METHOD FOR POSSIBILITY RULE INDICES IN FUZZY LINEAR PROGRAMMING

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ABSTRACT

This paper presents a suggested approach for solving a stochastic fuzzy linear programming problem. These problems can be applied for different types of fuzzy numbers, in addition to the trapezoidal and triangular fuzzy numbers that have been used in this paper. These criteria attributes may be both qualitative as well as quantitative factors. These factors and their interdependencies make the problem highly complex one. Also, any approximations that may exist, due to using another approach, there are generated according to each of the four rule indices. Each of the four rule indices, the equivalent deterministic-crisp linear programming problem is solved

Keywords: *fuzzy linear programming, rule, equivalent, functions.*

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1. INTRODUCTION

Comparison of fuzzy numbers is considered one of the most important topics in fuzzy logic theory. The early and most important work in the field of comparing fuzzy numbers has been presented by Dubois and Prade [1]. A comparison between their work and other attempts that have been made in this area has been given by Bortolan and Degani [2]. On the other hand, the dominance possibility indices, which have been introduced by Dubois and Prade, were utilized in the field of fuzzy mathematical programming [3,4]and the field of stochastic fuzzy mathematical programming [5,6]. The approach used in these field was based on formulating a possibility function, whether in the case of trapezoidal fuzzy numbers or the case of triangular fuzzy numbers. In this paper, we are going to utilize Dubois and Prade's dominance possibility and necessity indices, within a different approach, in the case of stochastic fuzzy linear programming problem. The dominance possibility and necessity, as well as the strict dominance possibility and necessity criteria, are utilized according to the chance-constrained method to transform the suggested problem to its deterministic-crisp equivalent. This approach helps avoiding any approximation that may exist due to comparing the inverse distribution function of fuzzy tolerance measures.

2. MODEL

In general, consider a fuzzy linear programming problem of the following form:

$$\text{Max} \quad \tilde{Z} = \sum_{j=1}^{n} \tilde{c}_j x_j \qquad \qquad \text{…1}$$

Subject to:

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \leq b_{i,} \quad i = 1,.......,m, \qquad \text{…2}$$

$$x_j \geq 0, \qquad j = 1,..............,n. \qquad \text{…3}$$

Here x_j , j =1,...,n are non-negative decision variables, c_j , j =1,...,n are fuzzy coefficients in the objective function, b_i ,i =1,...,m are independent random variables with known distribution functions, while a_{ij} represents the fuzzy coefficient of the j th decision variable in the i th stochastic constraint. Thus, by incorporating fuzzy tolerance measures $\tilde{\delta}_i, 0 \leq \tilde{\delta}_i \leq 1, i = 1......m$ and by utilizing the chance-constrained approach, the stochastic fuzzy constraints (2) can be transformed to their deterministic fuzzy equivalents as follows [5, 6].

$$\Pr\left(\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le b_i\right) \ge \tilde{\delta}_{i,} \qquad i = 1, \ldots\ldots, m, \qquad \ldots 4$$

Then,

$$\sum_{j=1}^{n} \tilde{a}_{ij} x_j \le F_i^{-1}\left(\tilde{\beta}_i\right), \quad i = 1, \ldots\ldots m, \qquad \ldots 5$$

Where $\tilde{\beta}i = 1 - \tilde{\delta}i$, and F_i^{-1} (.) is the inverse distribution function of the random variable bi, i = 1... m. It is apparent that this transformation requires the independent random variables to be continuous [6–8]. On the other hand, the deterministic fuzzy constraints set (5)is going to be represented by its crisp equivalent, according to each of the following four dominance indices that have been presented by Dubois and Prade [1]: Possibility of rule (PR), Possibility of Strict rule(PSR), Necessity of rule (NR), and Necessity of Strict rule (NSR). These indices for comparing fuzzy numbers are utilized whether $\tilde{a}_{ij}$ and $\tilde{\delta}_i$ are presented as trapezoidal or triangular fuzzy numbers.

3.   THE   EQUIVALENT   DETERMINISTIC-CRISP   LINEAR PROGRAMMING MODELS

In this section, the fuzzy objective function (1)as well as the set of deterministic fuzzy constraints (5) are transformed to their crisp equivalents. This transformation is applicable for different types of fuzzy numbers (trapezoidal or triangular).

