MATHEMATICAL IMPACT OF BLOOD CIRCULATION IN A VESSEL

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ABSTRACT

In the present paper depicts the mathematical impact of blood circulation in a vessel and also has been observed that the thickness of a blood vessel is proportional to the pressure. In section one contains a brief introduction to circulation of blood, while in section two introduce mathematical analysis and its graphical representation. In the section three, flow of blood through vessel of circulatory cylindrical system. In the section four, we obtain some mathematical examples of blood circulation in vessel. In the end, we are discussed the numerical calculation of blood and results are discussed graphically.

Key Words : Poiseville's Law, Newtonian fluid, Hooke's Law, exterior and interior pressure, blood circulation, Navier stokes equation.

Introduction

Consider such a vessel fluid with fluid at rest and surrounded by fluid. It radius will be determined by transmutable pressure difference between the interior and exterior pressure in the wall of vessel. Normally when exterior pressure equals interior pressure, the thickness of the blood through vessel wall is small compared to the resting radius of the vessel. So, we can treat the wall as a thin membrane for which the tension per unit length of tube and per unit thickness. But many researchers have been pointed out that the deposits of cholesterol, purely substance on the wall from plaques which grow in ward and restrict the flow of blood pumping by lungs, hence comprise one of the primary defense mechanism of lungs. It is assumed that blood in Newtonian fluid, density and viscosity are constant.

2. Mathematical Model

In the present paper, we have studied mathematical impact of blood circulation in human body is moved by pressure from lungs. Kidney is used to purify blood by urea and pure blood is again introduced into the parts of body. William Harvey seventeenth century that blood circulates from hearts and passes through the vessels in only one direction and returns from different parts of the body except the lungs through the veins into the right atrium.

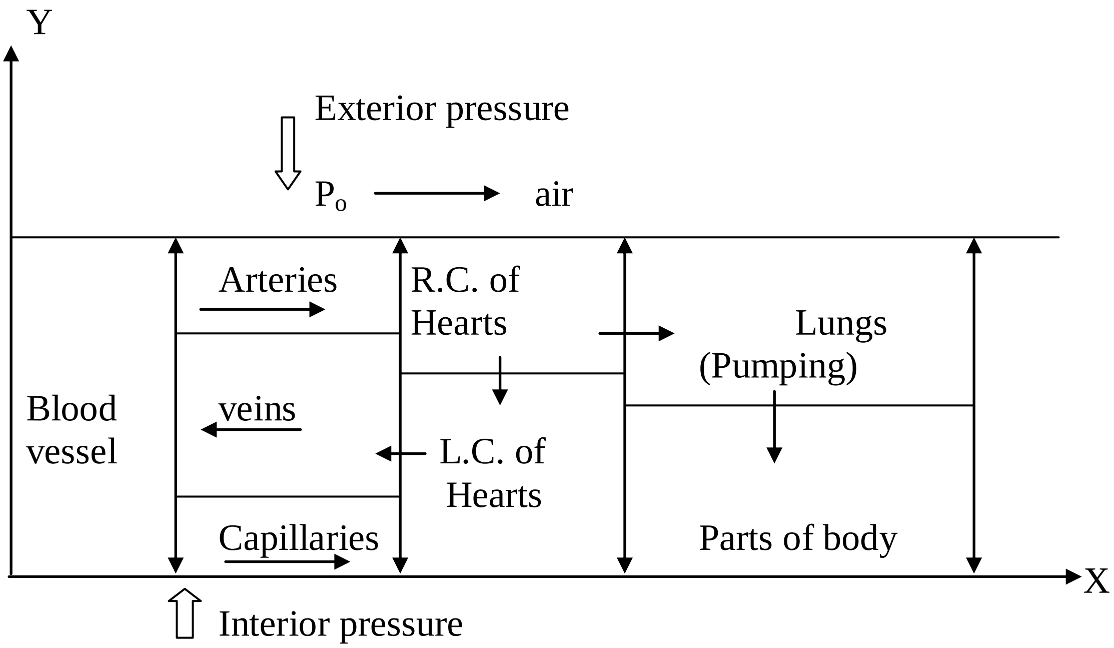

Fig. 1.1 Impact for blood circulation by vessel

Let the thickness and radius of the wall be X and h respectively. The interior fluid pressure is p and the exterior pressure p_o is given by

