International Journal of Research in Engineering and Applied Sciences(IJREAS)
Available online at http://euroasiapub.org/journals.php
Vol. 7 Issue 5, May-2017, pp. 231~236
ISSN (O): 2249-3905, ISSN(P): 2349-6525 | Impact Factor: 7.196 | Thomson Reuters Researcher ID: L-5236-2015

ANALYSIS OF CREEP IN TRANSVERSELY ISOTROPIC FGM CYLINDER

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Abstract: It is revealed by several investigators that aluminum/aluminum alloy matrix composites containing SiC whiskers or fibers have anisotropic properties, which is attributed to the different orientation of whiskers. In other words, creep properties obtained by conducting uniaxial test on specimens taken from the same composite material may differ depending on the orientation of specimens. In short fiber or whisker reinforced composites, material flow during processing such as forging and extrusion often leads to preferential alignment of short fibers or whiskers, resulting in anisotropic mechanical properties. As a result, the composite containing aligned fibers or whiskers exhibit different yield stresses in the direction of alignment and transverse direction.The experimental studies also reveal that mechanical strength of composite depends on the direction considered, because of the orientation of whiskers that contribute to reinforcing the composite to different degrees depending on their alignment in the direction of application of force (Lederich and Sastry, 1982; Crove *et al*, 1985; McDanels, 1985). The comparison of analytical results reported for creep in isotropic thick-walled cylinder subjected to internal pressure with the experimental results show discrepancies, which has been attributed to the development of anisotropy during creep (Davis, 1960 With these forethoughts, it is decided to investigate the consequence of anisotropy on the steady state creep behaviour of a long thick-walled circular cylinder made of FGM. For this purpose, the steady state creep is analyzed in a FGM cylinder composed of transversely isotropic 6061Al-SiCw composite, in which the SiC whiskers are assumed to align in the tangential direction. The content of SiCw in the FGM cylinder is assumed to decrease linearly from the inner to outer radius. The creep stresses and creep rates are estimated in the FGM cylinder for two different operating conditions: (i) cylinder subjected to internal pressure alone and (ii) The maximum SiCw content (V_{max}) in the FGM cylinder is assumed as 20 *vol%* at the inner radius while keeping the average SiCw content (V_{avg}) as 15 *vol*%. The results are estimated for varying degree of anisotropy, characterized by the ratio of radial (or axial) and tangential yield strength of the FGM. As a benchmark, the results are also estimated for a similar cylinder but made of isotropic FGM.

Key Words: Distribution of Creep Stresses and Creep Rates, *Cylinder Subjected to Internal Pressure, 2 Cylinder Subjected to Internal and External Pressures*

I. introduction

The analysis of steady state creep in thick-walled FGM cylinder made of transversely isotropic composite may be carried out in a similar way as described in chapter 4 for isotropic FGM cylinder. However, in this analysis the reinforcements are assumed to be in the form of whiskers rather than particles and the matrix is taken as 6061Al instead of pure Al. The assumptions made in the present study are similar to those described in chapter 4 for isotropic cylinder with an exception that in this study the FGM chosen is transversely isotropic *i.e.* its mechanical properties are same in the radial and axial directions but are different in the tangential direction.

The deformation compatibility equation is given by (refer Eqn. 3.5),

$$r\frac{d\dot{\varepsilon}_\theta}{dr} = \dot{\varepsilon}_r - \dot{\varepsilon}_\theta \qquad (5.5)$$

Assuming the FGM cylinder to operate under the following boundary conditions,

At $r = a$ $\quad \sigma_r = -p$ (5.6)

At $r = b$ $\quad \sigma_r = -q$ (5.7)

where the negative sign implies compressive nature of radial stress.

The equilibrium equation, as derived in chapter 3 (Eqn. 3.9), is written as,

$$r\frac{d\sigma_r}{dr} = d_\theta - d_r \qquad (5.8)$$

The constitutive equations for an anisotropic material are given by (Bhatnagar and Gupta, 1969),

$$\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{(G+H)\sigma_e}\left[G\{\sigma_r - \sigma_z\} + H\{\sigma_r - \sigma_\theta\}\right] \qquad (5.9)$$

$$\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{(G+H)\sigma_e}\left[F\{\sigma_\theta - \sigma_z\} + H\{\sigma_\theta - \sigma_r\}\right] \qquad (5.10)$$

$$\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{(G+H)\sigma_e}\left[F\{\sigma_z - \sigma_\theta\} + H\{\sigma_z - \sigma_r\}\right] \qquad (5.11)$$

where, F, G and H the anisotropic constants, $\dot{\varepsilon}_e$ and σ_e are respectively the effective strain rate and the effective stress.

