MANUFACTURING SETUP COST REDUCTION AND QUALITY IMPROVEMENT FOR THE DISTRIBUTION FREE CONTINUOUS-REVIEW INVENTORY MODEL WITH A SERVICE LEVEL CONSTRAINT

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ABSTRACT:

Over several decades, the continuous-review inventory model has been widely studied based on various assumptions and restrictions such as those related with quality improvement, service level constraint, and setup cost reduction. The proposed study investigates a continuous review inventory model with order quantity, reorder point, backorder price discount, process quality, and lead time as decision variables. The options of investing in process quality improvement and setup cost reduction are included, and lead time can be shortened at an extra crashing cost. The objective is to simultaneously optimize the lot size, the reorder point, the process quality, the setup cost, and the lead time. Two models are developed based on the probability distribution of lead time demand. The lead time demand follows a normal distribution in the first model and in the second model we apply the distribution free approach for the lead time demand. Finally, some numerical examples are given to illustrate the model.

KEYWORDS: Inventory; Quality improvement; Set up reduction; Backorder price discount; Lead time reduction; Distribution free approach.

1. INTRODUCTION:

In the classical production inventory models, such as the economic order quantity (EOQ) model, the setup/ordering cost and lead time are constant and quality is assumed to be fixed. In other words, these elements are treated as givens and not subject to control. However, this may not be realistic. For example, in many practical situations, lead time can be shortened at an added cost, and hence lead time is controllable. The Japanese successful experience of using just-in-time (JIT) production has evidenced that substantial advantages and benefits can be obtained from shortening the lead time. The underlying goal of JIT is to eliminate waste ; specifically, from an inventory standpoint, it is to produce small lot sizes with good-quality products. In addition to lead-time reduction, other actions such as setup cost reduction and quality improvement are also recognized as effective ways to achieve the JIT goal.

During the past two decades, the issues of investing in changing the levels of lead time, setup cost, and process quality in production inventory systems have received a great deal of attention. But in reality as just-in-time cannot be implemented in all situations, thus researchers should consider variable lead time. In practice, the distributional information about the demand is often limited. There is a tendency to use the normal distribution under these conditions by several researchers. However, the normal distribution does not offer the best shield against the occurrences of other distributions with the same mean and same variance. Therefore, it is a challenge to the managers to take a decision without having the idea of the distribution of the lead time demand. In order to solve this problem, Scarf [1] first developed the solution of distribution free newsboy problem with known mean and standard deviation of the lead time demand. Gallego and Moon [2] simplified the proof of the Scarf's [1] ordering rule. Liao and Shyu [3] discussed a stochastic inventory model in which the lead time is a unique decision variable and the order quantity is predetermined. Ben-Daya and Raouf [4] extended Liao and Shyu's [3] model by considering both lead time and order quantity as decision variables. Ouyang et al. [5] generalized Ben-Daya and Raouf's [4] model by assuming shortages, normally distributed lead time demand, and distribution free lead time demand.

Ouyang and Wu [6] used a service level constraint to improve an inventory model with variable lead time. Moon and Choi [7] extended Ouyang et al.'s [5] model by assuming lead time and reorder point as decision variables. Hariga and Ben-Daya [8] discussed some stochastic inventory models with deterministic lead times. Ouyang and Chang [9] developed an improved inventory model with imperfect production to study the joint effect of reducing lead time and setup cost. Pan et al. [10] analyzed an inventory model with a crashing cost which is assumed to be a function of order quantity and lead time. Ouyang et al. [11] derived an integrated inventory model with controllable lead time. Lee et al. [12] discussed an inventory model with lead time, order quantity, and reorder point as decision variables. Wu et al. [13] discussed a computational algorithmic procedure for the optimal inventory policy involving a negative exponential crashing cost and lead time demand. They considered lead time dependent backorder rate and a service level constraint. Cobb [14] explained a continuous review mixture inventory model with lead time demand. Sarkar and Majumder [15] investigated an integrated vendor–buyer supply chain model with vendor's setup cost reduction. Sarkar et al. [16] developed an inventory model for finite production rate with time dependent increasing demand. Sarkar et al. [17] discussed an economic manufacturing quantity model for the selling price and the time dependent demand pattern in an imperfect production process but they did not consider quality improvement of products for imperfect items. Sarkar et al. [18] used the min–max distribution free approach to develop a continuous-review inventory model for defective item with a service level constraint, delay-in-payments, and variable lead time. Moon et al.[19] developed the min–max distribution free continuous-review model with a service level constraint and variable lead time.