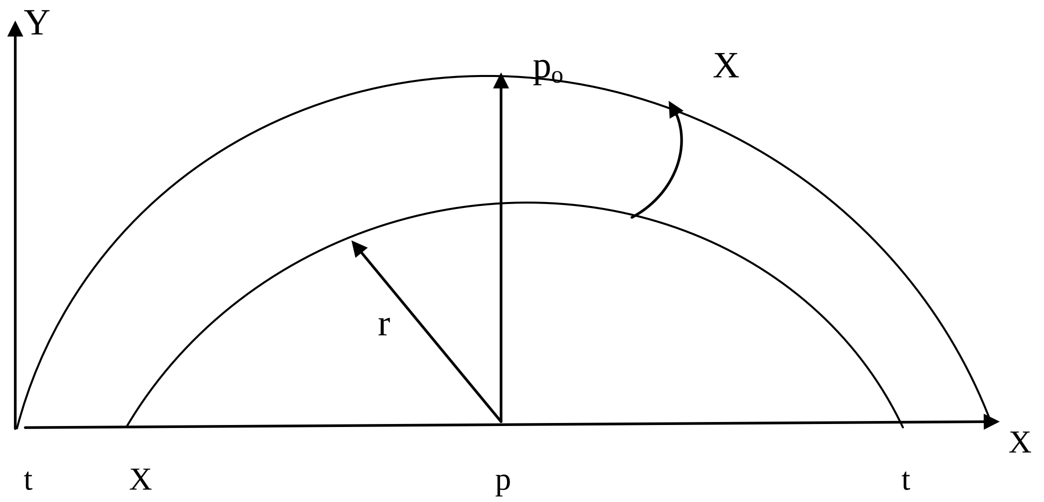

Fig. 1.2

Its wave equation is

$$\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}$$

Such that $Y = p_o$ Cos pt, when X = 0

and Y = 0, when X = 0

Cross section of one half of a long elastic cylinder containing a fluid with pressure

