International Journal of Research in Engineering and Applied Sciences (IJREAS)
Available online at http://euroasiapub.org/journals.php
Vol. 6 Issue 11, November - 2016, pp. 170~176
ISSN (O): 2249-3905, ISSN(P) : 2349-6525 | Impact Factor: 6.573 | Thomson Reuters ID: L-5236-2015

Differential Transform Method for Solving Berunouli Differential Equation

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Abstract

In this paper, a differantial transformation method (DTM) is used to the numerical solve Berunouli differential equation.This method is capable of greatly reducing the size of computational work while still accurately providing the series solution with fast convergence rate. Tow examples are considered for the numerical illustrations of this method. The results prove the efficiency of this method for such problems.

Key word: differantial transformation method , Berunouli differential equation

INTRODUCTION

The differential transformation technique is one of the numerical methods for ordinary differential equation. The basic idea of differential transform method (DTM) was initially introduced by Zhou [1] in 1986.Its main application there in was to solve both linear and nonlinear initial value problems arising in electrical circuit analysis. This method constructs a semi-analytical numerical technique that uses Taylor series for the solution of differential equations in the form of a polynomial.it is different from the high- order Taylor series method which requires symbolic computation of the necessary derivatives of the data functions. The Taylor series method is computationally time-consuming especially for high order equation. The differential transform is an iterative procedure for obtaining analytic Taylor series solutions of differential equations. It can be said that Differential transform method is a universal one, and is able to solve various kinds of functional equations. For example, it was applied to two point boundary value problems [2], to differential-algebraic equations [3], to the KdV and mKdV equations [4], to the Schrdinger equations [5] and to fractional differential equations [6] and to the Riccati differential equation [7]. The main advantage of this method is that it can be applied directly to nonlinear ODEs without requiring linearization, discretization or perturbation. Another important advantage is that this method is capable of greatly reducing the size of computational work while still accurately providing the series solution with fast convergence rate [8].

In this paper, we shall apply DTM to find the approximate analytical solution of the Berunouli differential equation.

Basic idea of differential transform method

An arbitrary function f(x) can be expanded in Taylor series about a point x = 0 as:

$$y(x)=\sum_{k=0}^{\infty}\frac{x^k}{k!}\left[\frac{d^k y}{dx^k}\right]_{x=0} \qquad (1)$$

The differential transformation of $y(x)$ is defined as:

$$Y(K)=\frac{1}{k!}\left[\frac{d^k y}{dx^k}\right]_{x=0} \qquad (2)$$

Then the inverse differential transform is

$$y(x)=\sum_{k=0}^{\infty}x^k Y(k) \qquad (3)$$

The fundamental mathematical operations performed by one-dimensional differential transform method can be readialy obtained and are listed in table1.

Table 1: The fundamental mathematical operations

Original function	**Transformed function**
$y(x)=g(x)\pm h(x)$	$Y(k)=G(k)\pm H(k)$
$y(x)=\alpha g(x)$	$Y(k)=\alpha G(k)$
$y(x)=\frac{dg(x)}{dx}$	$Y(k)=(k+1)G(k+1)$
$y(x)=\frac{d^2g(x)}{dx^2}$	$Y(k)=(k+1)(k+2)G(k+2)$
$y(x)=\frac{d^mg(x)}{dx^m}$	$Y(k)=(k+1)(k+2)\dots(k+m)G(k+m)$
$y(x)=1$	$Y(k)=\delta(k)$
$y(x)=x$	$Y(k)=\delta(k-1)$
$y(x)=x^m$	$Y(k)=\delta(k-m)=\begin{cases}1 & k=m\\ 0 & k\neq m\end{cases}$
$y(x)=f_1(x)f_2(x)$	$Y(k)=\sum_{m=0}^{k}F_1(m)F_2(k-m)$
$y(x)=e^{\lambda x}$	$Y(k)=\frac{\lambda^k}{k!}$
$y(x)=(1+x)^m$	$Y(k)=\frac{m(m-1)\dots(m-k+1)}{k!}$
$y(x)=f_1(x)f_2(x)\dots f_m(x)$	$Y(k)=\sum_{k_{m-1}=0}^{k}\dots\sum_{k_1=0}^{k_2}F_1(k_1)F_2(k_2-k_1)\dots F_m(k-k_{m-1})$

Numerical examples

To illustrate the ability and reliability of the method for the Berunouli differential equation some examples are provided. The results reveal that the method is very effective and simple.

Example 1: Consider the following Berunouli differential equation

$$\frac{dy}{dx}+2y=-e^{2x}y^{2} \qquad (4)$$

Subject to the initial condition,

$$y(0)=1 \qquad (5)$$

The analytical solution of the above problem is given by,

$$y(x)=\frac{1}{xe^{2x}+e^{2x}} \qquad (6)$$

By using the fundamental operations of differential transformation metod in table 1, we have;

$$(k+1)Y(k+1)+2Y(k+1)=-\left(\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}\frac{2^{k_1}}{k_1!}Y(k_2-k_1)Y(k-k_2)\right) \qquad (7)$$

From the initial condition given by Eq. (5) we have

$$Y(0)=1 \qquad (8)$$

Substituting Eq. (8) into Eq. (7) and by recursive method, the results are listed as follows

$$Y(1)=\frac{-1}{3}$$

$$Y(2)=\frac{-1}{3}$$

$$Y(3)=\frac{-5}{9}$$

$$Y(4)=\frac{-2}{27}$$

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Therefore, the closed form of the solution can be easily written as

$$y(x)=\sum_{k=0}^{\infty} Y(k)x^{k}=1-\frac{1}{3}x-\frac{1}{3}x^{2}-\frac{5}{9}x^{3}-\frac{2}{27}x^{4}+\ldots=\frac{1}{xe^{2x}+e^{2x}}$$

Which is the exact solution.

Example 2: Consider the following Berunouli differential equation

$$\frac{dy}{dx}-2xy=2xy^{2} \qquad (9)$$

Subject to the initial condition,

$$y(0)=1 \qquad (10)$$

The analytical solution of the above problem is given by,

$$y(x)=\frac{e^{x^{2}}}{-e^{x^{2}}+2} \qquad (11)$$

By using the fundamental operations of differential transformation metod in table 1, we have;

$$(k+1)Y(k+1)+2\sum_{m=0}^{k}\delta(m-1)Y(k-m)=2\left(\sum_{k_2=0}^{k}\sum_{k_1=0}^{k_2}\delta(k_1-1)Y(k_2-k_1)Y(k-k_2)\right) \qquad (12)$$

From the initial condition given by Eq. (10) we have

$$Y(0)=1 \qquad (13)$$

Substituting Eq. (13) into Eq. (12) and by recursive method, the results are listed as follows

$$Y(1)=0$$
$$Y(2)=2$$
$$Y(3)=\frac{2}{3}$$
$$Y(4)=3$$
$$Y(5)=\frac{4}{5}$$

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Therefore, the closed form of the solution can be easily written as

$$y(x)=\sum_{k=0}^{\infty}Y(k)x^{k}=1+2x^{2}+\frac{2}{3}x^{3}+3x^{4}+\frac{4}{5}x^{5}\ldots=\frac{e^{x^{2}}}{-e^{x^{2}}+2}$$

Which is the exact solution.

Conclusion

In this paper, differential transform method is proposed for solving Berunouli differential equation ; and show that this method is capable of greatly reducing the size of computational work while still accurately providing the series solution with fast convergence rate. It may be concluded that DTM is very powerful and efficient in finding analytical as well as numerical solutions for wide classes of linear differential equations.

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