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SEISMIC RESPONSE OF TALL STRUCTURES USING TUNED MASS DAMPERS

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ABSTRACT

Tuned mass dampers are known to control the seismic response of tall buildings when subjected to earthquake ground motions. It is certainly going to regulate the natural frequencies and mode shapes. Several investigations in this regard have been reported in the literature. It is very apparent that the efficiency of the dampers very strongly depends on the frequency content of the ground motions. The frequency content of any ground motion is a random quantity which varies with the earthquake. Of course; theoretically several stochastic models have simulated the random variation of the frequency content. However, it may still be better to evaluate certain different kinds of tuned mass dampers for different ground motions and conclude its effectiveness. More particularly; multiple tuned mass dampers need to be yet explored further. Therefore, this paper examines the effectiveness of tuned mass dampers (TMDs), both single and multiple to ease translational vibration when subjected to various earthquake ground accelerations. Three models of the structure have been modelled using ETABS. They are frame with single TMD, frame with multiple TMD and shear building with single TMD. Four external loading conditions have been applied and time history analysis has been carried out for appropriate ground motion and the variations of displacements in the structure are compared.

Keywords: TMD, Tuned frequency, Time history analysis, Seismic response, ETABS

1. Introduction

With the escalating demands for the safety, reliability, durability, and serviceability of structures, the vibration control theory and practices are leading for more and more concentration in civil engineering structures with a lot of practice being executed on current infrastructures such as high rise buildings, off-shore platforms and long span suspension bridges.

As we know vibration means mechanical oscillations about an equilibrium point. The oscillation can be cyclic or non-cyclic. The control of vibrations is necessary for machines, space shuttle, aircrafts, ships floating on water etc. With the innovations in engineering the vibration reduction technique has found its way to civil engineering and infrastructure field.

Currently innumerable high rise buildings have been constructed all round the world and the number goes on increasing every day. This not only because of high density of inhabitants in the cities, commercial zones and saving larger space area but also in establishing country's land mark and proving that their countries are up to the high level of standards.

As the load due to earthquake acting on a structure is a function of the self-weight of the structure, these structures are made comparatively light.

In every field they always try to follow the conservation of energy. If the energy or vibration acting on the structure by wind and earthquake load is completely dissipated in any way the structure will vibrate less.

As we all know some energy is naturally released in the structures via number of mechanisms like stressing internally, friction and plastic deformation. The total damping is about 5% in almost all large modern structures. Hence to dissipate the larger energy and to control the vibrations, artificial damping devices are installed in many tall structures all over the world. There are a number of methods to control the vibrations which include passive method, active method, semi-active method and hybrid method. The factors that govern the choice of a particular form of device to control vibrations are effectiveness, compactness and mass, initial cost, maintenance cost, maintenance supplies and safety.

A Tuned mass damper (TMD) is a system which utilizes a passive method to dissipate the higher energies by using an additional mass attached to the main structure generally through spring and dashpot to diminish the dynamic response of the structure. It is also known as pendulum damper as it is acts as a pendulum during a vibration and utilizes the same characteristic and properties. It is extensively used for the control of vibrations in mechanical operated systems. In the current era TMD theory is been utilized to control the vibrations of elevated buildings and other civil engineering structures. The additional mass system of the TMD is designed such that the natural frequency, which depends on the stiffness and mass, tuned to that of the main structure. When the frequency of that particular structure gets excited due to vibrations the TMD will resonate adjacent to its phase and move against the motion of the structure and thereby reducing its response. Then, the surplus energy that builds up in the structure due to the vibration can be transferred to the additional mass of the TMD and is released by the dashpot due to relative movement of the dashpot between them at a later Stage. The additional mass of the secondary system varies from 0.5-10% of the totaled load of the structure. When the structure vibrates due to earthquake, it contains a many number of frequencies, hence in recent day's multiple tuned mass dampers (MTMD) has been adopted to reduce the motion induced by seismic loads of tall structures where TMD of many numbers are installed and is tuned to various structural frequencies to control the modes in different frequencies.

The basic mathematical model system as shown in fig below consists of n number of oscillators attached to the primary structure. A wide band Excitation, modelled by white noise is applied to the mass of the main oscillator, and the root mean square displacement response of that mass is of interest. The analysis begins with the impedance of Multiple Tuned mass dampers defined as the force amplitude required to produce a unit harmonic velocity at the base of the TMD's. As per Newton's law, this impedance is the sum of impedances of the individual TMD's.