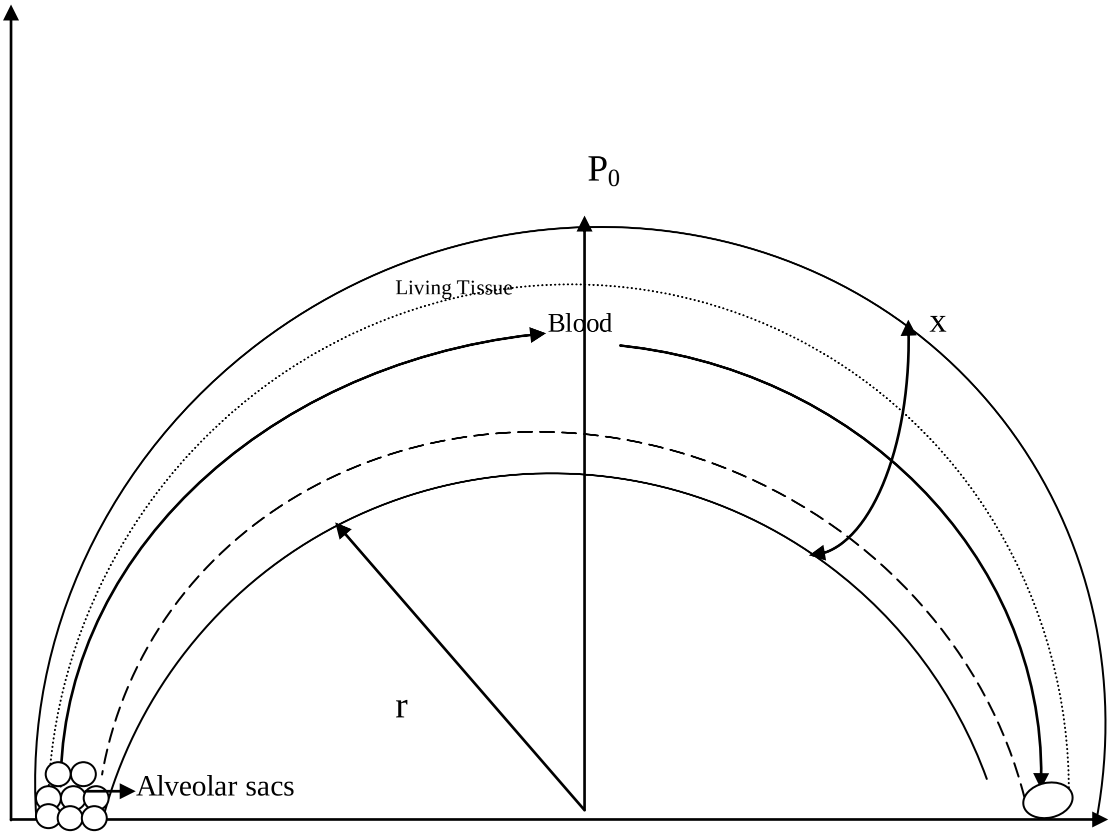

Fig. 1.3 Model for blood circulation

The net downward force per unit length on thin half cylinder is 2tx and in balanced by the net upward force per unit length. The net upward force per unit is equal to the product of he diameter and pressure difference between lower and upper surface.

(2.1) $2tx = (p - p_o)\, 2r$

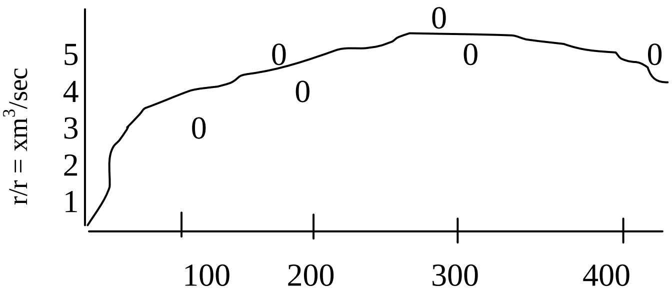

Fig. 1.4 $p - p_o$

The pressure develops due to the reaction to stretch of elastic wall. The pressure required to maintain the given radius of the tube can be obtained by plotting a pressure – radius diagram which relates the wall. This diagram for mammalian arteries is shown for external iliac thin wall in man shown by the figures (1.5)