In several inventory models, it is often assumed that shortages are either completely backlogged or completely lost. In reality, because of well behavior and reputation of the supplier, some customers are willing to wait until replenishment, especially if the waiting time is short, while others are more impatient and go elsewhere. That indicates the supplier has lost the opportunity to earn more profit, disappointed customers, and probably put some doubts in customer's mind about the nature of the storage capacity of the supplier. In this direction, Gallego [20] proposed a distribution free model with full backorder of unsatisfied demand and suggested an iterative procedure to obtain the optimal solution. Chuang et al. [21] investigated the periodic review inventory model with a mixture of backorders and lost sales by simultaneously con-trolling the lead time and setup cost. Alfares and Elmorra [22] discussed an extension of the distribution free newsvendor problem with shortages. Chu et al. [23] extended Ouyang and Wu's [6]model by considering backorders, lost sales, lead time, and order quantity as decision variables. Annadurai and Uthayakumar [24] formulated a mixture inventory model with backorders and lost sales in which order quantity, reorder point, lead time, and setup cost are decision variables. In the direction of backordering and rework cost, several researchers extended inventory models like

Cárdenas-Barrón [25], Sarkar et al. [26–28], Widyadana et al. [29],Sarkar [30], Sarkar and Sarkar [31], and others, but they did not consider distribution free approach. Chen [32] explained an optimal production and inspection strategy with preventive maintenance and rework cost. Cárdenas-Barrón et al. [33] extended an inven-tory model to determine jointly both the optimal replenishment lot size and the optimal number of shipments for an inventory model with rework and multiple shipments. Sarkar and Sarkar [34] developed an inventory model with deterioration and exponentialdemand in a production system over a finite time horizon under the effect of inflation and time value of money. Sarkar et al. [35] developed a single stage production model with random defective rate, rework process, and variable backorders. They introduced three random defective rates which follow three different continues distribution functions but the major contribution of that model is variable backorders rate instead of price discount during backorders.

In above-mentioned studies, authors considered backorder in inventory models but they did not consider backorder price discount. The backorder price discount may be provided during stock out situations which can make customers more willing to wait for desired items. In this direction, Pan and Hsiao [36] extended Ouyang et al.'s [5] model by assuming backorder price discount as a decision variable. Later, Ouyang et al. [37] considered a periodic review inventory model with review period and backorder discounts as decision variables, but the lead time is treated as a constant. Pan et al. [38] and Pan and Hsiao [39] discussed two inventory models with backorder price discounts. For both models, they considered lead time crashing cost as a function of reduced lead time. Lin [40] analyzed a continuous review inventory model with backorder price discount in which the lead time and ordering cost reductions are inter-related. Sarkar and Moon [41] investigated the effects of investment on quality improvement and setup cost reduction with variable backorder rate.

Due to long-run process, the manufacturing system shifts from in-control to out-of-control state as a result the manufacturing system produces perfect as well as imperfect (defective) quality items due to different machinery problems, labor problems. The imperfect items are reworked with some fixed costs to make perfect. Thus we consider an investment function as a logarithmic function to reduce the imperfect production in order to make the model more realistic. This paper extends the study of Lin [40] by adding an investment to improve the process quality. The decision makers have to decide about the quality of products. If the decision makers are aware about the quality, they should consider our strategy to improve the quality of products. Our goal is to reduce the total system cost of a mixture inventory model by using the distribution free approach. Rest of the paper is presented as follows: notation and assumptions are given in Section 2. Mathematical model is developed in Section 3. Numerical example is provided in Section 4. Finally, conclusions are given in Section 5.

2. NOTATION AND ASSUMPTIONS

To develop the model, we use the following notation and assumptions.