The paper presenting here proposes designing Multiple Tuned mass damper to reduce the unnecessary vibration of non-linear structures. It has been modelled and designed in the structural analysis software called ETABS which uses a direct integration method to obtain the Velocity and displacement of the structure by direct integration method. The method adopted using the structural analysis software is used to design multiple TMDs for MDOF lumped-mass structures subjected to wide-band of frequencies.

1.1 Working of TMD during Earthquake

When an earthquake occurs, the designed tuned mass dampers with a dashpot attached of heavy mass and tuned to the nearest frequency of the structure due to free vibration will move against the direction of the main structural vibrations caused by the lateral loads acting on the building

and due to movement against the direction of the lateral force an inertia force will be acting on the structure because of which the TMD mitigates the response of the structure.
From the studies it was observed that a single tuned mass damper can attenuate the response of the structure only in the first mode having its frequency being tuned to the fundamental frequency of the structure.

In a high rise building if only single tuned mass dampers is used it can attenuate only first mode of vibrations but for several higher modes of a structure are primary and the anticipated response of vibration reduction cannot be achieved.

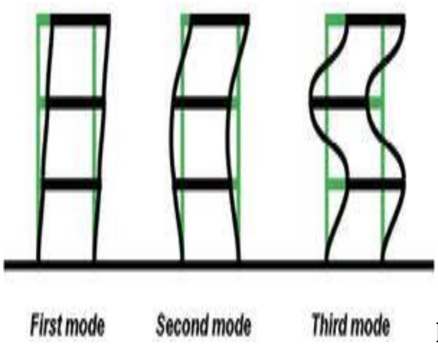


Fig 1.1

The researcher's Li and Wang studied and presented a new method of using multiple tuned mass dampers in the structure to control the higher multiple modes of structure due to vibrations and obtained the required

1.2AIM

The aim of the present project work is to study the behavior and effectiveness of TMD to dynamic response of tall frame structures which can be single or multiple tuned mass dampers.

1.3 Modelling of the Structure

A frame of 60m height with 20 stories of single bay has been modelled in ETABS having two dimensional multi degree of freedom systems with each node having three degrees of freedom.
The dynamic analysis was carried out for the determination of the dynamic response of simple structure by simple free vibration analysis system in which no external load has been applied, the time period and frequency of the structure has been obtained.
The building assumed to be located in seismic zone V having a seismic zone factor of0.36. The soil conditions are assumed to be medium stiff consider as a zone II medium and support at the base for the entire building is assumed as fixed at all joints. The R. C. frames are infilled with brick-masonry wall and the wall load is applied on the beams. The lumped weight due to dead loads is considered on floors and on the roof including a super imposed dead load of 1.2 kN/m^{2}has been considered. The floors are designed to cater to a live load of 3kN/m2 on floors. Dynamic analysis is carried out and the required unknowns are calculated.

1.5 Modal Periods and Frequencies

MODAL PERIODS AND FREQUENCIES			
20 Storey Structure			
MODE (Number)	**TIME PERIOD (Secs)**	**FREQUENCY Cycles/sec**	**CIRCULAR FREQ (Radians/sec)**
Mode 1	4.23924	0.23589	1.48215
Mode 2	3.86013	0.25906	1.62771
Mode 3	2.51843	0.39707	2.49488
Mode 4	1.36044	0.73505	4.61848
Mode 5	1.19764	0.83498	5.24633
Mode 6	0.82851	1.20698	7.58369
Mode 7	0.75552	1.32358	8.31632
Mode 8	0.63973	1.56315	9.82158
Mode 9	0.52451	1.90655	11.9792
Mode 10	0.4847	2.06314	12.9631
Mode 11	0.42549	2.35026	14.7671
Mode 12	0.39487	2.53246	15.91191

For the frame structure modelled, the dynamic properties of the structurelike natural time period and mode shapes for vibration inthe X and Y-directions have been obtained by carrying out a free vibration analysis.