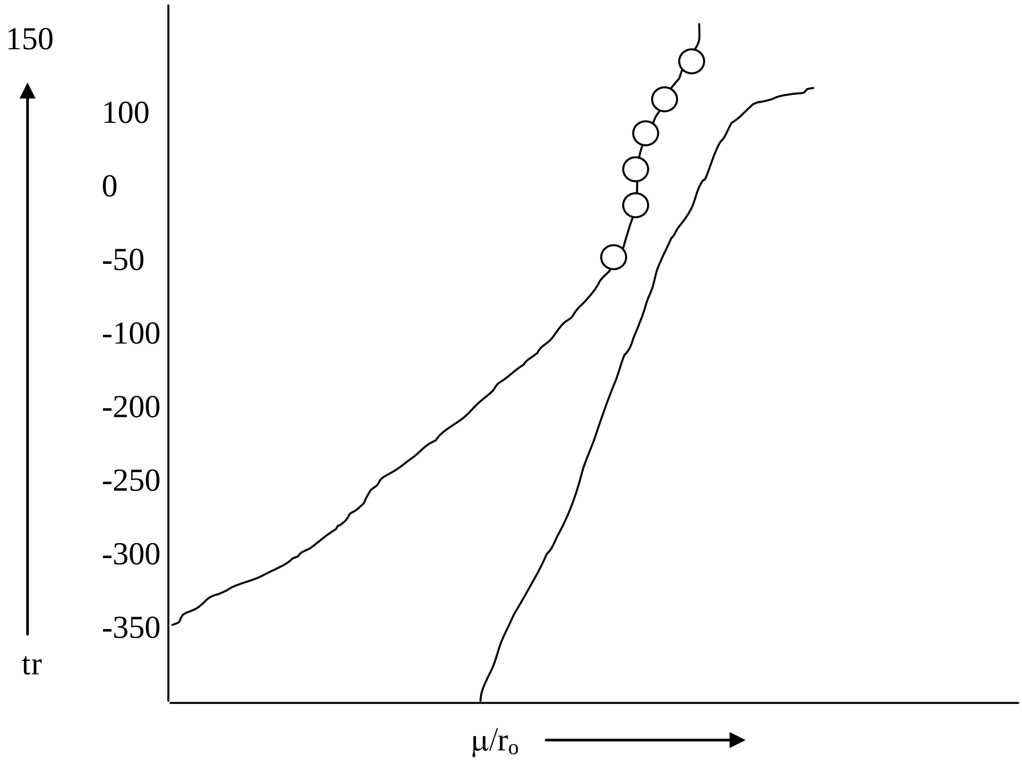

Fig. 1.5

Then the T(r) curve for a section of external lilac artery in Man, here

$$T(r) = tx$$

and

$$p - p_o = Tr$$

(2.2) $(p - p_o) = tx$

We know that the Hooke's law, we have

(2.3) $t = E\left(\frac{r - r_o}{r_o}\right)$

Hence the value of t from (2.3) and putting in (2.1), we have

$$(2.4) \quad p - p_o = \frac{2E}{r_o} \text{ X } \left(1 - \frac{r_o}{r}\right)$$

Hence the equating the volume on assuming the wall in thin, we have

$$4 \pi r^2 X = 4\pi r^2_o X_o$$

$$(2.5) \quad X = \left(\frac{r^2_o}{r}\right)^2 X_o$$

From (2.5) and (2.4), we have

$$Þ- p_o = \frac{2E}{r_o} \left(\frac{r_o}{r}\right)^2 X_o \left(1 - \frac{r_o}{r}\right)$$

$$(2.6) \quad p - p_o = \frac{2E}{r_o} X_o \left(\frac{r_o^2}{r^2} - \frac{r_o^3}{r^3}\right)$$

Then three cases are arise

(i) when $r > r_o$ => $p - p_o > 0$

(ii) when $r = r_o$ => $p - p_o = 0$

(iii) when $r < r_o$ => $p - p_o < 0$

On differentiating (2.6), we have

$$(2.7) \quad \frac{dp}{dr} = \frac{2E}{r_o} X_o \left(\frac{-2r^2_o}{r^3} + \frac{3r_o^3}{r^4}\right)$$

Solving, we have

$$r = \frac{3r_o}{2} \quad => \quad \frac{r_o}{r} = \frac{2}{3}$$

$$=> \frac{r_o}{r} < 1$$

Hence, it is clear that the growth of ovarian follicle occurs without increasing the internal pressure.

The flow of blood through an vessel in the lungs of artery with mild stenosis under the pressure due to magnetic field. It is also assumed that the blood is Newtonian field and electromagnetic is very small.

The geometry of the stenosis is shown in the given fig. 1.6 and radius of the artery is given by Young (1968) as

$$(2.8) \quad \frac{r}{r_o} = \begin{cases} 1 - \dfrac{\delta}{\delta_o} \left\{ 1 + \text{Cos} \dfrac{2\pi}{\ell_o} \left(Z - X - \dfrac{\ell_o}{2}\right); x \le z \le \ell_o + X \right. \\ 2, \quad \text{otherwise} \end{cases}$$

