MATHEMATICAL MODEL OF CHILDREN'S AND ADULTS MEMORY DUE TO STRESS

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ABSTRACT

Weibull distribution is a well-known, common distribution. A new distribution of weibull is used in this paper, namely, weighted weibull (WW) and it is obtained by introducing a skewness parameter to a weibull distribution. In the application part, we discussed about whether Cortisol reactivity to a stressful situation was connected to children's memory and also that to adults memory. 9 to 12-year olds were selected for this study. They were completed an impromptu speech and math task during which repeated cortisol samples and self –reported stress ratings were collected. Two weeks later participants' memory for this task was examined. It is known that greater Cortisol reactivity was associated with enhanced memory, most prominently in children.

KEYWORDS: Weibull distribution, HPA Axis, Memory, Cortisol

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1. INTRODUCTION

Weighted weibull distribution has been obtained by extending Azzalini's concept in skew-normal distribution [1], which was the first introduced skew-normal distribution. Let X is a non-negative random variable; w is non negative function defined on the real line. Suppose that the realization x of X will be recorded with probability proportional to w (t(x)), t(x) is not an observation on X but it is an observation on the so–called weighted random variable x/ {w}. The density function is,

$$f_{X/\{w\}}(x) = \frac{w(t(X))f_X(x)}{E[w(t(X))]}, -\infty < x < \infty. \text{ Where } 0 < E[w(t(X))] < \infty.$$

The random variable X/ {w} is the weighted version of X, and its distribution relative to X is known as the weighted distribution of X with the weight function w [2, 5]. When applying the Azzalini's concept to the weibull, then it produces a new class of weibull distribution, called weighted weibull WW (λ, β, α), with an additional parameter [6, 7, 12, 13].

1.1. WW DISTRIBUTION

Now the weighted weibull (WW) distribution is defined as,

$f_{X/\{\alpha\}}(x) = \frac{F_X(\alpha x) f_X(x)}{E[F(\alpha x)]}, x > 0.$ ---------- (1).

With $w(t(x)) = F(\alpha x)$.

From (1), weighted weibull can be obtained as follows.

Let X be distributed according to weibull distribution with parameters λ, β, the density function is,

$f_{X/\{\lambda,\beta\}} = \lambda\beta x^{\beta-1} e^{-\lambda x^{\beta}}$ ---------- (2). The distribution function of $X/\{\lambda, \beta\}$ is $F_{X/\{\lambda,\beta\}}(x) = 1 - e^{-\lambda x^{\beta}}$.

Thus $F_{X/\{\lambda,\beta\}}(\alpha x) = 1 - e^{-\lambda(\alpha x)^{\beta}}$,

And, $E[F_{X/\{\lambda,\beta\}}(\alpha x)] = \frac{\alpha^{\beta}}{1+\alpha^{\beta}}$.

Hence (1) implies,

$f_{X/\{\lambda,\beta,\alpha\}}(x) = \frac{\lambda\beta(1+\alpha^{\beta})x^{\beta-1}e^{-\lambda x^{\beta}}(1-e^{-\lambda(\alpha x)^{\beta}})}{\alpha^{\beta}}$, $x > 0$. ---------- (3).

And $f_{X/\{\lambda,\beta,\alpha\}}(x) = 0$ otherwise.

Next, the distribution function, survival function, hazard function, reversed hazard function, for the WW (λ, β, α), distribution are given below,

The distribution function $F_{WW(}\lambda, \beta, \alpha_{)}$ of WW (λ, β, α) is

$$FWW(\lambda, \beta, \alpha)(X) = \frac{1}{\alpha^{\beta}} [(1 + \alpha^{\beta}) (1 - e^{-\lambda x^{\beta}}) + e^{-\lambda x^{\beta}(1+\alpha^{\beta})} - 1]$$

The survival function $FWW(\lambda, \beta, \alpha)(X)$ of WW (λ, β, α) is

$$FWW(\lambda, \beta, \alpha)(X) = 1 - \frac{1}{\alpha^{\beta}} [(1 + \alpha^{\beta}) (1 - e^{-\lambda x^{\beta}}) + e^{-\lambda x^{\beta}(1+\alpha^{\beta})} - 1]$$