Table 1.1

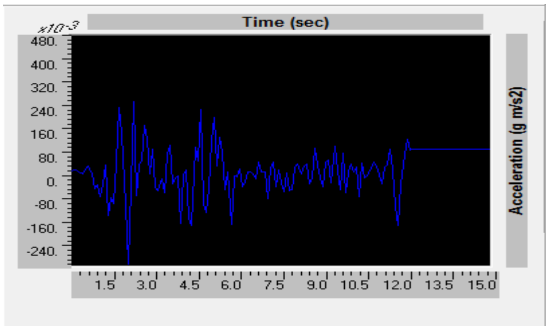
1.6 Time History Analysis

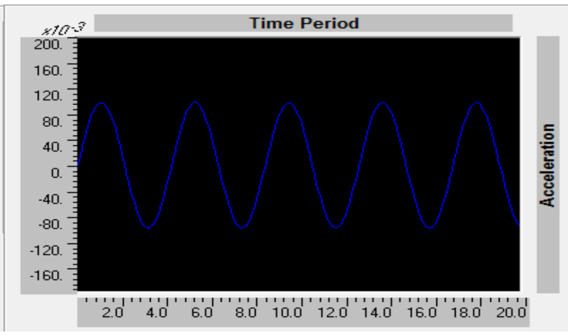
Time history analysis is an analysis for evaluating the dynamic response of the structure under random lateral loadings which may vary to a particular time function. The time history analysis can be obtain for linear or non-linear evaluation of the structure under dynamic loadings.
The Time history is obtained using ETABS based on the equation

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = F(t)$$

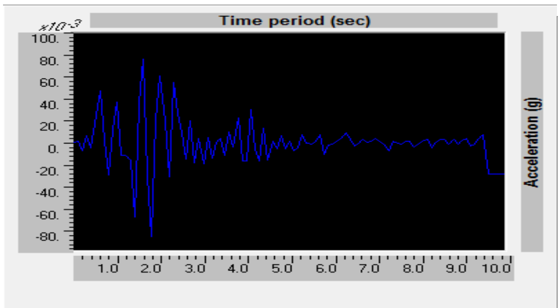
The Equation is solved using direct integration method.

1.7 Random Earthquake Ground Accelerogram

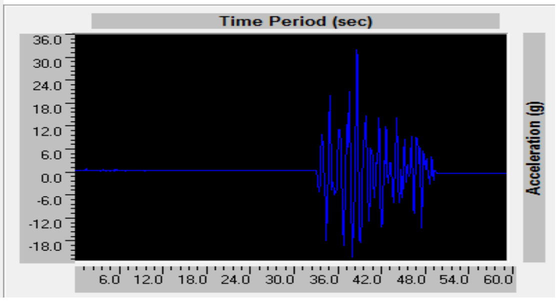




1. 2 El-Centro earthquake 1.3 Sinusoidal ground acceleration



1.4 KOBE Earthquake



1.5 BHUJ Earthquake

2.0 Structure Equipped With Single Tuned Mass Damper'

2.1 Modelling of TMD

Tuned mass damper is a passive energy device installed in the structure generally on the top floor which has a maximum displacement. In ETABS tuned mass damper has been modelled as a 2D element using a spring mass system with damping having a spring stiffness attached with a spring and link property having a lumped mass attached to a 1m thick beam which can carry a load upto 1000kN

The damper is modelled as a 2D steel rod having a diameter of 150mm and based on the frequency obtained from free vibration analysis the length of the steel rod is modelled. It is also designed in such a way that the dampers shall have enough space around to oscillate without damping the structure.

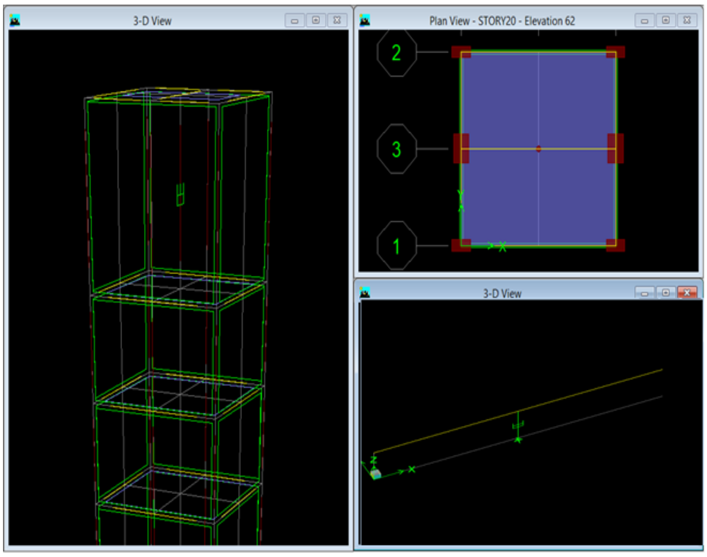
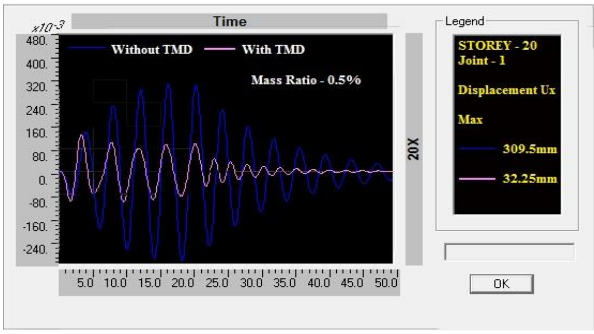


