INDUCED VARIATION AND SENSITIVITY ROBUST OPTIMUM DESIGN – AN ILLUSTRATION

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ABSTRACT

Solution of any design optimization problem is in form of a set of values for each individual design variable. Optimum Design is a rigid design and no change or variation in the set values is tolerated. A change or variation in set values of design variables and / or design parameters may cause improper functioning or even failure in some critical cases. The Optimum Design is generally so sensitive to change or variation that it does not tolerate any change or variation though the change is very small and even of the level of geometric tolerances. This all happens because the variation in design variables and parameters get induced in design function causing variation of design function. Variation of design function may cause improper functioning or even failure. In this context Robust Optimum Design is that optimum design which tolerates variations. The variations (which also include the geometric tolerances) are the expected deviation of design variables and / or parameters from their set values. If any how the induced variation is reduced to such an extent that it is hardly noticeable then the variations become tolerable up to certain extent and proper functioning and no failure is ensured up to certain extent. Thus the design becomes a robust one and it is called as a 'Sensitivity Robust Optimum Design'.

***Keywords**: Induced variation, Robust Optimum Design, Sensitivity Robustness, Transmitted variation.*

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I. INTRODUCTION

For most of the design problems, one can find an infinite number of possible design solutions for the overall objective of the design. Any one of such possible design solutions can be designated as an 'Adequate Design' because it represents a synthesis which merely satisfies the functional requirements while remaining within the confines of existing limitations.

Associated with any mechanical element are certain inherently unavoidable undesirable effects like stresses, deflections, vibrations, weight, cost etc. and certain desirable effects like power transmission capability, energy absorption capability, usable length of life, etc. As any design problem can have number of adequate design solutions, to find the best out of them which will result in maximum benefit or in order to obtain a more explicit method of designing, an overall objective of the design should be defined clearly. Depending upon the problem in hand, an objective of the design in form of either to minimize the most significant undesirable effect or to maximize the most significant desirable effect can be defined. Once an objective of the design is clearly defined, it results in an explicit design procedure to arrive at a solution which is the best possible and this design is called as an 'Optimum Design'. The Optimum Design Solution is in form of a set of values for each individual design variable. An 'Optimum Design' is a rigid design and no change or variation in the set values of design variables is tolerated. A change or variation in set values of design variables and / or design parameters may cause improper functioning or even failure in some critical cases. The Optimum Design is generally so sensitive to the changes or variations that it does not tolerate any change or variation though the change is very small and even of the level of geometric tolerances. In this context 'Robust Optimum Design' is that Optimum Design which tolerates variations. The variations (which also include the geometric tolerances) are the expected deviation of design variables and / or parameters form their set values.

One of the key concepts of robust optimum design is, variations in variables and parameters get transmitted to the design function causing variations in it. The variation of the design function, which is due to variation of design function, is called as 'Induced Variation' or 'Transmitted Variation'.

If any how the effect of this transmitted variation in design function. i.e. the 'Induced Variation' is taken care of such that despite variations no improper functioning and no failure is ensured up to certain extent then the resultant design can be assumed to be tolerant to variations and hence a 'Robust Optimum Design'. If any how the Induced Variation is reduced to minimum possible level and hence variation of design function became hardly noticeable. Then it can be assumed that the variations in variables and parameter have become tolerable up to certain extent and the resultant design is a robust design. The act of minimizing the Induced Variation is nothing but to reduce the sensitivity of the design to the variability and hence the design is referred as a 'Sensitivity Robust Optimum Design'.

II. AN ILLUSTRATION OF FORMULATING A SENSITIVITY ROBUST OPTIMUM DESIGN

A. Problem Definition

A liquid storage tank is required to be designed having a storage capacity approximately equal to 531450 m^3. The height of the tank should not exceed a maximum value of 243 m and a thin taller tank is expected. The tolerance on the base dimension (i.e. on diameter if the tank is having a circular base and on length of the side if the tank is having a square base) is $\pm$ 0.6 m. It is expected that the tank will have least surface area so as to reduce convective heat transfer from the tank to the surroundings. As it is comparatively difficult to manufacture a tank having spherical shape so the spherical shape should be avoided as far as possible. It is expected to have least change in calculated surface area once manufactured (as some further critical calculations are based on the value of the surface area of the tank).