NOTATIONS:

Decision Variables

Q - Order Quantity (units)

A - Setup cost per setup ($/setup)

r - Reorder point

k	-	Safety factor
L	-	Replenishment lead time (weeks)
ϕ	-	Probability of the production process which may go to out of control state
π_x	-	Backorder price discount per unit offered by the supplier ($/unit)

Parameters

D	-	Average demand per year (units/year)
A_0	-	Initial setup cost ($/setup)
ϕ_0	-	Initial probability of the production process which may go to out of control state.
μ	-	Mean of the lead time demand
σ	-	Standard deviation of the lead time demand
h	-	Holding cost per unit per year ($/unit/year)
α	-	Annual fractional cost of capital investment
x	-	Lead time demand which has a distribution function F
$E(x)$	-	Expected value of x
x^+	-	max{x , 0}
$E(x-r)^+$	-	Expected shortage per replenishment cycle
m	-	Cost of replacing a defective unit ($/defective unit)
B	-	Percentage decrease in setup cost per dollar increase in the investment to reduce the setup cost
b	-	Percentage decrease in out of control probability per dollar increase in the investment to reduce the out of control probability
β	-	Backorder ratio, $0 < \beta < 1$
β_0	-	Upper bound of the backorder ratio
π_0	-	Marginal Profit per unit ($/unit)
L_i	-	Length of the lead time with components i = 1, 2. . . n (weeks)
u_i	-	Component of the lead time with u_i as the minimum duration (days)
v_i	-	Component of the lead time with v_i as normal duration (days)
c_i	-	Component of the lead time with c_i as crashing cost per unit time ($/days)
$I_A(A)$	-	Investment for setup cost reduction

$I_\phi(\phi)$ - Investment for quality improvement

$I(A, \phi)$ - Setup cost reduction and capital investment required to reduce setup cost from A_0 to A and the out-of-control probability from ϕ_0 to ϕ.

The following assumptions are considered to develop this model.

1. Inventory is continuously reviewed. Replenishment are made whenever the inventory level falls to the reorder point r.

2. For any continuous model, the reorder point r can be defined as the sum of expected demand during lead time and safety stock ie., $r = DL + K\sigma\sqrt{L}$ where DL = expected demand during lead time, $K\sigma\sqrt{L}$ = safety stock and k = safety factor.

3. The Lead time L has n mutually independent components, each having a different crashing cost for reducing lead time. The i^{th} component has a normal duration v_i and the minimum duration u_i with crashing cost per unit time c_i with $c_1 \le c_2 \le c_3 \le \ldots \le c_n$. The lead time demand x follows a normal distribution with mean DL and standard deviation $\sigma\sqrt{L}$.

4. Let $L_0 = \sum_{j=1}^{n} v_j$ and L_i be the length of the lead time with components 1, 2, . . . i, crashed to their minimum duration. Then, L_i can be written as $L_i = L_0 - \sum_{j=1}^{i}(v_j - u_j)$ and the lead time crashing cost per cycle R(L) is expressed as $R(L) = C_i\left(L_i - L\right) + \sum_{j=1}^{i-1} C_j(v_j - u_j)$ for i = 1, 2, 3, . . . , n.

5. The backorder ratio β is considered as variable and is a proportion to the backorder price discount (π_x) offered by the supplier. As β is the backorder rate and we consider price discount on the backorder, thus we assume $\beta = \frac{\beta_0 \pi_x}{\pi_0}$, , $0 \le \beta_0 < 1$ and $0 \le \pi_x < \pi_0$ where π_x is the price discount during the maximum of backorder β_0 and π_0 is the marginal profit per unit. The supplier makes decisions in order to obtain profits. Therefore, if the backorder price discount π_x is greater than the marginal profit π_0 then the supplier may decide against offering the price discount.

6. The relationship between lot size and quality of product is assumed while producing a lot, the process may shift to out of control state. The process is assumed to be in control before beginning production of lot. Once the process is in out-of control sate, it produces defective units and continues to do so until the entire lot is produced.