Fig – 2.1 Model of Structure equipped with single TMD on the 20th Floor

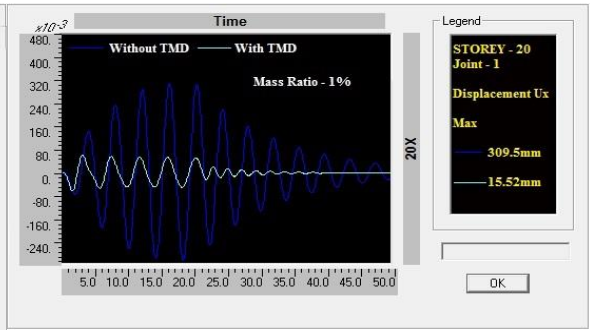
2.2 Non-Linear Time History Analysis of Frame with Single TMD with variation in mass ratios

2.2.1 Comparative study on the Maximum displacement of the frame structure at the 20th floor with and without single TMD with variation of mass ratio.

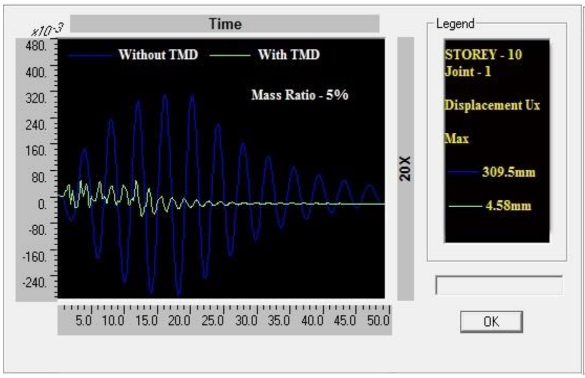
(a) Sinusoidal Ground Acceleration



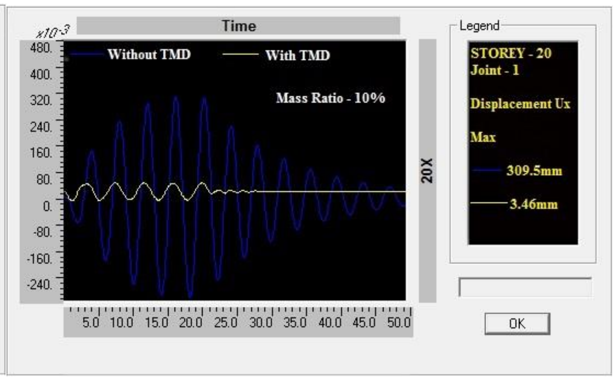
Mass ratio 0.5%



Mass ratio 1%

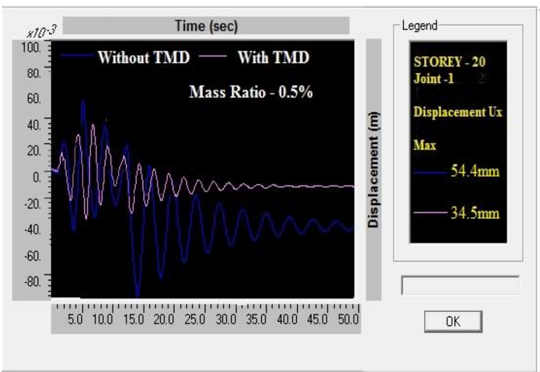


Mass ratio 5%

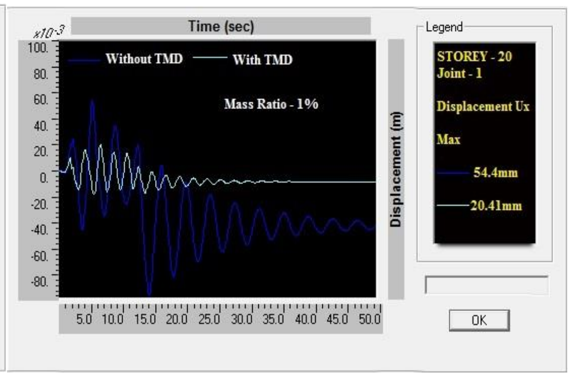