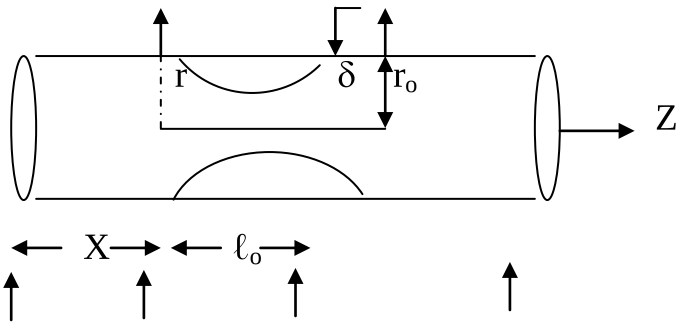

Fig. 1.6 . Geometry of mild stenosis

(2.9) (a) $\frac{\delta}{\delta_o} << \ell$

(b) $\frac{r_o}{r} \sim 0 (\ell)$

where r_o = radius of unobstructed tube
r = radius of obstructed tube
ℓ_o = length of the stenosis
δ = maximum height of stenoic growth

The equation of motion and continuity for steady state in compressible laminar blood circulation through a vessel in lungs under no body forces in the cylindrical coordinates (r, θ, Z) are given by

(2.10) $$0 = - \frac{1}{\rho} \frac{dp}{dz} + \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{\mu}{\rho} r \frac{\partial \nu}{\partial r} \right) + B_o 6_o v$$

(2.11) $$0 = \frac{\partial \rho}{\partial r}$$

(2.12) $$\frac{\partial v}{\partial z} = 0$$

Where V = axial velocity of blood

Þ = fluid pressure

$\frac{\partial \rho}{\partial z}$ = pressure gradient

ρ = (constant) density of blood,

μ = coefficient of viscosity

$$B_o = \frac{\mu e}{H_o} = \text{electromagnetic induction}$$

μe = magnetic permeability

H_o = intensity of magnetic field

6e = conductivity of fluid

The boundary conditions are

(2.13) V = 0 at R = r (0)

(2.14) $\frac{\partial v}{\partial r} = 0$ at r = 0

3. Analysis of Blood Circulation:

We analysis of blood circulation through vessel of circulatory system but due to the clothing of blood on exposure to air, he took the water in place of blood and then firstly investigate in a flow of water through glass pipes.

Let the cylindrical tube of length ℓ and radius R, exterior pressure p_o and interior pressure p, where $p > p_1$, by

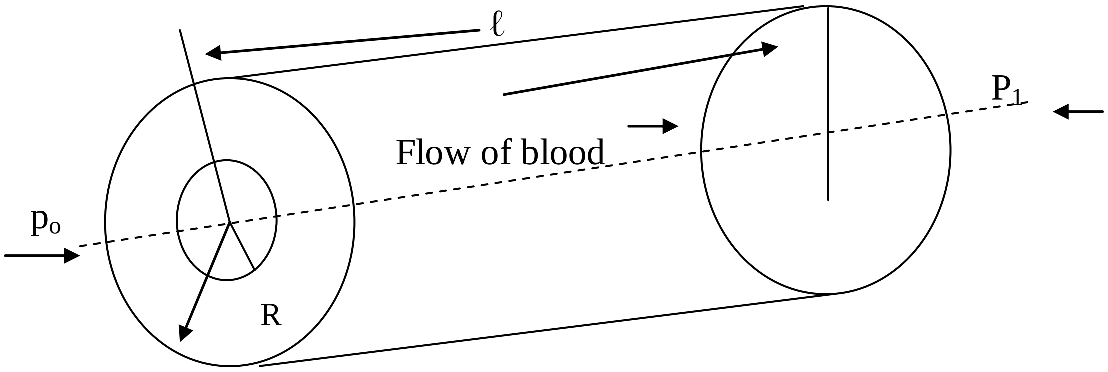

Fig. 1.7. Blood circulation in cylindrical tube

Let the velocity u does not depend upon θ, so that

$$u = u\,(n, r)$$

Now,

$$\frac{\partial u}{\partial x} = 0 \quad \Rightarrow u = u\,(r)$$

Navier stokes equation neglecting the body forces is

$$(3.1)\quad \rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial n}\right) = \frac{-\partial p}{\partial x} + \mu \nabla^2 u$$

$$(3.2)\quad u \nabla^2 u = \frac{\partial \rho}{\partial x} \qquad \because u\,(r) = u$$