7. Additional investment is a good strategy to reduce imperfect production during the out of control state for instance, to reduce an out of control probability from 0.00002 to 0.000018, an investment of $200 may be needed and again another $200 can be used to reduce it to 0.000016 and so on. Therefore the best way to reduce the imperfect production is by using some initial investment. We assume a capital investment $I_\phi(\phi)$ to improve the process

quality and reduce out of control probability as $I_\phi(\phi) = b \ln\left(\frac{\phi_0}{\phi}\right)$, for $0 < \phi < \phi_0$ where $b = \frac{1}{\xi}$ and $b = \xi$ the percentage decrease in A per dollar increase in $I_\phi(\phi)$. From the investment function if $I_\phi(\phi_0) = 0$ then there is no investment for quality improvement. If there is atleast some investment, then the value of ϕ will be reduced for every stage, which indicates the improvement of product quality. The benefit of using the logarithmic function is that it is convex within the range defined for the investment function.

8. The basic inventory model is generally based on the assumption of fixed setup cost. By using investment; we can reduce the setup cost of the model. The initial investment may be high but total cost will be reduced in each stage by using the initial investment function we assume a logarithmic investment function for this purpose.

9. The service level constraint is used to make the model more realistic.

3. MATHEMATICAL MODEL

This model considers a continuous review inventory model with variable lead time, order quantity and backorder price discount. The inventory is reviewed after reorder point r along with a lead time L units of time. We assume that the lead time demand X follows a normal distribution with mean DL and standard deviation $\sigma\sqrt{L}$. When the inventory reaches to reorder point r, and order quantity Q is placed for filling up the inventory, before receipt an order the expected inventory is r – DL and after delivery of Q quantity expected inventory is r – DL + Q. Thus the average inventory is $\frac{Q}{2} + r - DL$.

We assume that there is a backorder rate β and the expected number of backorder rate β and the expected number of backorders per cycle is $\beta E(x - r)^+$ where $E(x - r)^+$ is the expected shortage at the end of cycle as well as the expected number of lost sales per cycle is $(1 - \beta)E(x - r)^+$.

Therefore, the annual stockout cost is $\frac{D}{Q}\left(\pi_x\beta + \pi_0(1-\beta)\right)E(x-r)^+$ and the annual expected holding cost is $h\left[\frac{Q}{2} + r - DL + (1-\beta)E(x-r)^+\right]$.

Most continuous review inventory models are based on a fixed setup cost. However, an initial investment can be used to reduce the setup cost of the whole system. We use the logarithmic expression is as follows:

$$I_A(A) = B\ln\left(\frac{A_0}{A}\right) \text{ for } 0 < A \le A_0 \quad \ldots (1)$$

Where $B = \frac{1}{\tau}$, τ is the percentage decrease in A per dollar increase in $I_A(A)$.

During long production process, machinery systems may produce low quality products, which may result in revenue loss and an impugned industry reputation. Therefore, to maintain the brand image of the industry, firms may choose to make some investments that improve the quality of products. Although this strategy for quality improvement may lead to increases in total system costs, setup cost reduction counter balances the added expenses such that the total cost of the system is maintained. The investment for quality improvement is as follows :

$$I_\phi(\phi) = b \ln\left(\frac{\phi_0}{\phi}\right), \text{ for } 0 < \phi < \phi_0 \quad \ldots (2)$$

where $b = \frac{1}{\xi}$ and ξ is the percentage decrease in ϕ per dollar increase in $I_\phi(\phi)$. Hence, total investment for quality improvement and setup cost reduction can be written as follows :

$$I(A, \phi) = I_\phi(\phi) + I_A(A)$$

$$= G - b \ln\phi + B \ln A$$

where $G = b\ln\phi_0 + B\ln A_0$, $0 < A \le A_0$ and $0 < \phi \le \phi_0$ . . . (3)

After a long production process, the system can get out of control state with a probability ϕ (generally, ϕ is very small and close to zero) and once it is out of control, it products imperfect items continuously until the entire lot is produced. Due to this relationship between lot size and out of control product quality, the expected numbers of defective items during a production run cycle is approximated by $\frac{DQ\phi}{2}$ and the cost of replacing a defective item is m. Therefore, the expected annual defective cost is $\frac{mDQ\phi}{2}$.