Mass ratio 10%

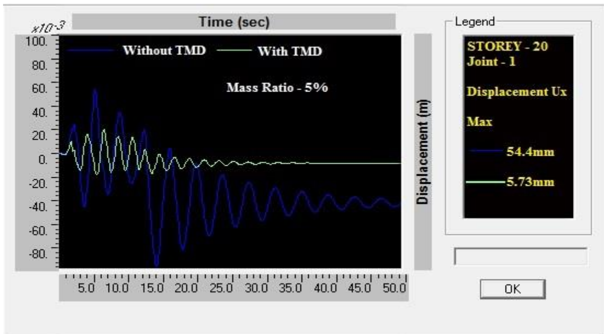
b. El-Centro Earthquake

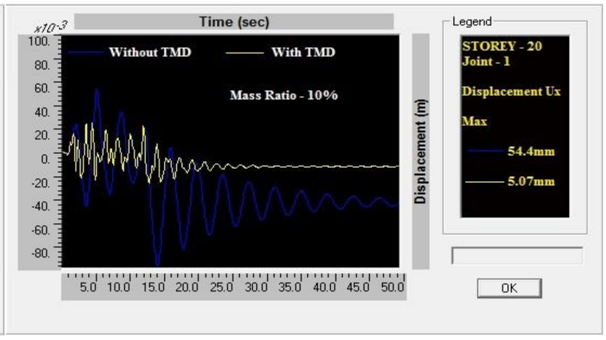


Mass ratio 0.5%



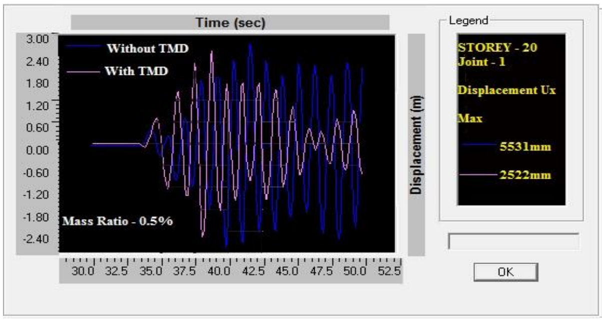
Mass ratio 1%

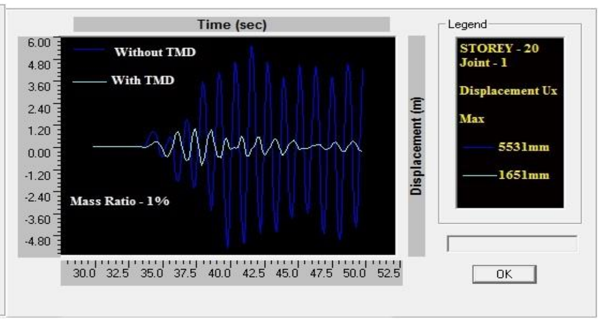




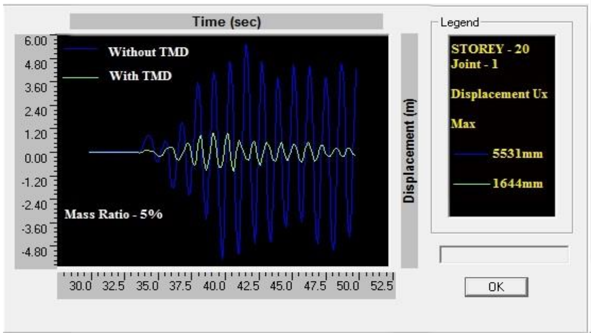
Mass ratio 5% **Mass ratio 10%**

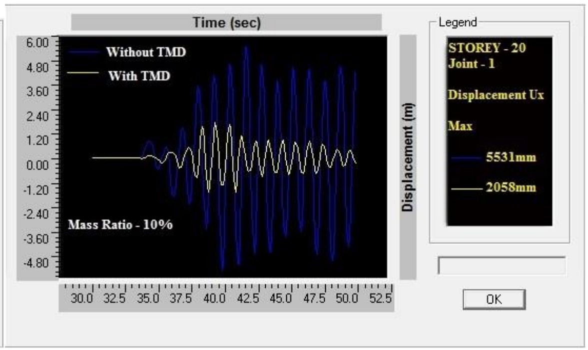
c. BHUJ Earthquake





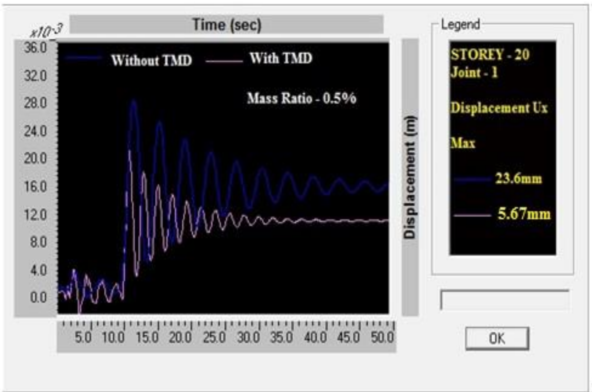
Mass ratio 0.5% **Mass ratio 1%**

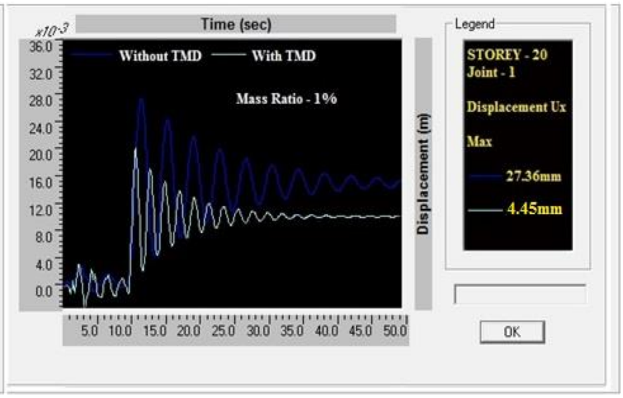




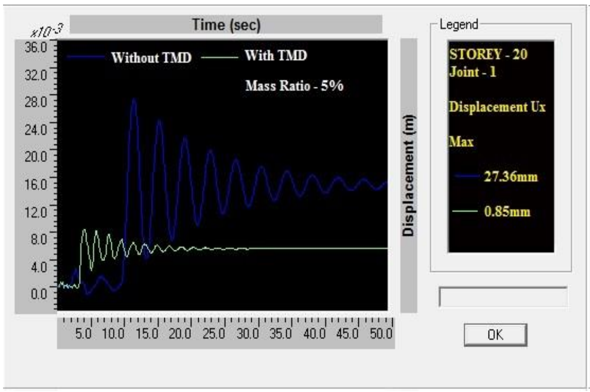
Mass ratio 5% **Mass ratio 1%**

d. KOBE Earthquake