Where

$$(3.3)\quad \nabla^2 = \frac{\partial^2}{\partial \rho^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial r^2}$$

From (3.2), we have

$$(3.4)\quad u.\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{\partial \rho}{\partial x}$$

from Navier stokes equation

$$(3.5)\quad \rho\left(\frac{\partial}{\partial t} + q.\nabla\right) qr = -\,(\nabla t)r + \mu\,(\nabla^2 qr)$$

reduce to

$$(3.6)\quad \frac{\partial \rho}{\partial r} = 0 \Rightarrow \rho = \rho\,(x)$$

using the equation (3.6) and (3.4) becomes

$$(3.7)\quad u.\frac{1}{r}\frac{d}{dr}\left(r\frac{\partial u}{\partial r}\right) = \frac{d\rho}{\partial x}$$

Solving above equation, we have

$$(3.8)\quad u = -\frac{1}{4\mu}\frac{d\rho}{dx}(a^2 - r^2)$$

The volume flux θ is given by

$$(3.9)\quad Q = \int_0^R 2\pi ru\, dr$$

Now putting $\text{Þ} = \text{Þ}_0 + \frac{\text{Þ}_1 - \text{Þ}_0}{\ell}$ x and $u = -\frac{\ell}{4\mu}\frac{d\rho}{dx}(a^2 - r^2)$

In equation (3.1), we have

$$(3.10)\quad Q = \frac{\pi}{8}\frac{\text{Þ}_0 - \text{Þ}_1}{\mu\ell}r^4$$

(3.11) $T = \mu\nu$ is Newtonian fluids

Where
T = Shear stress rate
ν = Shear strain rate

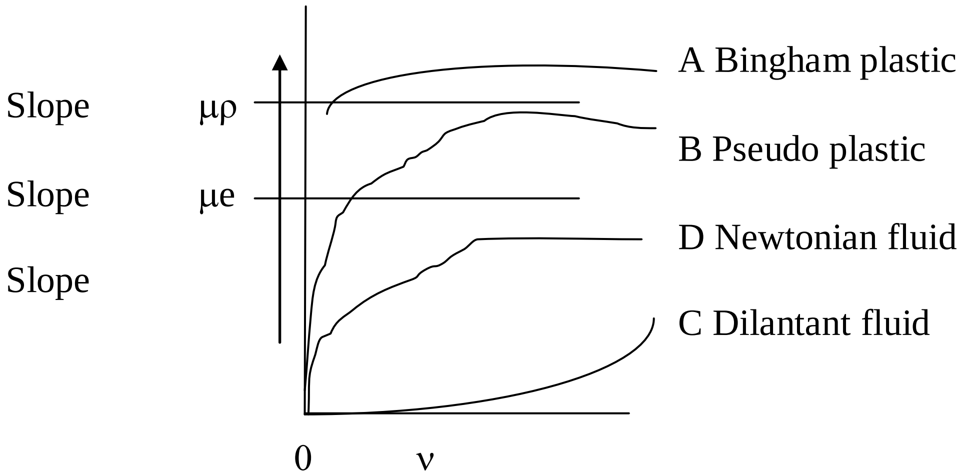

Fig. 1.8 Shear strain rate

$$(3.12)\ Qx = \frac{\pi}{8\mu\ell} \int_{p2-p0}^{p1-p0} (p - p_0)\, dp$$

4. Result and Discussion

Mathematical results for blood blow through vessel have been obtained in numerical form. For numerical calculations, we take