The hazard function $h\,ww(\lambda, \beta, \alpha)$ of WW (λ, β, α) is

$$h\,ww(\lambda, \beta, \alpha) = \frac{(1+\alpha^{\beta})\lambda\beta x^{\beta-1}\exp(-\lambda x^{\beta})[\,1-\exp(-\lambda(x\alpha)^{\beta})]}{[\,(1+\alpha^{\beta})\exp(-\lambda x^{\beta})-\exp(-\lambda x^{\beta}(1+\alpha^{\beta}))-a1+1\,]}$$

The reversed hazard function is $rww(\lambda, \beta, \alpha)$ of WW (λ, β, α) is

$$rww(\lambda, \beta, \alpha) = \frac{(1 + \alpha^{\beta})\lambda\beta x^{\beta-1}\exp(-\lambda x^{\beta})[\,1 - \exp(-\lambda(x\alpha)^{\beta})]}{(1 + \alpha^{\beta})(1 - \exp(-\lambda x^{\beta}) + \exp(-\lambda x^{\beta})(1 + \alpha^{\beta}) - 1}$$

Also, two special cases of WW distribution have been given below.

1.2. WEIGHTED EXTREME VALUE DISTRIBUTION

Let random variable X has a Weibull distribution in (2), then the weighted probability density function of $Y = -\beta \log(X\lambda^{1/\beta})$ is

$$f\,Y/\{\beta,\alpha\}\,(y) = \frac{(1+\alpha^{\beta})}{\alpha^{\beta}} e^{-y} e^{-e^{-y}} \left(1 - e^{-\alpha^{\beta} e^{-y}}\right) \text{ ---------- (4)}$$

This is known as weighted extreme value distribution.

1.3. THE WEIGHTED EXPONENTIAL DISTRIBUTION

This distribution can be obtained following Gupta and Kundu [7] who introduced a new weighted exponential distribution but not fixed λ . Suppose $\beta = 1$ in (2), then we get this distribution of the random variable $Y/\{\lambda,\alpha\}$ as

$f\,Y/\{\lambda,\alpha\}\,(y) = \lambda(1+\alpha)\exp(-\lambda y)\,(1-\exp(-\lambda y\alpha))\,/\alpha$ -------- (5) for $y > 0$. and $f\,Y/\{\lambda,\alpha\}$ otherwise.

2. APPLICATION

The HPA axis is a key biological system respective to stress exposure.CRH releases when HPA axis activated, which stimulates ACTH from pituitary. ACTH binding on the adrenal cortex leads to the release of cortisol, the most important glucocorticoid in humans. In the key regions of brain relevant for emotional memory, including the hippocampus and amygdale where high concentrations of glucocorticoid receptors are found [3, 4]. In spatial and declaration memory and in the consolidation of information for transfer to long term memory, the hippocampus plays vital role. The amygdale is activated in case of personally silent events, including those that are threating or stressful. The associations between HPA axis activation and memory for emotional information have investigated in adults by large number of research. In this study, HPA axis activation is typically induced endogenously or pharmacologically before, during or after exposure to emotional information [11]. Findings reveal positive associations between HPA axis activation and memory. Greater cortisol responses to the speech task were associated with enhanced memory for trait words but not other words. The results suggest that the memory enhancing effects of HPA axis activation may be strongest when this content overlaps between the stress-inducing event and to-be-remembered information.

Cortisol and memory in children have examined by very few studies. Although among the studies, children's memory was tested for the precise event that elicited the cortisol response in the first place[12]. Children's cortisol levels during invasive medical procedures were compared by two such studies. Also saliva samples from 3-to-7 years olds just after a medical test and on another day at the same time when children at home. Cortisol difference scores were unrelated to children's memory for the test, when memory was tested just afterward and a month later. Also saliva samples collected in chronically ill 3-to-18 year olds before and after undergoing treatment for leukaemia, children's memory memory for the treatment was examined a week later. Pre-to-post cortisol changes were unrelated to memory.

2.1. METHODS

The sample included 28 children, 9-to12-year –olds &29 adults, 18-to23-year-olds. Both children & adults have ongoing experience in school increasing the comparable relevance of the TSST-M

instructions across age which requires participants imagine entering a new class room and introducing themselves.