3.1. The equivalent crisp objective function

Assume that the fuzzy coefficients are considered trapezoidal fuzzy numbers, i.e., $\tilde{c}j = (c_{j,}$ $c_{j1}, c_{j2}, c_j)$.By utilizing the α-cut approach, where α is any predetermined value, $\alpha \in (0, 1]$, then according to Islander [4], the crisp objective function that is equivalent to (1) can be presented as

$$\text{Max} \quad Z = \sum_{j=1}^{n}\left((1-\alpha)\overline{c}_j + \alpha c_{j2}\right)x_{j,} \qquad \ldots 6$$

Subject to (5)and (3).

3.2. The equivalent crisp constraints

The crisp equivalent to (5) should be formulated according to each rule index. The suggested approach, for formulating this set of crisp constraints, depends on utilizing the α-cut

approach, for the membership functions of a˜ij and $\tilde{\beta}i$ .Assuming that a˜ij and $\tilde{\beta}i$ are presented as trapezoidal fuzzy numbers, i.e., a˜ij = (a $_{ij}$, aij 1, aij 2, aij) and $\tilde{\beta}i$ = (β_{i} ,βi1,βi2 ,β_{i}) = (1 − δi , 1 − δi2, 1 − δi1, 1 − δ$_{i}$),then the α-cut for each of the membership functions of a˜$_{ij}$ and $\tilde{\beta}i$ should, respectively, derivate the following two closed crisp intervals, [(1 − α)a $_{ij}$ + αaij 1,(1 − α)aij + αaij 2] and [(1 − α)β $_{i}$ + αβi1 ,(1 − α)β$_{i}$ + αβi2]. And since the decision variables are non-negative, then the set of deterministic fuzzy constraints (5) can be transformed to its deterministic-crisp equivalent, according to each of the four rule indices, which is based on comparing closed crisp intervals [1], as follows:

(a)   According to PR:

$$\sum_{j=1}^{n}\left((1-\alpha)a_{ij}+\alpha a_{ij}1\right)x_{j} \leq F_{i}^{-1}\left((1-\alpha)\tilde{\beta}_{i}+\alpha\beta_{i2}\right),\ i=1,\ldots\ldots,m,$$
..7

(b)   According to PSR:

$$\sum_{j=1}^{n}\left((1-\alpha)\overline{a}_{ij}+\alpha a_{ij}2\right)x_{j} \leq F_{i}^{-1}\left((1-\alpha)\tilde{\beta}_{i}+\alpha\beta_{i2}\right),\ i=1,\ldots\ldots,m,$$
…8

(c)   According to NR:

$$\sum_{j=1}^{n}\left((1-\alpha)a_{ij}+\alpha a_{ij}1\right)x_{j} \leq F_{i}^{-1}\left((1-\alpha)\tilde{\beta}_{i}+\alpha\beta 1\right),\quad i=1,\ldots\ldots,m,$$
…9

(d)   According to NSR:

$$\sum_{j=1}^{n}\left((1-\alpha)\overline{a}_{ij}+\alpha a_{ij}2\right)x_{j} \leq F_{i}^{-1}\left((1-\alpha)\tilde{\beta}_{i}+\alpha\beta 1\right),\quad i=1,\ldots\ldots,m,$$
…10

Table 1 Results of the four rule indices

α	0.2	0.4	0.8
PR	$x_1 = 0$, $x_2 = 3.576$ Z = 31.114	$x_1 = 0$, $x_2 = 3.154$ Z = 26.808	$x_1 = 0$, $x_2 = 2.649$ Z = 21.719
PSR	$x_1 = 0$, $x_2 = 2.703$ Z = 23.519	$x_1 = 2.216$, $x_2 = 0$ Z = 18.838	$x_1 = 2.227$, $x_2 = 0$ Z = 15.591
NR	$x_1 = 0$, $x_2 = 2.703$ Z = 23.519	$x_1 = 0$, $x_2 = 2.500$ Z = 21.250	$x_1 = 0$, $x_2 = 2.257$ Z = 18.505
NSR	$x_1 = 1.670$, $x_2 = 0$ Z = 15.866	$x_1 = 1.757$, $x_2 = 0$ Z = 14.932	$x_1 = 1.898$, $x_2 = 0$ Z = 13.284

Also, the sensitivity of the results can be tested for other values of α.