After the absorption, the expected annual total cost can be expressed as

$$TEAC(Q, r, \beta, \phi, A, L) = \frac{AD}{Q} + h\left[\frac{Q}{2} + r - DL + (1-\beta)E(x-r)^+\right]$$

$$+ \frac{D}{Q}\left[\pi_x\beta + \pi_0(1-\beta)\right]E(x-r)^+ + \frac{D}{Q}R(L) + \alpha I(A,\phi) + \frac{mDQ\phi}{2} \quad \ldots (4)$$

Moreover, by assumptions (4) and (5), we have R(L) and $\beta = \frac{\beta_0\pi_x}{\pi_0}$. Therefore (4) becomes

$$TEAC(Q, r, \beta, \phi, A, L) = \frac{AD}{Q} + h\left[\frac{Q}{2} + r - DL\right]$$

$$+\left[h\left(1-\frac{\beta_0\pi_x}{\pi_0}\right)+\frac{D}{Q}\left(\pi_0-\beta_0\pi_x+\frac{\beta_0\pi_x^2}{\pi_0}\right)\right]E(x-r)^+$$

$$+\frac{D}{Q}\left[C_i\left(L_i-L\right)+\sum_{j=1}^{i-1}C_j(v_j-u_j)+\alpha\left(G-b\ln\phi-B\ln A\right)+\frac{mDQ\phi}{2}\right]$$

. . . (5)

for $0 < \phi \le \phi_0$ and $0 < A \le A_0$, Thus the backorder price discount π_x offered by the supplier can be treated as a decision variable instead of the backorder ratio β.

3.1. Normal Distribution Model

We first assume that lead time demand X follows a normal distribution with mean DL and standard deviation $\sigma\sqrt{L}$. We note that $r = DL + k\sigma\sqrt{L}$ and

$E(x-r)^+ = \int_r^\infty (x-r)dF(x) = \sigma\sqrt{L}\left\{\phi(k)-k[1-\phi(k)\right\} = \sigma\sqrt{L}\chi(R)$ and ϕ and Φ denote the standard normal probability density function(p.d.f) and d.f. respectively and $\chi(K) = \phi(k) - k[1 - \Phi(k)]$. Therefore, we can treat the safety factor R, as a decision variable instead of the reorder point r. Thus, our problem can be transformed to

$$\text{Min } TEAC^N(Q, R, \pi_x, \phi, A, L) = \frac{AD}{Q}+h\left[\frac{Q}{2}+k\sigma\sqrt{L}\right]+\left[h\left(1-\frac{\beta_0\pi_x}{\pi_0}\right)+\frac{D}{Q}\bar{\pi}\right]\chi(k)\sigma\sqrt{L}+$$

$$+\frac{D}{Q}R(L)+\alpha\left(G-b\ln\phi-B\ln A\right)+\frac{mDQ\phi}{2} \quad \ldots (6)$$

Subject to $0 < \phi \le \phi_0$

$0 < A \le A_0$

where $\bar{\pi} = \pi_0-\beta_0\pi_x+\frac{\beta_0\pi_x^2}{\pi_0}$ and $TEAC^N(Q, k, \pi_x, \phi, A, L)$ is the expected annual total cost when demand during lead time is normally distributed.

In order to solve the non-linear programming problem, we first ignore the restrictions $0 < \phi \le \phi_0$ and $0 < A \le A_0$ and take the first order partial derivatives of $TEAC^N(Q, k, \pi_x, \phi, A, L)$ with respect to Q, k, π_x, ϕ, A and $L \in (L_i, L_{i-1})$ respectively we obtain

$$\frac{\partial TEAC^N}{\partial Q} = \frac{-AD}{Q^2}+\frac{h}{2}-\frac{D\bar{\pi}\chi(k)\sigma\sqrt{L}}{Q^2}-\frac{DR(L)}{Q^2}+\frac{mD\phi}{2} \quad \ldots (7)$$

$$\frac{\partial TEAC^N}{\partial k} = h\sigma\sqrt{L}-\left[h\left(1-\frac{\beta_0\pi_x}{\pi_0}\right)+\frac{D}{Q}\bar{\pi}\right]\sigma\sqrt{L}\left[1-\phi(k\right] \quad \ldots (8)$$

$$\frac{\partial TEAC^N}{\partial \pi_x} = \left[\frac{D}{Q}\left(\frac{2\beta_0\pi_x}{\pi_0} - \beta_0\right) - \frac{h\beta_0}{\pi_0}\right]\sigma\sqrt{L}\chi(k) \quad \ldots (9)$$