2.2. CORTISOL REACTIVITY TO THE TSST-M, AGE& MEMORY

The links between cortisol trajectories to the TSST-M &memory were examined by main analyses &also evaluated whether these differed for children & adults [8, 9, 10]. Free recall was considered first significant interactions were observed between free-recall & the TSST-M cortisol trajectory. To interpret these, cortisol trajectories were plotted for individuals who reported more versus less correct information in free recall and for children separate plots were done. Overall, larger responses to TSST-M were associated with providing more correct information in free recall, with this pattern being more robust in children than adults. Later participants direct question accuracy was examined. This case also, larger cortisol responses associated in children. Similar associations were not uncovered among adults.

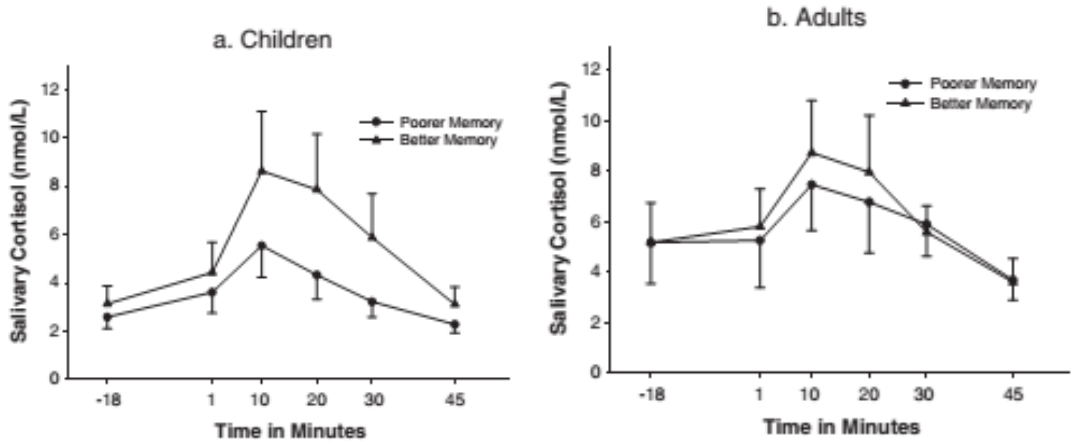
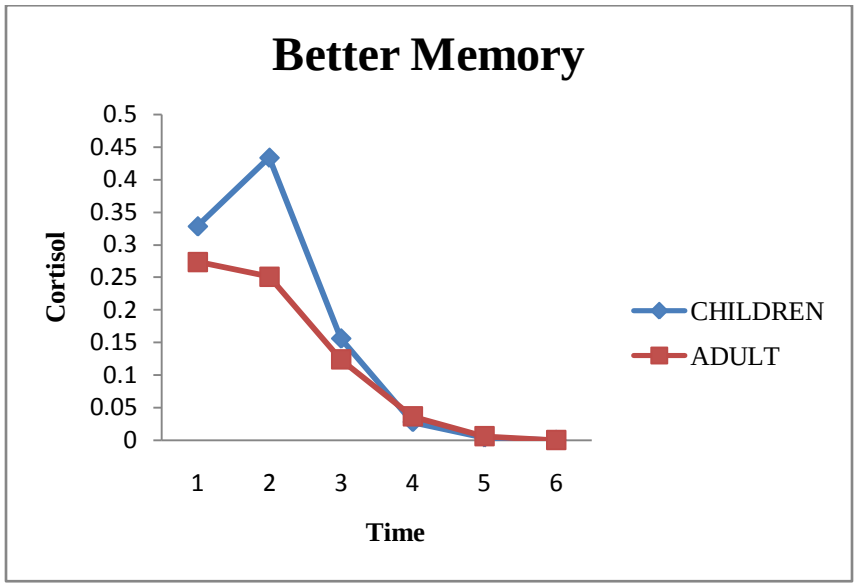
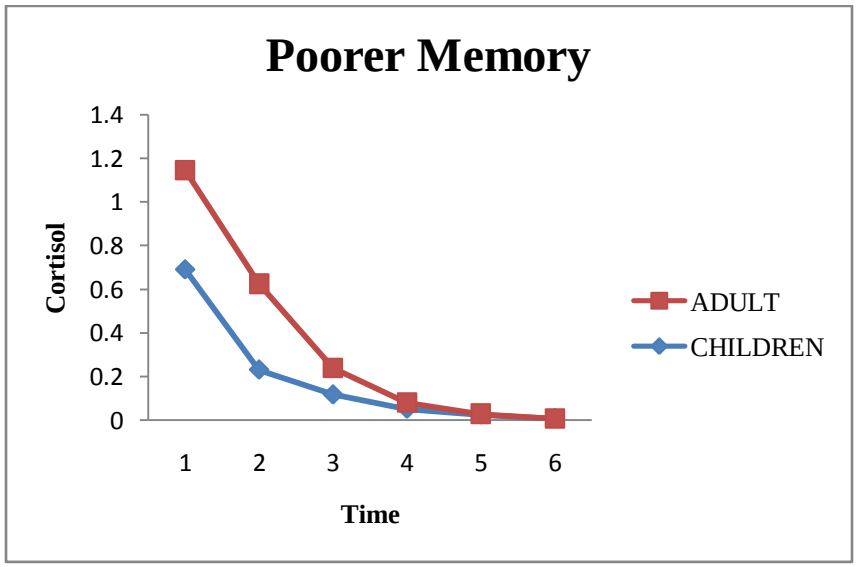


