BIANCHI TYPE-IX STRING COSMOLOGICAL MODEL WITH VISCOUS FLUID AND MAGNETIC FLUX

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Abstract: ***In this paper, we have examined the nature of Bianchi type-IX string cosmological model in presence of viscous fluid and electromagnetic field in general relativity. We have considered the magnetic permeability as variable quantity. The magnetic field is due to an electric current produced along x-axis, F_{23} is the only non-vanishing component of electromagnetic field tensor F_{ij}. To get the deterministic solution, it has been assumed the metric potential relation $a = b^n$ in which, n is constant. The physical and geometrical properties of this model were discussed.***

Keywords: Bianchi type-IX space-time, Viscous fluid, Electromagnetic field, Magnetic permeability.

1. Introduction:

It is interesting to study the Bianchi type-IX cosmological models because these models allow not only expansion but also rotation and shear, and in general these are anisotropic. Many relativists have taken keen interest in studying Bianchi type-IX universe because familiar solutions like Robertson-Walker universe, the de Sitter universe, the Taub-Nut solutions etc. are of Bianchi type-IX space-times. The homogeneous and isotropic FRW cosmological models, which are used to describe standard cosmological models, are the particular cases of Bianchi type I, V and IX space-times. According to the constant curvature of the physical three-space, t = constant, i.e. zero, negative or positive. In these models, neutrino viscosity explains the large radiation entropy in the universe and the degree of isotropy of the cosmic background radiation. Vaidya and Patel [1] have studied spatially homogeneous space-time of Bianchi type-IX, and they have outlined a general scheme forth derivation of exact solutions of Einstein's field equations in the presence of perfect fluid and pure radiation fields. And many other researchers namely, Uggla and Zur-Muhlen [2], Burd et al. [3], King [4], and Paternoga and Graham et al. [5] have studied Bianchi type-IX space-time in different context. Bali and Yadav [6] have investigated Bianchi type-IX viscous fluid cosmological models. Pradhan et al. [7] derived Bianchi type-IX viscous fluid cosmological models with a varying cosmological constant. The dynamical effects of spatially homogeneous electromagnetic field on anisotropic Bianchi type-IX models are studied by Waller [8]. Brill [9] studied Einstein Maxwell's equations in a homogeneous and non-isotropic universe. Bali and Dave [10] have investigated Bianchi type-IX string cosmological model in General Relativity. Bali and Kumawat [11] have investigated Bianchi type-IX stiff fluid tilted cosmological model with bulk viscosity. Tyagi and Sharma [12] investigated Bianchi type-IX string cosmological model for perfect fluid distribution in General Relativity. Tyagi et al.

[13] have studied homogeneous anisotropic Bianchi type cosmological model for perfect fluid distribution and electromagnetic field tensor.

In this paper, we have investigated homogeneous anisotropic Bianchi type-IX string cosmological model with viscous fluid distribution and electro-magnetic field. We have assumed that F_{23} is the only non-vanishing component of electro-magnetic field tensor F_{ij}. To get a determinate model, we have assumed the metric potentials relation $a = b^n$, where n is constant. The various physical and geometrical aspects of the model are also discussed.

2. Formations of Field Equations:

We have consider the homogeneous anisotropic Bianchi type-IX metric,

$$ds^2 = -dt^2 + a^2(t)dx^2 + b^2(t)dy^2 + \left(b^2 sin^2 y + a^2 cos^2 y\right)dz^2 - 2a^2 cosy\, dx\, dz \quad (1)$$

Where metric potentials a and b are functions of 't' alone.

The energy momentum tensor for viscous fluid distribution and electromagnetic field is taken in the form

$$T_j^i = \rho v^i v_j - \lambda x^i x_j - \xi\theta\left(v^i v_j + g_j^i\right) + E_j^i \quad (2)$$

The comoving coordinates for metric (1) are considered as,

$v^1 = 0 = v^2 = v^3\,,\ v^0 = 1\,.$

The electromagnetic field E_j^i is defined as,

$$E_j^i = \frac{1}{4\pi}\left[-F_{jl}F^{il} + \frac{1}{4}g_i^j F_{lm}F^{lm}\right] \quad (3)$$

With the Maxwell's equation

$$\frac{\partial}{\partial x^i}\left(F^{ij}\sqrt{-g}\right) = 0 \quad (4)$$

Equation (4) leads to,

$$F_{23} = H\, siny \quad (5)$$

3. Solution of Field Equation:

The Einstein's field equation is given by

$$R_j^i - \frac{1}{2}R\, g_j^i = -8\pi\, T_j^i \quad (6)$$

For the line element (1), equation (6) implies,

$$2\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{3}{4}\frac{a^2}{b^4} = 8\pi(\lambda + \xi\theta) - \frac{H^2}{b^4} \quad (7)$$

$$2\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{\dot{b}^2}{b^2} + \frac{1}{b^2} - \frac{1}{4}\frac{a^2}{b^4} = 8\pi\varrho - \frac{H^2}{b^4} \quad (8)$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{1}{4}\frac{a^2}{b^4} = 8\pi(\xi\theta) + \frac{H^2}{b^4} \quad (9)$$

