Unusual Flow of Blood Through a Stenosed Artery: A Theoretical Investigation

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Abstract:

An attempt has been made to study the slip effect on Bingham-plastic flow of blood through a constricted artery. In the present study I have considered the steady flow of blood through an axially symmetric but radially non-symmetric stenosed artery. The variation of flux, resistance to flow and skin friction with different stenosis height and yield stress in presence of slip velocity have been incorporated here. The results are shown graphically and discussed.

AMS Mathematics Subject Classification : 76Z05

Keywords: Yield stress, Bingham plastic flow, flow rate, resistance to flow, Skin friction.

Introduction:

The knowledge of blood flow through an artery is very much important for better understanding the anatomy and physiology of an organic system. The blood flow characteristics in arteries can be altered significantly by the arterial disease, such as stenosis or aneurysm. The medical term stenosis means narrowing of body passage or tube which may occur in various type of cardiovascular disorder. Stenosis is most widely spread cardiovascular disease which is caused due to the unnatural growth in the lumen of the artery. Though actual formation of stenosis in a conclusive manner is unknown to us, it is believed that stenosis is formed by the deposition of various substances like cholesterol or fats/lipids on the inner wall of the artery and unusual growth of the connective tissue. Stenosis developed in the arteries can cause several diseases like blood pressure, atherosclerosis, heart attack and brain haemorrhage. This may be caused by unhealthy living conditions such as exposure of tobacco smoke, lack of physical activity and improper dietary habits.

In view of this several mathematicians have presented various types of mathematical models to get insight for various types of cardiovascular disease relating to blood flow problems. Many researchers have studied mathematical models for blood flow through stenosed or constricted arteries (Texon [1], Forrester and Young [2], Fry [3], Young [4], Lee and Fung [5], Shukla et. al [6] and Young and Tsai [7]).

In all these studies the behaviour of blood has been considered as a Newtonian fluid. However it may be noted that blood does not behave as a Newtonian fluid under certain conditions. Since blood is consists of suspension of cells in plasma, Majhi and Nair [8] suggested that blood behaves like a non-Newtonian fluid at low shear rate.

A mathematical model of blood flow through an irregular arterial mild stenosis is developed by Jain et. al [9] and they have found that if the viscosity of fluid increases, the velocity of fluid decreases in the presence of stenosis. Nanda and Bose [10] developed a mathematical model for studying blood flow through narrow artery with multiple stenosis and they have observed that stenosis height and axial velocity of flow very much influence the shear stress in a stenosed artery. Singh and Singh [11] developed a mathematical model to study the effect of stenosis an d shape on resistance to flow. Some researchers study the power law fluid model of blood by

giving reason that under certain conditions blood behaves like a power law fluid. Casson [12] examine the validity of Casson model in studies the flow characteristics of blood and reported that at low shear rate the yield stress for blood is nonzero. Some authors (Maruthiprasad and Radhakrishnamacharya [13] Maruthiprasad et. al [14], Siddiqui et. al [15], Biswas et. al [16], Misra and Shit [17]) have analysed various types of mathematical models by considering blood as Herschel-Bulkley type non-Newtonian fluid. Blair and Spanner [18] reported that blood behaves like a Casson fluid in the case of moderate shear rate.

In all the above mentioned studies traditional no slip boundary condition has been employed. But many mathematician (Biswas and Chakraborty[19], Verma et. al [20], Ponalgusamy [21]) have presented mathematical model for blood flow through stenosed artery with slip velocity at the flow boundary or in their immediate neighbourhood. So it is clear from the above observation that slip velocity at the stenosedwall vessel plays a vital role in blood flow modelling.

In the present study I propose to discuss the slip effect on Bingham plastic flow of blood through a constricted arterial tube.

The problem and its solution:

Let us considered the steady flow of blood through an axially symmetric but radially non-symmetric constricted artery.