$$\frac{\partial TEAC^N}{\partial \phi} = \frac{-\alpha b}{\phi} + \frac{mDQ}{2} \quad \ldots (10)$$

$$\frac{\partial TEAC^N}{\partial A} = \frac{-\alpha B}{A} + \frac{D}{Q} \quad \ldots (11)$$

$$\frac{\partial TEAC^N}{\partial L} = \frac{1}{2}hk\sigma L^{\frac{-1}{2}} + \frac{1}{2}\left[h\left(1 - \frac{\beta_0\pi_x}{\pi_0}\right) + \frac{D}{Q}\bar{\pi}\right]\sigma L^{\frac{-1}{2}}\chi(k) - \frac{D}{Q}C_i \quad \ldots (12)$$

By examining the second order sufficient conditions for a minimum value, it can be verified that $TEAC^N(Q, k, \pi_x, \phi, A, L)$ is not a convex function of $(Q, k, \pi_x, \phi, A, L)$, since the second derivative of $TEAC^N(Q, k, \pi_x, \phi, A, L)$ with respect to L is negative, that is

$$\frac{\partial^2 TEAC^N(Q,k,\pi_x,\phi,A,L)}{\partial L^2} = -\frac{1}{4}hk\sigma L^{\frac{-3}{2}} - \frac{1}{4}\left[h\left(1 - \frac{\beta_0\pi_x}{\pi_0}\right) + \frac{D}{Q}\bar{\pi}\right]\sigma L^{\frac{-3}{2}}\chi(k) < 0 \quad \ldots (13)$$

Also, from (13) we note that for fixed Q, k, π_x, ϕ, A and L, $TEAC^N(Q, k, \pi_x, \phi, A, L)$ is concave in L. Therefore, for fixed Q, k, π_x, ϕ and A, the minimum expected annual total cost will occur at the end points of the interval $[L_i, L_{i-1}]$.

On the other hand, by setting (7)–(11) equal to zero, for a given value of $L \in (L_i, L_{i-1})$, we obtain

$$Q = \sqrt{\frac{2D\left(A + \bar{\pi}\sigma\sqrt{L}\chi(k) + R(L)\right)}{h+mD\phi}} \quad \ldots (14)$$

$$\phi(k) = 1 - \frac{Qh}{hQ\left(1 - \frac{\beta_0\pi_x}{\pi_0}\right) + D\bar{\pi}} \quad \ldots (15)$$

$$\pi_x = \frac{hQ}{2D} + \frac{\pi_0}{2} \quad \ldots (16)$$

$$\phi = \frac{2\alpha b}{mDQ} \quad \ldots (17)$$

and $$A = \frac{\alpha BQ}{D} \quad \ldots (18)$$

From (14) – (18) for fixed $L \in (L_i, L_{i-1})$, we can get the values of Q^*, k^*, π_x^*, ϕ^* and A^*. Therefore, when the constraints $0 < \phi \le \phi_0$ & $0 < A \le A_0$ are ignored, the point $(Q^*, k^*, \pi_x^*, \phi^*, A^*)$ is the optimal solution such that the total expected annual cost is minimum.

3.2. Distribution Free Model

In the previous section, a model is developed and the assumption that the lead time demand follows a normal distribution. However, in practical situations, it is very difficult to know the nature of lead time demand distribution. Thus, the assumption about the normal distribution of the lead time demand is relaxed and the model only assumes that the density function of lead time demand belongs to the class of least favorable distribution function F with mean DL and standard deviation $\sigma\sqrt{L}$. As the distributional form of lead time demand X is unknown, the exact value of $E(X-r)^+$ cannot be calculated. Therefore, we consider the min-max distribution free approach to solve this problem.

$$\text{Min} \max_{F \in F} \text{TEAC}(Q, k, \pi_x, \phi, A, L) \quad \ldots (19)$$

subject to $0 < \phi \le \phi_0$

$0 < A \le A_0$

From (5), we first note that the larger that $E(X-r)^+$ (whose value depends on the d.f in F), the larger the expected total annual cost TEAC$(Q, k, \pi_x, \phi, A, L)$. Therefore, to find the least favorable d.f. in F for (19) such that TEAC$(Q, k, \pi_x, \phi, A, L)$ has a maximum is equivalent to finding the worst case for $E(X-r)^+$ in model (5).