B. Problem Solution

For the problem specified, the dimensions of the tank are to be calculated with an aim of minimizing the surface area of the tank while it has to be made sure in the design that the volume of the tank is approximately equal to 531450 m^3 and length of the tank is $\leq$ 243 m. Up to this point it is a simple optimization problem. In the later part of the problem statement it has been asked to make sure in the design that the variability induced in the surface area due to tolerance on base dimension of the tank is to be reduced to minimum possible level and so as a whole it is a 'Sensitivity Robust Optimum Design' problem.

While solving this problem first of all a simple 'Optimum Design' solution is to be formulated and later sensitivity robustness is induced in the design, the procedure is as illustrated below.

1) Simple Optimum Design: Using the conditions as stated in the problem statement first of all a mathematical model of the problem is formulated as below

Minimize surface area 'S' for a liquid storage tank.

Volume of the tank 'V' should be approximately equal to 531450 m^3 (a tolerance of $\pm$ 100 m^3 is allowed in volume).

Length / Height of the tank 'H' should be $\leq$ 243 m.

In short the mathematical model will be

Minimize S

When $V \approx 531450\ m^3$

and $H \leq 243\ m$

To start solving the problem, various possible shapes for the tank are enlisted as below

- Cube
- Cylinder
- Cone
- Square Prism
- Square Pyramid
- Sphere

Some possible shapes like rectangular prism, rectangular pyramid, triangular pyramid, et al. are omitted for the sake of simplicity of the illustration. The Sphere enlisted in above is only for comparison with others sake and as per the problem statement it has to be avoided as far as possible.

Remembering the fact that a thin taller tank is expected and so assuming maximum possible height of the tank, H = 243m and using volume of the tank, $V \approx 531450\ m^3$, the dimension of the base (i.e. base size) for different shapes is calculated as shown in Table 1.

(For simplicity of calculations, value of Π is assumed as 3 every where).

Table I: Dimension Of The Base For Different Shapes

Shape	**Dimension of the Base (m)**	**Actual Volume of the tank (m^3)**	**Difference Between the Actual Volume and Expected Volume (m^3)**
Cube	Width, b = 81	531441	- 9
Cylinder	Diameter, d = 54	531441	- 9
Cone	Diameter, d = 93.54	531546	+ 96.19
Square Prism	Width, b = 93.54	531546	+ 96.19
Square Pyramid	Width, b = 81	531441	- 9
Sphere	Diameter, d = 102.06	531540	+ 90

The above results show that the difference between actual volume and expected volume is marginal and hence any of the shapes is a suitable one. As far as confining to the expected volume is concern, the shapes, the Cube, the Cylinder and the Square Pyramid render less variability in volume and hence any of these three will be declared as an optimum (amongst the remaining) if confinement to the expected volume is an objective of the design.

Referring the problem statement and the mathematical model prepared based on that, it can be realized that the actual objective of the design is to minimize the surface area of the tank and hence a shape resulting least surface area will be declared as an optimum shape for the current problem. Surface area for each of the shape is calculated as shown in Table 2.

Table III: Surface Area Calculation For Different Shapes

Shape	**Surface Area, 'S' (m^2)**
Cube	39366.00
Cylinder	43740.00
Cone	34721.10
Square Prism	49835.30
Square Pyramid	46470.33
Sphere	31248.73

From the above results it can be realized that the Sphere renders least surface area (it is obvious) but it has to be avoided hence the shape rendering least surface area (though greater than that of sphere) the Cone will be declared as an optimum shape for the problem in hand. And this is the simple optimum design with an objective of minimizing the surface area.

2) Effect of Variability*:* It has been stated that The tolerance on the base dimension (i.e. on diameter if the tank is having a circular base and on length of the side if the tank is having a square base) is ± 0.6 m. Nothing has been stated regarding tolerance on the height so in this illustration, tolerance or variability of height and its effect is neglected. The variability in base dimension will cause variability in surface area and also in volume. This means variation in design parameter (the base dimension) will cause variation in objective function (the surface area) and in functional requirement of the design (the volume). This can be realized by referring Table 3. and Table 4.

Table IIIII: Effect Of Variability In Base Dimension On Surface Area

Shape	†Surface Area, 'S' (m^2)	Basic Dimension Added With Maximum Tolerance	††Surface Area, 'S_1' (m^2)	(Induced) Variability in Surface Area (m^2) i.e. S_1- S
Cube	39366.00	b = 81.6	39951.36	585.36
Cylinder	43740.00	d = 54.6	44275.14	535.14
Cone	34721.10	d = 94.14	34951.85	230.75
Square Prism	49835.30	b = 94.14	50531.47	696.17
Square Pyramid	46470.33	b = 81.6	46879.34	409.01
Sphere	31248.73	d = 102.66	31617.22	368.48