When the principal axes of anisotropy are the axes of reference, the Hill's yield criterion (Dieter, 1988) for anisotropic material is given by,

$$\sigma_e = \left[\frac{1}{(G+H)}\{F(\sigma_\theta - \sigma_z)^2 + G(\sigma_z - \sigma_r)^2 + H(\sigma_r - \sigma_\theta)^2\}\right]^{1/2} \qquad (5.12)$$

Under the assumption of incompressibility and plane strain condition, Eqn. (5.11) yields,

$$\sigma_e = \frac{G\sigma_r + F\sigma_\theta}{(F+G)} \qquad (5.13)$$

Substituting σ_z from Eqn. (5.13) into Eqn. (5.12) and simplifying, we get,

$$\sigma_e = \frac{(\sigma_\theta - \sigma_r)}{\sqrt{(H+G)}}\left[\frac{(FG+GH+HF)}{(F+G)}\right]^{1/2} \qquad (5.14)$$

Putting $\dot{\varepsilon}_r = \frac{C}{r^2}$ (Eqn. 3.24) and σ_z from Eqn. (5.13) into Eqn. (5.9) and simplifying, we obtain,

$$\sigma_\theta - \sigma_r = \frac{(F+G)(H+G)}{(FG+GH+HF)} \frac{\sigma_e}{\dot{\varepsilon}_e} \frac{C}{r^2} \qquad (5.15)$$

Using Eqn. (5.14) and creep law given by Eqn. (3.2) in Eqn. (5.15), we get,

$$\sigma_\theta - \sigma_r = \left[\frac{(F+G)(H+G)}{(FG+GH+HF)}\right]^{\frac{n+1}{2n}} \frac{C^{1/n}}{r^{2/n}} \frac{1}{M} \sigma_o \sqrt{\frac{(F+G)(H+G)}{(FG+GH+HF)}}$$

or,

$$\sigma_\theta - \sigma_r = \frac{I_1}{r^{2/n}} + I_2 \qquad (5.16)$$

where,

$$I_1 = \left[\frac{(F+G)(H+G)}{(FG+GH+HF)}\right]^{\frac{n+1}{2n}} \frac{C^{1/n}}{M} \qquad (5.17)$$

and

$$I_2 = \sigma_o \sqrt{\frac{(F+G)(H+G)}{(FG+GH+HF)}} \qquad (5.18)$$

Substituting Eqn. (5.16) in equilibrium Eqn. (5.8) and Integrating, we obtain,

$$\sigma_r = \int_a^r \frac{I_1}{\frac{n+1}{r^n}} dr \int_a^r \frac{I_2}{r} dr - p \qquad (5.19)$$

or,

$$\sigma_r = X_1 + X_2 - p \qquad (5.20)$$

where,

$$X_1 = \int_a^r \frac{I_1}{\frac{n+2}{r^n}} dr \quad \text{and} \quad X_2 = \int_a^r \frac{I_2}{r} dr \qquad (5.21)$$

Subsuming Eqn. (5.19) into Eqn. (5.8), we get,

$$\sigma_\theta = \int_a^r \frac{I_1}{\frac{n+2}{r^n}} dr \int_a^r \frac{I_2}{r} dr + \frac{I_2}{r^{2/n}} + I_2 - p$$

or,

$$\sigma_\theta = X_1 + X_2 + \frac{I_1}{r^{2/n}} + I_2 - p \qquad (5.22)$$

To evaluate the value of constant C, required for estimating I_1 given by Eqn. (5.17), the boundary conditions given in Eqs. (5.6) and (5.7) are substituted in Eqn. (5.19) to obtain,

$$\int_a^b \frac{I_1}{\frac{n+2}{r^n}} dr + \int_a^r \frac{I_2}{r} dr - p = -q$$

Substituting the values of I_1 and I_2 respectively from Eqs. (5.17) and (5.18) into above equation and simplifying, we get, n

$$C = \left[\frac{p-q-X_4}{X_3}\right] \quad \text{n} \tag{5.23}$$

where,

$$X_3 = \int_a^b \frac{(K)^{\frac{n+1}{n}} dr}{r^{\frac{n+2}{n}}.M} \; ; \; X_4 = \int_a^b \frac{K\sigma_o dr}{r} \text{ and } K = \sqrt{\frac{(F+G)(H+G)}{(FG+GH+HF)}} \tag{5.24}$$

Using Eqs. (5.13) and (5.14) in Eqs. (5.9) and (5.10), we obtain,

$$\dot{\varepsilon}_\theta = -\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{\sqrt{\frac{(F+G)(H+G)}{(FG+GH+HF)}}} \tag{5.25}$$

The strain rates in an isotropic FGM cylinder may be obtained from the above equation by setting *F = G = H*, as given below,

$$\dot{\varepsilon}_\theta = -\dot{\varepsilon}_r = 0.87\,\dot{\varepsilon}_e \tag{5.26}$$

Therefore, the relationship between radial and tangential strain rates, as described above, similar to that obtain in chapter 3 (Eqn. 3.34).