The geometry of bell-shaped stenosis is given by

$$\frac{R(z)}{R_0} = 1-\frac{\delta}{R_0}\exp\left(-\frac{m^2\epsilon^2 z^2}{R_0^2}\right), \quad \text{.....(1)}$$

where R_0 stands for the radius of the arterial tube outside the stenosis, R(z) is the radius in the stenotic region, δ is the depth of the stenosis, m is a parametric constant and ϵ characterises the relative length of the constriction, defined as the ratio of the radius to half-length of stenosis.

$$\epsilon = \frac{R_0}{L_0}$$

The geometry of stenosis can be written as (cf. Fig. 1)

$$\frac{R(z)}{R_0} = 1-ae^{-bz^2}, \quad \text{.....(2)}$$

where $a = \frac{\delta}{R_0}$ and $b = \frac{m^2\epsilon^2}{R_0^2}$

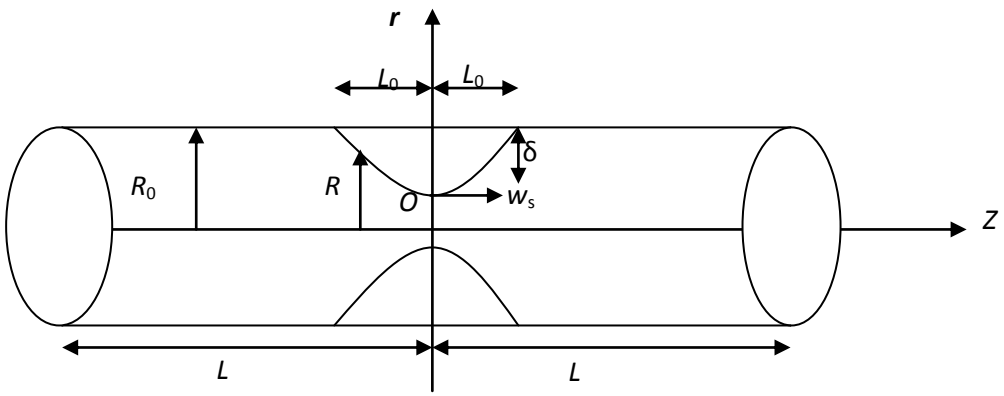

Fig. 1. Geometry of the arterial segment with stenosis

The equation governing the flow is given by

$$-\frac{dp}{dz}=\frac{1}{r}\frac{d(r\tau)}{dr}, \qquad \text{.....(3)}$$

in which τ represents the shear stress of blood considered as Casson fluid and p is the pressure at any point.

The relationship between shear stress and shear rate is given by

$$-\frac{\partial w}{\partial r} = f(\tau) = \frac{1}{k}(\tau - \tau_0); \qquad \tau \geq \tau_0$$

$$= 0; \qquad \tau < \tau_0 \ , \quad \text{.....(4)}$$

where w stands for the axial velocity of blood, $\tau_0$is the yield stress and k is the coefficient of viscosity.

The boundary conditions are

$$\text{w} = w_s \text{ at r = R(z) (slip condition) .....(5)}$$

$$\tau \text{ is finite at r = 0 (regularity condition) .....(6)}$$

Integrating (3) and using the boundary condition (6) we get

$$\tau = -\frac{r}{2}\frac{dp}{dz}, \qquad \text{.....(7)}$$

The skin-friction τ_R is given by

$$\tau_R = -\frac{R}{2}\frac{dp}{dz}, \text{ where R = R(z)} \qquad \text{.....(8)}$$

The volumetric flow rate i.e., the flux is given by

$$\text{Q} = \int_0^R 2\pi r w dr \text{.....(9)}$$

Integrating (9) and using slip condition (5) we get

$$\text{Q} = \pi R^2 w_s + \pi \int_0^R (-\frac{\partial w}{\partial r}) r^2 dr$$

$$= \pi R^2 w_s + \frac{\pi}{k}\int_0^R (\tau - \tau_0) r^2 dr \text{.....(10)}$$

From (7) and (8) we get $\frac{\tau}{\tau_R} = \frac{r}{R}$

From which we get $\text{r} = \frac{R\tau}{\tau_R}$ and $dr = \frac{R}{\tau_R} d\tau$

Thus $\text{Q} = \pi R^2 w_s + \frac{\pi R^3}{k\tau_R^3}\int_0^{\tau_R} \tau^2(\tau - \tau_0)\, d\tau$

$$= \pi R^2 w_s + \frac{\pi R^3}{k}\left[\frac{\tau_R}{4} - \frac{\tau_0}{3}\right] \text{.....(11)}$$

From which we get $\tau_R = \frac{4}{3}\tau_0 + \frac{4kQ}{\pi R^3} - \frac{4kw_s}{R}$ .....(12)