FIGURE 1 Free-recall performance based on the trajectories of Cortisol across the TSST-M in children (a) and adults (b).

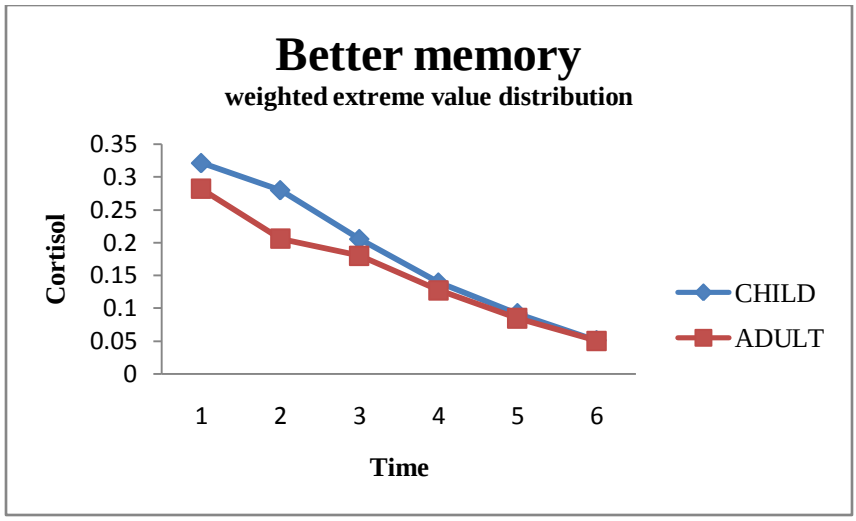
3. MATHEMATICAL RESULTS

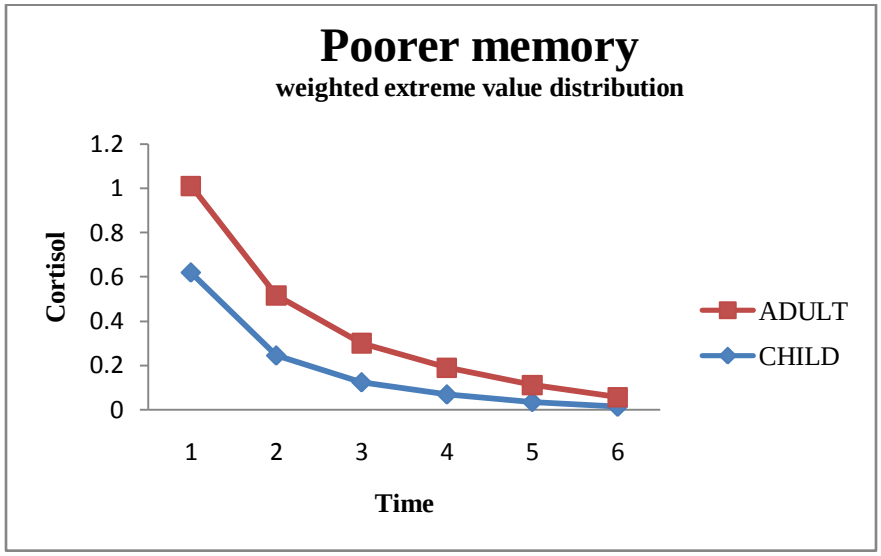
Mathematical results of WW distribution is given below