II ESTIMATION OF ANISOTROPIC CONSTANTS

The Hill's yield criterion for orthotropic material, as given by Eqn. (5.12), involves constants *F, G* and *H*, the values of which are required for estimating creep response of the cylinder. Hill described a procedure for estimating these anisotropic constants, when principal axes of anisotropy are the axes of reference (Hill, 1948) and the form of Eqn. (5.12) remains valid. If *X, Y* and *Z* are the tensile stresses in the principal directions of anisotropy, then according to Hill (1950), one may write,

$$\left\{\begin{array}{l} \frac{1}{X^2} = G + H \\ \frac{1}{Y^2} = H + F \\ \frac{1}{Z^2} = F + G \end{array}\right\} \tag{5.27}$$

The above set of equations may be solved to obtain the values of anisotropic constants as given below,

$$\left\{\begin{array}{l} 2F = \left(\frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2}\right) \\ 2G = \left(\frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2}\right) \\ 2H = \left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right) \end{array}\right\} \tag{5.28}$$

If the material of cylinder is subjected to uniaxial loading in r and θ directions, the corresponding stress invariant may be expressed in terms of observed tensile strength and Hill's anisotropic constants as given below,

$$\sigma_e = \sqrt{\frac{G+H}{2}}\,\sigma_{r_y} \tag{5.29}$$

$$\sigma_e = \sqrt{\frac{F+H}{2}}\,\sigma_{\theta_y} \tag{5.30}$$

where σ_{r_y} and σ_{θ_y} are respectively the yield strength of composite in r and θ directions and σ_e is the isotropic yield stress. For isotropic case the ratio of anisotropic constants is unity *i.e. F/G = G/H = H/F = 1*.

If the material of cylinder is tested under uniaxial loading in z direction, the stress invariant may similarly be written as,

$$\sigma_e = \sqrt{\frac{F+G}{2}}\,\sigma_{z_y} \tag{5.31}$$

In the present analysis, it is assumed that during processing of FGM cylinder the whiskers get aligned in the tangential (θ) direction, leading to anisotropic behaviour. As a result, in FGM cylinder the direction θ becomes longitudinal direction and the remaining directions (*i.e.* r and z) may be taken as transverse directions. For axisymmetric problems like cylinder, the directions r, θ and z may be taken as the principal directions. Thus, the anisotropic constants given by Eqn. (5.28), may be expressed as,

$$F = \left(\frac{1}{\sigma_{\theta_y}^{\,2}} + \frac{1}{\sigma_{z_y}^{\,2}} - \frac{1}{\sigma_{r_y}^{\,2}}\right)\sigma_e^{\,2} \tag{5.32}$$

$$G = \left(\frac{1}{\sigma_{z_y}^{\,2}} + \frac{1}{\sigma_{r_y}^{\,2}} - \frac{1}{\sigma_{\theta_y}^{\,2}}\right)\sigma_e^{\,2} \tag{5.33}$$

$$H = \left(\frac{1}{\sigma_{r_y}^{\,2}} + \frac{1}{\sigma_{\theta_y}^{\,2}} - \frac{1}{\sigma_{z_y}^{\,2}}\right)\sigma_e^{\,2} \tag{5.34}$$

An orthotropic material having one of its plane as the plane of isotropy is referred as transversely isotropic material. In a transversely isotropic material at every point there is a plane in which the mechanical properties remain same in every direction. Many unidirectional composites with fibers packed in hexagonal array, or close to it, can be considered as transversely isotropic with the direction normal to the whisker as the plane of isotropy (Danial and Ishai, 2005).

For transversely isotropic FGM cylinder, we have $\sigma_{r_y} = \sigma_{z_y} = \sigma_e$. As a consequence of this, from Eqs. (5.32) and (5.34) we get $F/H = 1$. The yield strength of the FGM cylinder will be different in circumferential and radial/axial directions. The extent of anisotropy in the FGM cylinder may be characterized by ratio (α) as defined below,

$$\alpha = \frac{\sigma_{r_y}}{\sigma_{\theta_y}} \qquad (5.35)$$

Substituting σ_{r_y} and σ_{θ_y} respectively from Eqs. (5.29) and Eqn. (5.30), and noting that *F/H* = 1, we get,

$$\frac{G}{H} = \frac{2}{\alpha^2} - 1 \qquad (5.36)$$

In the present study, the extent of anisotropy (α) for FGM is assumed to lie in the range 0.7 to 1.3. The value α = 1 will correspond to isotropic FGM. The following section investigates the effect of anisotropy (*i.e.* α deviating from unity) on the distribution of creep stresses and creep rates in the FGM cylinder. For a transversely isotropic FGM cylinder, the ratio *F/H = 1* and the value of *G/H* is estimated from Eqn. (5.36) for a given value of α.

II Conclusions

The magnitude of maximum a minimum value of strain rates (tangential/radial), observed respectively at the inner and outer radii of the cylinder, increases with the increase in α, as shown in Fig. 5.13. The tangential/radial strain rate inhomogenity increases significantly with the increase in α, Fig. 5.14. Though, the inhomogenity in axial and tangential stresses are minimum corresponding to α = 1.3 (Fig. 5.12), but the y in strain in the FGM cylinder.

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