Thus $\frac{dp}{dz} = -\frac{8\tau_0}{3R} - \frac{8kQ}{\pi R^4} + \frac{8kw_s}{R^2}$.....(13)

Integrating (13) along the length of the artery and using the conditions p = p_1 at z = $-L$ and p = p_2 at z = L we obtain

$$p_2 - p_1 = -\frac{8\tau_0}{3R_0}\int_{-L}^{L}\left(\frac{R}{R_0}\right)^{-1} dz - \frac{8kQ}{\pi R_0^4}\int_{-L}^{L}\left(\frac{R}{R_0}\right)^{-4} dz + \frac{8kw_s}{R_0^2}\int_{-L}^{L}\left(\frac{R}{R_0}\right)^{-2} dz$$

……,……..(14)

Where $\frac{R}{R_0}$ can be obtained by using equation (4)

Thus the resistance to flow λ defined by

$$\lambda = \frac{p_2 - p_1}{Q}$$

$$= -\frac{16\tau_0}{3R_0 Q}\left[(L - L_0) + \int_0^{L_0}\left(\frac{R}{R_0}\right)^{-1} dz\right] - \frac{16k}{\pi R_0^4}\left[(L - L_0) + \int_0^{L_0}\left(\frac{R}{R_0}\right)^{-4} dz\right]$$

$$+ \frac{16kw_s}{QR_0^2}\sqrt{\frac{\tau_c k}{\pi Q R_0^5}}\left[(L - L_0) + \int_0^{L_0}\left(\frac{R}{R_0}\right)^{-2} dz\right]$$.....(15)

In the absence of stenosis the resistance to flow λ_N may be expressed as

$$\lambda_N = \left(-\frac{16\tau_0}{3R_0 Q} - \frac{16k}{\pi R_0^4} + \frac{16kw_s}{QR_0^2}\right)L$$ .....(16)

In dimensionless form, the resistance to flow may be expressed as

$$\overline{\lambda} = \frac{\lambda}{\lambda_N} = 1 - \frac{L_0}{L} + \frac{1}{L}\frac{(f_1 I_1 + f_2 I_2 + f_3 I_3)}{(f_1 + f_2 + f_3)}$$ .....(17)

Where $I_1 = \int_0^{L_0}\left(\frac{R}{R_0}\right)^{-1} dz$, $I_2 = \int_0^{L_0}\left(\frac{R}{R_0}\right)^{-4} dz$, $I_3 = \int_0^{L_0}\left(\frac{R}{R_0}\right)^{-2} dz$

and $f_1 = -\frac{16\tau_0}{3R_0 Q}$, $f_2 = -\frac{16k}{\pi R_0^4}$, $f_3 = \frac{16kw_s}{QR_0^2}$

Substituting the expression for $\frac{R}{R_0}$, the integrals I_1, I_2,I_3 takes the form

$$I_1 = \int_0^{L_0}\frac{dz}{1 - ae^{-bz^2}}, I_2 = \int_0^{L_0}\frac{dz}{(1 - ae^{-bz^2})^4}, I_3 = \int_0^{L_0}\frac{dz}{(1 - ae^{-bz^2})^2}$$

Numerical Discussions:

To illustrate the flow analysis the results are shown graphically with the help of MATLAB-7.6.

To attain the numerical results for flux, resistance to flow and skin-friction, some parameters have been taken constant with the values

L = 1; k = 4; m = 1; ∈= 1; Q = 0.002 ; R_0 = 0.05.

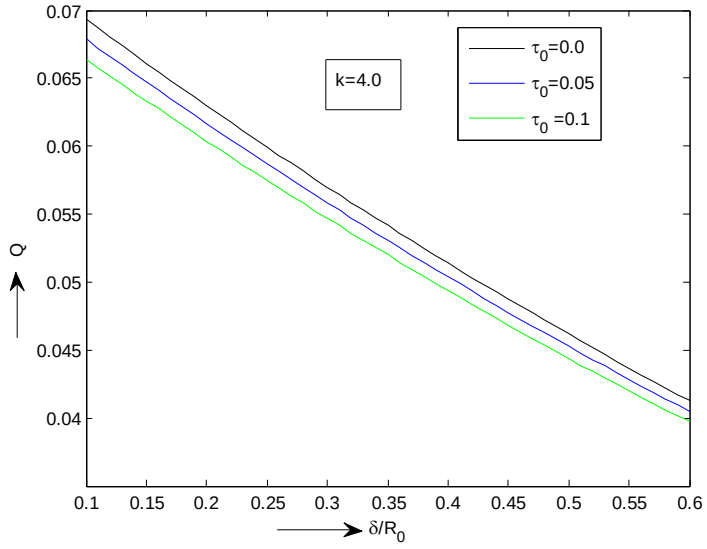
k=4.0
τ_0=0.0
τ_0=0.05
τ_0 =0.1
Q
δ/R_0
0.07
0.065
0.06
0.055
0.05
0.045
0.04
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6