Fortunately, this task can be achieved by utilizing $r = DL + K\sigma\sqrt{L}$. That is, we have

$$E(X-r)^+ \le \frac{1}{2}\sigma\sqrt{L}(1+k^2-k) \text{ for any } F \in F \quad \ldots (20)$$

Now using (6) and inequality (20), (19) reduces to

Min $\text{TEAC}^W(Q, k, \pi_x, \phi, A, L)$

$$= \frac{AD}{Q} + h\left[\frac{Q}{2} + k\sigma\sqrt{L}\right] + \frac{1}{2}\left[h\left(1-\frac{\beta_0\pi_x}{\pi_0}\right) + \frac{D}{Q}\bar{\pi}\right]\sigma\sqrt{L}\left[1+k^2-k\right]$$

$$+\frac{D}{Q}R(L) + \alpha\left(G - b\ln\phi - B\ln A\right) + \frac{mDQ\phi}{2} \quad \ldots (21)$$

subject to $0 < \phi \le \phi_0$

$0 < A \le A_0$

As on before, we first ignore two constraints and for minimization, taking partial derivatives of $\text{TEAC}^W(Q, k, \pi_x, \phi, A, L)$ with respect to Q, k, π_x, ϕ, A and L we obtain

$$\frac{\partial TEAC^W}{\partial Q} = \frac{-AD}{Q^2} + \frac{h}{2} - \frac{D\bar{\pi}\sigma\sqrt{L}}{2Q^2}\left(1+k^2 - k\right) - \frac{DR(L)}{Q^2} + \frac{mD\phi}{2} \quad \ldots (22)$$

$$\frac{\partial TEAC^W}{\partial k} = h\sigma\sqrt{L} + \frac{1}{2}\left[h\left(1-\frac{\beta_0\pi_x}{\pi_0}\right) + \frac{D}{Q}\bar{\pi}\right]\sigma\sqrt{L}\left[\frac{k}{\sqrt{1+k^2}} - 1\right] \quad \ldots (23)$$

$$\frac{\partial TEAC^W}{\partial \pi_x} = \left[\frac{D}{2Q}\left(\frac{2\beta_0\pi_x}{\pi_0} - \beta_0\right) - \frac{h\beta_0}{2\pi_0}\right]\sigma\sqrt{L}(1+k^2-k) \quad \ldots (24)$$

$$\frac{\partial TEAC^W}{\partial \phi} = \frac{-\alpha b}{\phi} + \frac{mDQ}{2} \quad \ldots (25)$$

$$\frac{\partial TEAC^W}{\partial A} = \frac{-\alpha B}{A} + \frac{D}{Q} \quad \ldots (26)$$

and $$\frac{\partial TEAC^W}{\partial L} = \frac{1}{2}hk\sigma L^{\frac{-1}{2}} + \frac{1}{4}\left[h\left(1-\frac{\beta_0\pi_x}{\pi_0}\right) + \frac{D}{Q}\bar{\pi}\right]\sigma L^{\frac{-1}{2}}\left(\sqrt{1+k^2} - k\right) - \frac{D}{Q}C_i \quad \ldots (27)$$

We first check the concavity or convexity of L

$$\frac{\partial^2 TEAC^W}{\partial L^2} = -\frac{1}{4}hk\sigma L^{\frac{-3}{2}} - \frac{1}{8}\left[h\left(1-\frac{\beta_0\pi_x}{\pi_0}\right) + \frac{D}{Q}\bar{\pi}\right]\left(\sqrt{1+k^2} - k\right)\sigma L^{\frac{-3}{2}} < 0 \quad \ldots (28)$$

From (28), it is clear that for fixed $(Q, k, \pi_x, \phi, A, L)$ $TEAC^W(Q, k, \pi_x, \phi, A, L)$ is concave in L. Therefore, the minimum total expected annual cost will occur at the end points of the interval $[L_i, L_{i-1}]$.