$\rho = 1.05$, $\mu = 0.004$ poise

$\frac{\delta}{\delta_0} = 0.1$, $\ell_0 = 1$, $X = 0$ then

from (2.8), we obtain the table

2	0	0.2	0.4	0.5	0.6	0.8	1.0
r/r_0	0.998	0.956	0.908	0.900	0.908	0.956	0.998

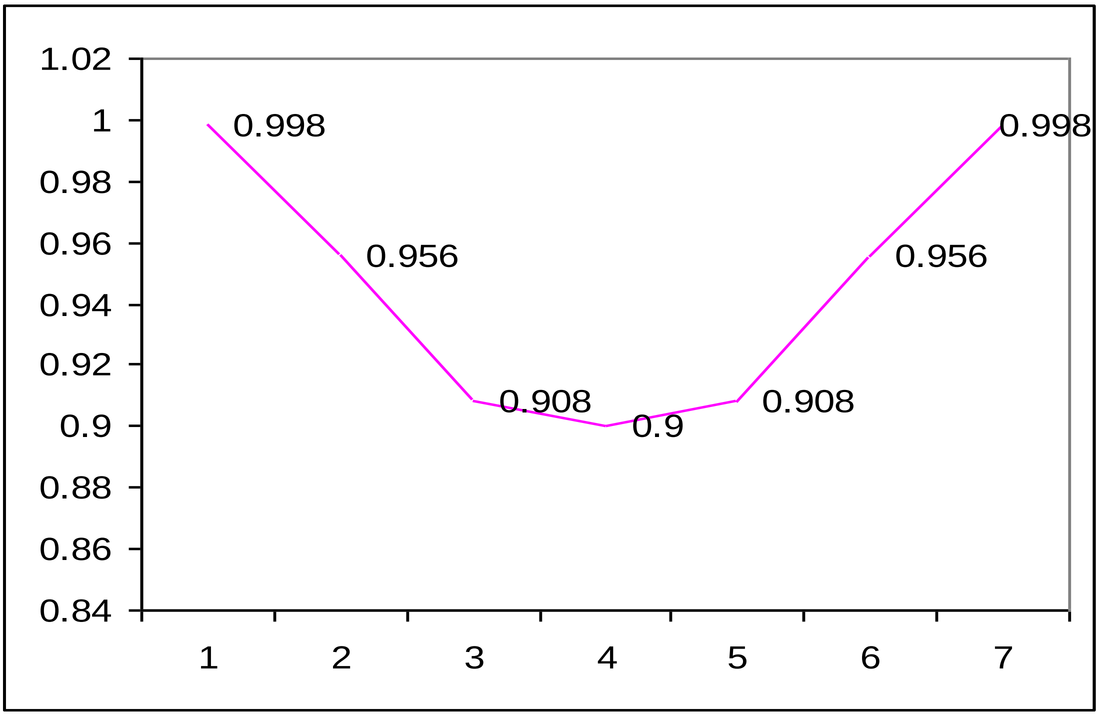

Fig. 1.9

The curve for flux shown in fig. (1.9), physically these curves imply that if a flexible tube is used as drinking straw, we observe that flow through a thin elastic tube and Þ $_1$ – Þ $_2$ from equation (3.10) as shown figure (1.10) and (1.11). Actually after this limit veins or tubes (elastic start buckling under normal physiological conditions which is not considered in this theory. This consideration does not affect the qualitative features.