Fig.2

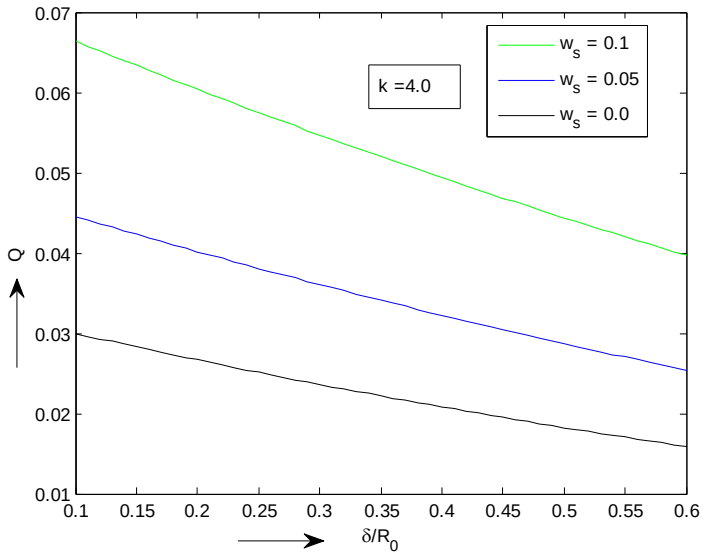
k =4.0
w_s = 0.1
w_s = 0.05
w_s = 0.0
Q
δ/R_0
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6

