 3a & 2a^2 & -3a & 4a^2 \end{bmatrix} \quad (2)$$

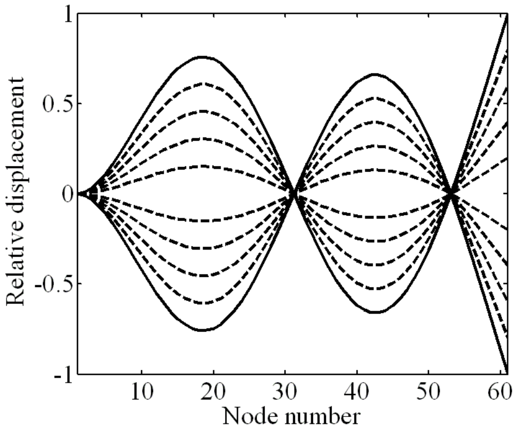
Where m_e, ρ, A, a, k_e, E and I are respectively the element mass matrix, density, area of cross-section of element, half-length of element, element stiffness matrix, Young's modulus of elasticity and moment of inertia of cross-section of beam element. Subsequently, the individual elements are assembled to form their global counterparts, which are also jointly known as system matrices. These system matrices along with certain boundary conditions are used to formulate a set of governing equations, which are then processed on a computer to evaluate dynamic characteristics of the system. Thus it is seen that the dynamic response of the structure depends upon a number of input parameters belonging to material, structural, finite element and computational categories. In the present research work, the FE model of the beam is processed in Matlab to produce the theoretical dynamic response of the cantilever beam. First six natural frequencies predicted by the FE model are 4.9, 30.9, 86.6, 169.8, 280.7, and 419.4 Hz. The extreme and intermediate positions of different points along the length of beam during first four modes of vibration are also drawn in Fig. 2. The points, having larger displacements from their mean or neutral position, will vibrate at higher average speeds than those points which are displaced to a smaller distance from their mean positions.



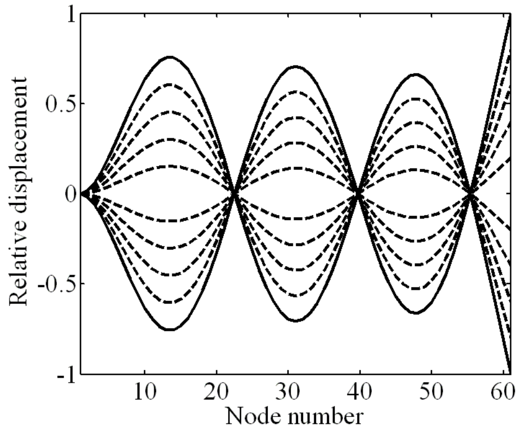
(a) First mode



(b) Second mode

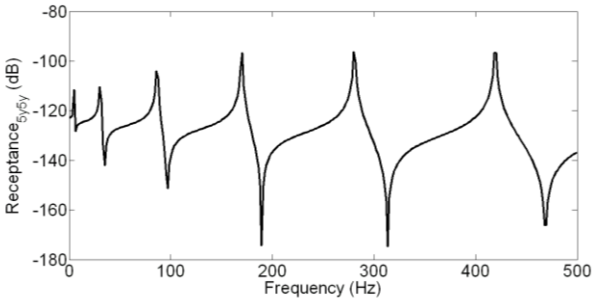


(c) Third mode

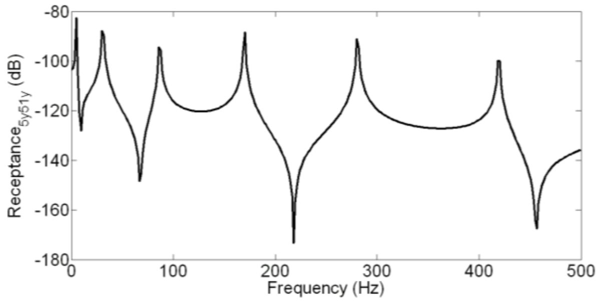


(d) Fourth mode

Fig. 2: Extreme and intermediate positions drawn using solid and dashed lines respectively



(a) Point FRF



(b) Transient FRF

Fig. 3: Frequency response functions of cantilever beam structure

Receptance FRFs obtained during structural dynamic analysis are drawn in Fig. 3. Receptance '5y5y' is drawn in Fig. 3(a), which is a function of displacement signal in 'y' direction at node '5' and force signal in 'y' direction at node '5'. This type of FRF, where displacement and force signals are measured at same points is called as point FRFs. Another category of FRFs is transient FRFs as drawn in Fig. 3(b), where displacement and force signals are measured at separated nodes. For example, receptance '5y54y' represents a receptance FRF generated using displacement signal at node '5' and force signal at node '54', both in 'y' direction. It is seen that in a point FRF each resonance point (peak) in the curve is followed by an anti-resonance point (valley).

3. CONCLUSION

A cantilever beam structure has been analyzed for its dynamic behavior. Eigenvalues, modeshapes and point as well as transient FRFs have been drawn and analyzed through the application of FE method. The analysis is very useful in the field of structural dynamic modification, dynamic design of structures and finite element model updating. Future efforts will be directed towards use of the results of present research work for finite element model updating of a cantilever beam structure with the help of techniques of design of experiments.

REFERENCES

1. N. M. M. Maia and J. M. Silva, Theoretical and Experimental Modal Analysis, Research Studies Press Limited, England, 1997.
2. M. Petyt, Introduction to Finite Element Vibration Analysis, Cambridge University Press, Cambridge, United Kingdom, 1998.
3. M. I. Friswell and J. E. Mottershead, Finite Element Model Updating in Structural Dynamics, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1995.
4. R. S. Bais, A. K. Gupta, B. C. Nakra and T. K. kundra, Studies in dynamic design of drilling machine using updated finite element models, Mechanism and Machine Theory, 39, 2004, pp. 1307-1320.
5. S. V. Modak, T. K. Kundra and B. C. Nakra, Studies in dynamic design using updated models, Journal of Sound and Vibration, 281, 2005, pp. 943-964.
6. V. Arora, S. P. Singh and T. K. Kundra, Finite element model updating with damping identification, Journal of Sound and Vibration, 324, 2009, pp. 1111-1123.