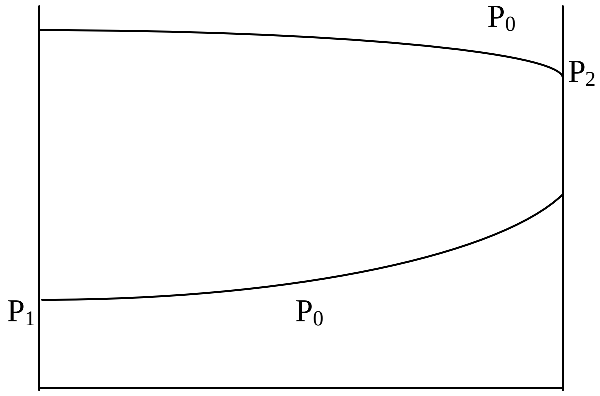

Fig. 1.10

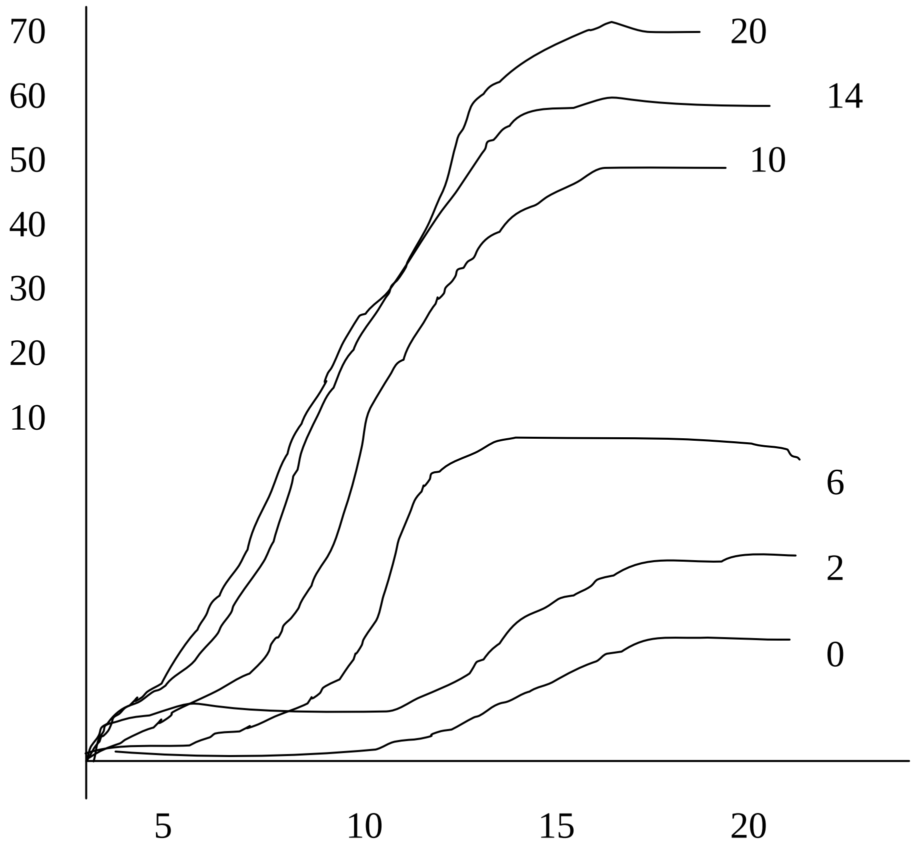

Fig. 1.11

Flow through a thin elastic tube

REFERNCES

1. **Bitoum, J.P. and Bellet, D**. : Biorheology, 23 (1986), Sl.
2. **Black, J.R**.: On the movement of mucus in the long. J. Biomechanics, 8 (1975), 179-190.
3. **Boyed, T.J.M. and Sanderson, J.J**. : Plasma dynamics Thomas Nelson and Sons Ltd., London (1969).
4. **Couston, D.R.:** A Biological Mathematics, Edward, Arnold.
5. **J.N. Kapoor**: Mathematical Modeling in Biology & Medicine, East West Press (1985).
6. **Levin, S.A.** : Frontiers in Mathematical Biology, Springer Verlog (1994).
7. **Lotka, A.S**. : Elements of Mathematical Biology, Dover Publication, New York.
8. **Ross, S.M. and Lorrsin, S**. : Results of an analytical model musociliary pumping, J. Appl. Physical, 37 (1974), 333-340.
9. **Sud, V.K. and Sekhon, G.S**.: Phys. Med. Biol., 34 (1989), 795.
10. **Uardanyan, U.A.:** Biophysics, 18 (1973), 575.
11. **Weibel, E.R**.: Morphometry of the human lung. Academic Press Inc., New York (1963).
12. **Young, D.F**.: J. Engang, Ind., 90 (1968), 249.