Again, for fixed $L \in [L_i, L_{i-1}]$ by taking (22) – (25), we obtain

$$Q = \sqrt{\frac{D\left(2[A+R(L)] + \bar{\pi}\sigma\sqrt{L}(1+k^2 - k)\right)}{h + mD\phi}} \quad \ldots (29)$$

$$\frac{k}{\sqrt{1+k^2}} = 1 - \frac{2Qh}{hQ\left(1-\frac{\beta_0\pi_x}{\pi_0}\right) + D\bar{\pi}} \quad \ldots (30)$$

$$\pi_x = \frac{hQ}{2D} + \frac{\pi_0}{2} \quad \ldots (31)$$

$$\phi = \frac{2\alpha b}{mDQ} \quad \ldots (32)$$

and $$A = \frac{\alpha BQ}{D} \qquad \ldots (33)$$

Again, we note that, as in the normal distribution model, for a given value of L, it can be shown that the values Q_w, k_w, π_{x_w}, ϕ_w and A_w determined from (29) – (33) satisfy the second order sufficient condition for a minimum value. And hence for fixed $L \in (L_i, L_{i-1})$, $TEAC^W(Q, k, \pi_x, \phi, A, L)$ has a relative minimum value at point $(Q_w, k_w, \pi_{x_w}, \phi_w, A_w)$.

4. NUMERICAL EXAMPLE:

Consider the following parametric values D = \$600/unit, A_0 = \$200/order, h = \$20/unit/year, π_0 = \$150/unit, σ = 7 units, ϕ_0 = 0.0002, m = \$75/defective unit, α = \$0.1/year, b = 400, B = 5800, π = \$ 50/unit, μ = 11 units/week and the lead time has three components with data as shown in Table 1.

Table – 1

LEAD TIME DATA

Lead Time Component i	**Normal Duration V_i (days)**	**Minimum Duration u_i (days)**	**Unit Crashing cost m_i (\$/day)**
1	20	6	0.4
2	20	6	1.2
3	16	9	5.0

Table – 2

The optimal solution of Example 1 for normal distribution and distribution free model

λ	Normal Distribution Model			Distribution Free Model		
	(A^*, Q^*, π_x^*, k^*, ϕ^*, L^*)	**$TEAC^N(.)$**	**Savings**	**(A^*, Q^*, π_x^*, k^*, ϕ^*, L^*)**	**$TEAC^W(.)$**	**Savings**
0.75	(66.67, 76.31, 76.27, 2.01, 0.0000233, 4)	2257.40	26.62	(33.33, 112.15, 76.87, 2.65, 0.0000158, 3)	3079.01	19.59
1.00	(100.00, 88.99, 76.48, 1.95, 0.0000199, 4)	2499.41	18.75	(75.00, 124.24, 77.07, 2.51, 0.0000143, 3)	3290.48	14.06
1.25	(120.00, 88.99, 76.48, 1.95, 0.0000199, 4)	2629.23	14.52	(100.00, 130.85, 77.18, 2.44, 0.0000136, 3)	3408.08	10.99
2.50	(160.00, 108.03, 76.8, 1.87, 0.0000164, 4)	2864.83	6.87	(150.00, 143.02, 77.38, 2.32, 0.0000124, 3)	3627.14	5.27
5.00	(180.00, 113.64, 76.89, 1.84, 0.0000156, 4)	2973.02	3.35	(175.00, 148.67, 77.48, 2.27, 0.0000119, 3)	3729.98	2.59

Example 1 :

We consider the relationship between lead time and ordering cost. We solve the case when the upper bound of the backorder ratio, β_0 = 0.95 and the scaling parameter λ = 0.75, 1.00, 1.25, 2.50, 5.00. The optimal solutions of example 1 for both normal distribution and distribution free cases are summarized in Table 2. Furthermore, to investigate the effect of lead time reduction with interaction of ordering cost, we list the results of fixed ordering cost in Table 2. We observe when the value of λ decreases, the total expected annual cost increases.

5. CONCLUSION:

This study discussed about two models as first model with normally distributed lead time demand and second model without any specific distribution with known mean and standard deviation. The purpose of this study was to minimize the total expected cost with order quantity, reorder point, backorder price discount, process quality, and lead time as decision variables. We simultaneously optimize lot size, reorder point, process quality, setup cost, and lead time, with the objective of minimizing the total relevant cost. The models are utilized with complete and partial information about the lead-time demand distribution .A logarithmic investment function was considered to improve the quality of products. Furthermore, numerical example were given to compare with the existing models.

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