Fig.3

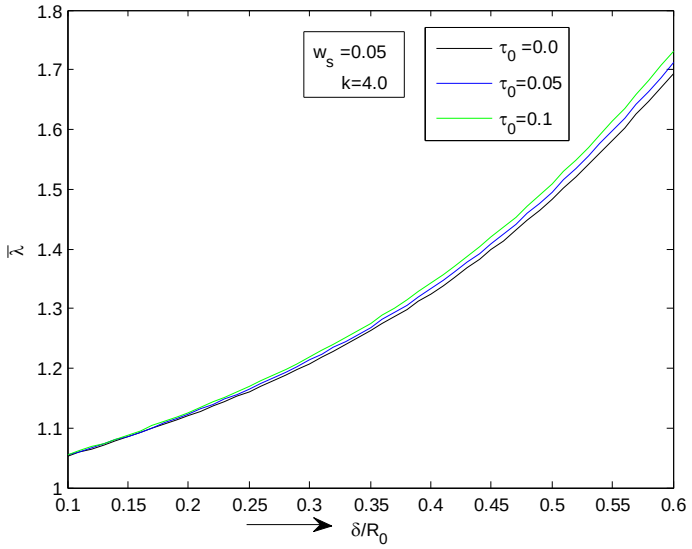

Fig.4

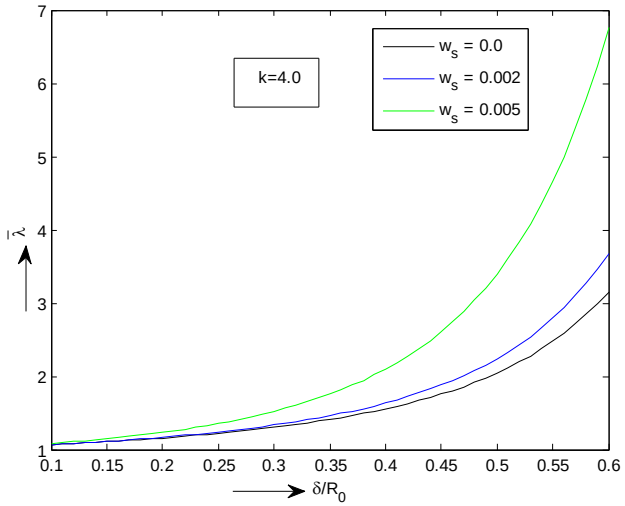

Fig.5

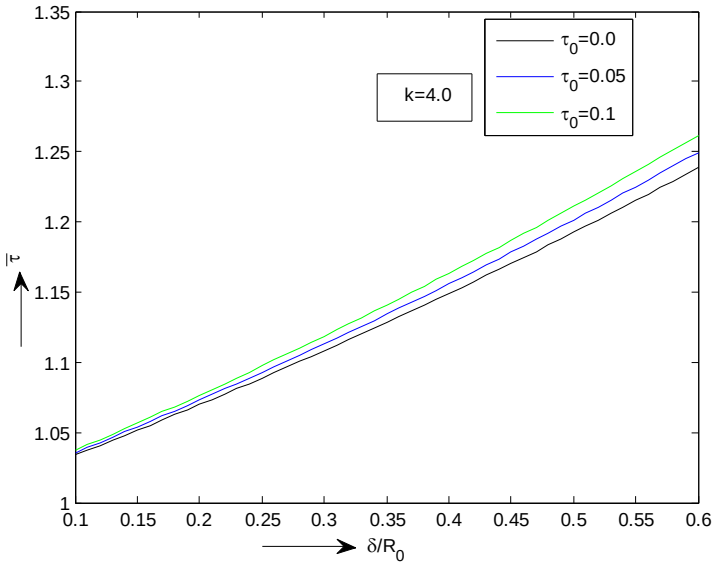

Fig.6

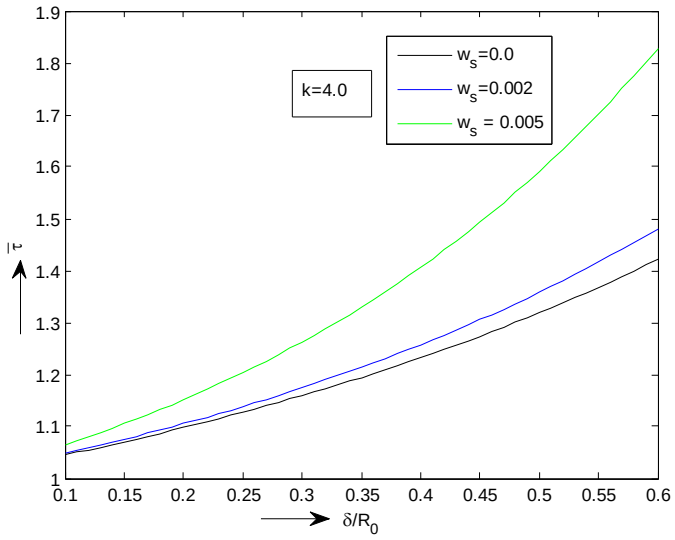

Fig.7

Figures 2, 3 give the variation of flow rate for different values of yield stress and slip velocity, with the variations of $\frac{\delta}{R_0}$. It is observed that Q decreases with the increase of $\frac{\delta}{R_0}$ and yield stress, but the reverse effect occurs when slip velocity increases. Figures 4 and 5 describe the effect of yield stress and slip velocity on resistance to flow against $\frac{\delta}{R_0}$. It is found that for fixed values of k, resistance to flow increases with the increase of $\frac{\delta}{R_0}$ .It also increases when both yield stress and slip velocity increase. Figures 6 and 7 depict the variation of skin-friction for different values of yield stress and slip velocity. It is observed that skin-friction increases with the increase of $\frac{\delta}{R_0}$. Skin-friction increases with the increase of yield stress and slip velocity for fixed values of k.

Conclusions:

Blood flow through an artery mainly depends on the resistance to flow and wall shear stress. It is clear that resistance to flow increases for irregular growth of stenosis whose consequences cause several diseases like hypertension, stroke, heart disease and brain haemorrhage. It is also clear from the observation that resistance to flow and wall shear stress increase with the increase of slip velocity. We cannot ignore this problem occurred in blood flow through human arteries while we present it by a model. So that slip at the wall of the stenosed arterial boundary wall plays an important role in modelling of blood flow. Thus we arrive at the conclusion that with the help of slip velocity damages of the arterial wall may be repaired.

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