† Surface Area 'S' when no tolerance on base dimension is considered

††Surface Area 'S_1' when maximum tolerance is added on base dimension

Table IVV: Effect Of Variability In Base Dimension On Volume

Shape	˜Volume, 'V' (m^3)	Basic Dimension Added With Maximum Tolerance	˜ ˜Volume, 'V_1' (m^3)	(Induced) Variability in Volume (m^3) i.e. V_1- V
Cube	531441.00	b = 81.6	547343.43	15902.43
Cylinder	531441.00	d = 54.6	543316.41	11875.41
Cone	531546.00	d = 94.14	538387.13	6841.13
Square Prism	531546.00	b = 94.14	531546.19	0.1947
Square Pyramid	531441.00	b = 81.6	539343.36	7902.36
Sphere	531540.91	d = 102.66	540970.75	9429.83

˜ Volume 'V' when no tolerance on base dimension is considered

˜˜Volume 'V_1' when maximum tolerance is added on base dimension

3) Sensitivity Robust Optimum Design*:* It can be realized from Table 3. that the change in surface area or variability of surface area due to tolerance (variability) on basic dimension is maximum for the shape Square Prism and it is least for the shape Cone. The objective of the design is to minimize the surface area and variation in surface area due to variation (tolerance) of the design variable (basic dimension) is least for the shape Cone this means the shape Cone is least sensitive to variability. Variability of design function (the objective function, the surface area) due to variability of design variables (basic dimension) is least for Cone and so Cone can be considered as a Sensitivity Robust shape. Besides having least surface area i.e. being optimum shape, the shape Cone has least induced variability (which is due to variability of design variable i.e. the base dimension) so the Cone besides optimum is sensitivity robust hence it is sensitivity robust optimum shape or design.

4) Discussion of the Results from Table 4. *:* It can be realized from Table 4. that the change in volume or variability of volume due to tolerance (variability) on basic dimension is maximum for the shape Cube and it is least for the shape Square Prism. If variability of volume due to variability of base dimension is the only point of concern then the Square Prism being least sensitive is the Sensitivity Robust shape. Referring the problem statement, it can be realized that minimum surface area and minimum variability of surface area are the points of concern and variability of volume is not the point of concern there fore the shape Square Prism bear no merit it has largest surface area and largest variability of surface area. The shape cone has least surface area and least variability of surface area hence it is truly a Sensitivity Robust Optimum Shape or Design also it has second smallest Induced Variability in volume (just next to Square Prism) so lower Induced variability in volume (another design function or a functional requirement) is an added advantage with the shape Cone. Thus in all the respects for the problem in hand the shape Cone will be declared as the Sensitivity Robust Optimum Design.

The shape Sphere is having least surface area and second smallest Induced Variation in surface area (just next to cone) but due to difficulty in manufacture it is advised to be avoided. It has been considered in the illustration for comparison sake only.

5) **Loss of Optimality While Achieving Robustness** *:* It this illustration we realized that the shape Cone has least surface area and least variability of surface area but this was a mere coincidence and always it may not happen that the optimum design decided in first step in least sensitive to variation and it itself is sensitivity robust optimum design in such case we have to choose amongst the designs which results in less Induced variability though it is not the true optimum design. Thus the optimality is sacrificed up to certain extent for the sake of less sensitivity or sensitivity robustness of the design. A trade-off is made between loss of

optimality and achievement of robustness in the design in such cases. The trade-off is based on cost of loss of optimality and need of robustness of the design.

III. CONCLUSION

A Robust Optimum Design is that Optimum Design which tolerates the variations where variations are expected deviation of design variables and parameters from their set values.

Variations in design variables and parameters get transmitted to design function. Design function means the objective function of the design, the functional requirements of the design and undesirable effects in the design. The variation getting transmitted to the design function is called as Induced Variation. Induced variation may cause violation of constraint boundaries of the design; it may cause improper functioning or even failure in some critical cases. Thus Induced variation is of concern.

An Optimum Design is generally rigid and it does not tolerate Induced variation (which is due to variability of the design variables and parameters). If the effect of Induced Variation is nullified in such a way that despite variations the design is feasible and no improper functioning or no failure is ensured then the design can be considered to be capable of tolerating the variations and it is a 'Feasibility Robust Optimum Design'. If the effect of Induced Variation is reduced to such an extent that variation of design function is hardly noticeable and hardly matters then as the sensitivity of the design is minimized therefore it is assumed that the design tolerates variations and hence it is robust. Such a design is called as a 'Sensitivity Robust Optimum Design'.

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