It should be noticed that, according to this approach for comparing closed crisp intervals, we can avoid the approximation that may exist due to comparing the inverse distribution function of trapezoidal or triangular fuzzy tolerance measures. This approximation has been

presented by using rule possibility functions for making such comparisons [6]. According to the concept of extension principle, the inverse distribution function of trapezoidal or triangular fuzzy tolerance measures is not exactly trapezoidal or triangular, respectively, except the case of the uniform distribution. Thus, for the other distributions, comparing the inverse distribution function of trapezoidal or triangular fuzzy tolerance measures by utilizing rule possibility functions, for comparing traditional trapezoidal or triangular fuzzy numbers, is an approximation, although it forms an ideal compromise [3].

On the other hand, the suggested approach can be applied in the case of triangular fuzzy numbers for It is obvious that a general comparison between the four rule indices, according to the value of the objective function (6), shows that $ZNSR \leq ZPSR \leq ZPR$ and $ZNSR \leq ZNR \leq ZPR$, where ZPR, $ZPSR$, ZNR, and $ZNSR$ are the values of the objective function according to PR, PSR, NR, and NSR, respectively. A numerical comparison is given by the following example

4. NUMERICAL EXAMPLE

Let the fuzzy linear programming problem (1)–(3) be presented as follows:

Maximize Z= (2, 6, 8, 12) x_1 + (4, 6, 8, 10) x_2

Subject to (12, 15, 17, 20) x_1 + (10, 16, 20, 25) $x_2 \leq b_1$,

(0.04, 0.07, 0.08) x_1 + (0.03, 0.06, 0.1) $x_2 \leq b_2$,

$x_1, x_2 \geq 0$.

Here b_1 and b_2 are independent random variables, with b_1 having a uniform distribution on the interval [30, 50], while b2 is exponentially distributed with mean equals one. As well, let $\tilde{\delta}_1 = (0.3, 0.6, 0.8, 0.95)$ and $\tilde{\delta}_2 = (0.5, 0.7, 0.9)$.Then, for each of the four rule indices, the equivalent deterministic-crisp linear programming problem is solved, using the GAMS package [9], when $\alpha = 0.3, 0.5$, and 0.8. The results are presented in Table 1.

5. CONCLUSION

The suggested approach for comparing fuzzy numbers in the case of stochastic fuzzy linear programming problems can be applied for different types of fuzzy numbers, in addition to the trapezoidal and triangular fuzzy numbers that have been used in this paper. Also, any approximation that may exist, due to using another approach, can be avoided. Utilizing the α-cut technique for the membership functions to derive closed crisp intervals represents the main step in our approach. Thus, for different values of α, and by comparing the closed crisp intervals, results are generated according to each of the four rule indices, whereas the most convenient one can be chosen.

REFERENCES

[1] D. Dubois, H. Prade, Ranking fuzzy numbers in the setting of possibility theory, Information Sciences 30 (1983) 183–224. [2]G.Bortolan, R. Degani, A review of some methods for ranking fuzzy subsets, Fuzzy Sets and Systems 15 (1985) 1–19.

[2] Bellman, R. E., & Zadeh, L. A. (1970), "Decision making in a fuzzy environment" Management Sciences 17: B

[3] D.S. Negi, E.S. Lee, Possibility programming by the comparison of fuzzy numbers, Computers and Mathematics with Applications 25 (1993) 43–50.

[4] M.G. Islander, Comparison of fuzzy numbers using possibility programming: comments and new concepts, Computers and Mathematics with Applications 43 (2002) 833–840.

[5] Tahriri et al. (2008), Selection methods concerning supplier selection. The advantages and disadvantages of selection methods European Journal of Operational Research, Vol.50,5-10

[6] Alexios-patapios kontis and vassilionvyagotis (2011), supplier Selection problem multi- Criteria Approaches based on DEA, international Scientific Advances in Management & Applied Economics, vol1.1, no.2, 207- 219

[7] Amit karani and Zhiling Guo (2012), apply fuzzy logic approach to intergraded qualitative Survey Data into Traditional MCDM, Hawaii International conference on System Sciences vol.7 ;(IEEE)

[8] Tu-chung Chu, Rangnath varma (2012), Evaluating supplier via a multiple criteria Decision making method under Fuzzy environment, international Journal of computers & Industrial Engineering vol, 62; 653-660

[9] Li-Xin Wang (2009), a course in fuzzy systems and control." Prentice Hall International journal of Management science)

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