To obtain the solution of field equations (7), (8) and (9), we have

$$\theta = (n+2)\frac{\dot{b}}{b} \quad (10)$$

Assuming the metric potential relation,

$$a = b^n \quad (11a)$$

and the relation between coefficient of viscosity and scalar expansion is,

$$\xi \propto \theta \qquad \text{i.e.} \qquad \xi = k\theta \quad (11b)$$

Where k is proportionality constant .

To solve the field equations, we assume that

$k = \frac{1}{8\pi(n+2)^2}$, provided that $n \neq -2$

Therefore

$$\xi = \frac{1}{8\pi(n+2)}\frac{\dot{b}}{b} \quad (11c)$$

Using equations (11a), (11b) and (11c) in equation (9), we obtain,

$$\dot{b} = \frac{db}{dt} = \left(\frac{2}{(n+1)}\left(\frac{H^2}{2(n-2)b^2} - \frac{b^{2n-2}}{16(n-1)}\right)\right)^{1/2}$$

Using above equation,the equation (1) takes the form,

$$ds^2 = -\left(\frac{dt}{db}\right)^2 db^2 + b^{2n}(t)dx^2 + b^2(t)dy^2 + \left(b^2 sin^2 y + b^{2n} cos^2 y\right)dz^2 - 2b^{2n} cosy\, dx\, dz \quad (12)$$

Substitute $b = T$ in equation (12), Equation (12) becomes,

$$ds^2 = -\left[\frac{2}{(n+1)}\left(\frac{H^2}{2(n-2)T^2} - \frac{T^{2n-2}}{16(n-1)}\right)\right]^{-1} dT^2 + T^{2n}(t)dx^2 + T^2(t)dy^2 + \left(T^2 sin^2 y + T^{2n} cos^2 y\right)dz^2 - 2T^{2n} cosy\, dx\, dz. \quad (13)$$

For physical and geometrical interpretation,

$$8\pi\lambda = \frac{H^2}{T^4}\left(\frac{n+5}{n+1}\right) - \frac{T^{2n-4}}{(n+1)} - \frac{2(n-1)H^2}{(n+1)(n-2)T^5} + \frac{T^{2n-5}}{4} + \frac{1}{T^2} - \frac{3}{4} T^{2n-4} \quad (14)$$

$$8\pi\varrho = \frac{(2n+1)}{(n+1)}\left(\frac{H^2}{(n-2)T^4} - \frac{T^{2n-4}}{8(n-1)}\right) + \frac{1}{T^2} - \frac{T^{2n-4}}{4} + \frac{H^2}{T^4} \quad (15)$$

$$\xi = \frac{1}{T(n+2)8\pi}\left[\frac{2}{(n+1)}\left(\frac{H^2T^{-2}}{2(n-2)} - \frac{T^{2n-2}}{16(n-1)}\right)\right]^{1/2} \quad (16)$$

$$\theta = \frac{(n+2)}{T}\left[\frac{2}{(n+1)}\left(\frac{H^2T^{-2}}{2(n-2)} - \frac{T^{2n-2}}{16(n-1)}\right)\right]^{1/2} \quad (17)$$

And we have obtain,

$$\sigma^2 = \frac{1}{2}\theta^2$$

$$\sigma^2 = \frac{1}{2}(n+2)^2/T^2 \left[\frac{2}{(n+1)}\left(\frac{H^2T^{-2}}{2(n-2)} - \frac{T^{2n-2}}{16(n-1)}\right)\right] \quad (18)$$

4. Conclusion:

The model starts to expand with the big bang at $T = 0$ and stops at $T = \infty$, The model is expanding , shearing, rotating and anisotropic in general , Since $\lim_{T\to\infty}\left(\frac{\sigma}{\theta}\right)^2 = \frac{1}{2} \neq 0$, then the model does not approach to isotropy for large value of T, here we observe that, when $n = -2$, then the scalar expansion θ tends to zero. Presence of magnetic field expands the model and absences of magnetic field it reduces. The density ϱ decreases as time increases and it tends to infinite as time tends to zero and also for $n = -1,2,1$. Also, λ and ξ tends to ∞ as time tends to zero and for $n = -1,2$ and $n = -2,-1,1,2$ respectively.

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