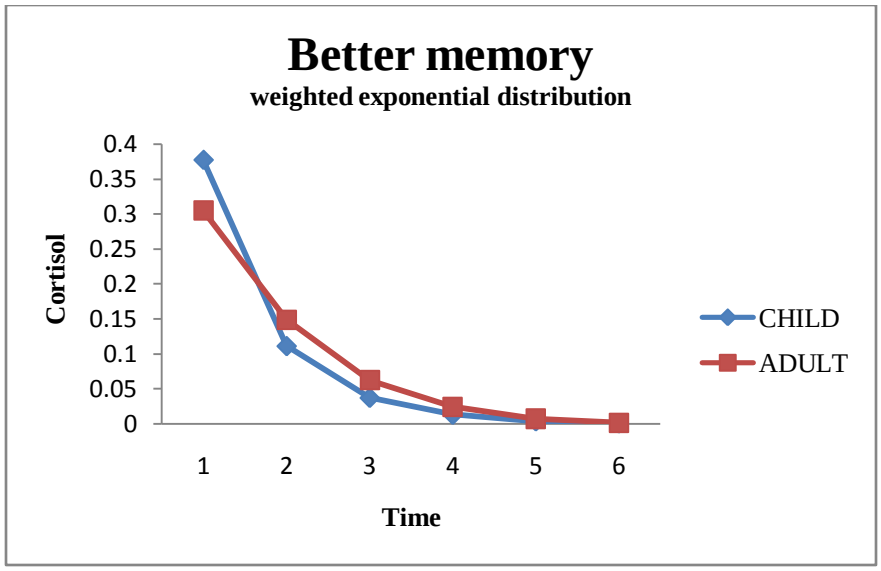


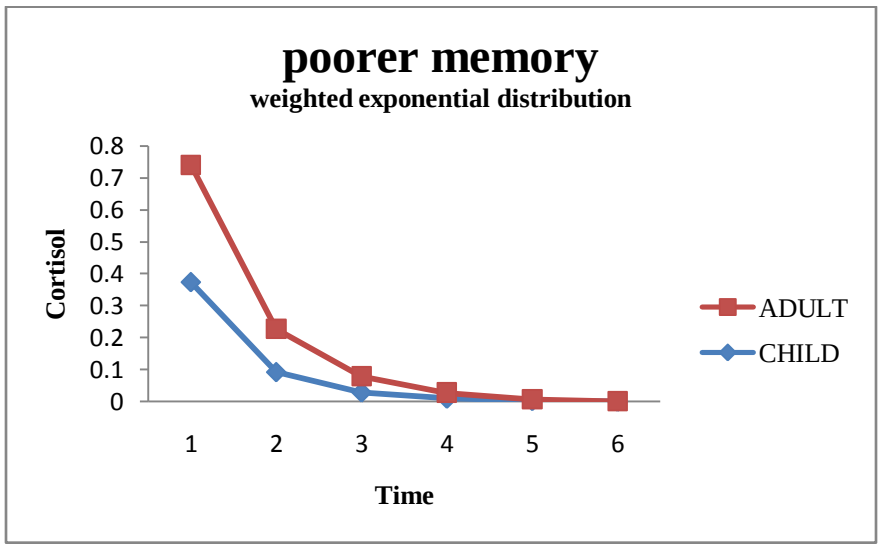


Also, Mathematical results of weighted extreme value distribution and weighted exponential distribution also given as follows.


Better memory
weighted extreme value distribution
Cortisol
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5
6
Time
CHILD
ADULT


Poorer memory
weighted extreme value distribution
Cortisol
1.2
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
Time
ADULT
CHILD





4. CONCLUSION

Greater Cortisol reactivity was associated with enhanced memory, most prominently in children. Self-reported stress was unrelated to memory. Findings reveal that an important mechanism underlying the association between emotion and memory in adults, namely activation of the hypothalamic pituitary adrenal axis, appears to operate similarly in late childhood. We found Mathematical results of WW distribution and also its special cases ,it is shown that children's had notably increased better memory than adults and adults had more poorer memory than children. Hence it is conclude that, in case of stress children had better memory than adults.

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