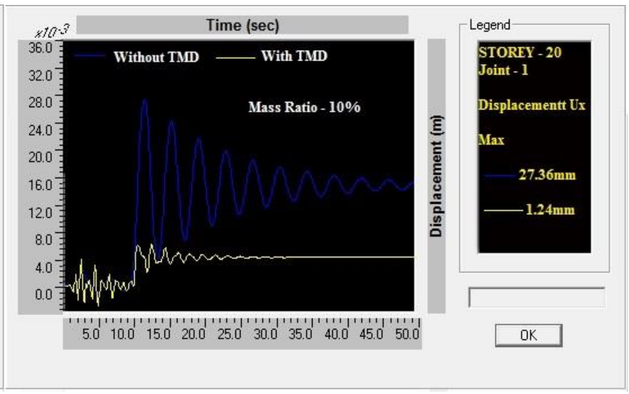




Mass ratio 0.5% **Mass ratio 1%**



Mass ratio 5%



Mass ratio 1%

Loading conditions	Displacement (mm)					% Reduction			
	Without TMD (A)	With TMD of mass ratio's				0.50%	1%	5%	10%
		0.5% (B)	1% (C)	5% (D)	10% (E)				
Sinusoidal acceleration	309.5	32.25	15.52	4.58	3.46	90%	95%	98.5%	98.9%
El-Centro Earthquake	54.4	34.5	20.41	5.73	5.07	37%	62%	89%	91%
BHUJ Earthquake	5531	2522	1651	1644	2058	54%	70%	70%	63%
KOBE earthquake	27.36	5.67	4.45	0.85	1.24	79%	84%	97%	95%

Table-1 Tabulation of Comparative study on the Maximum displacement of the frame structure

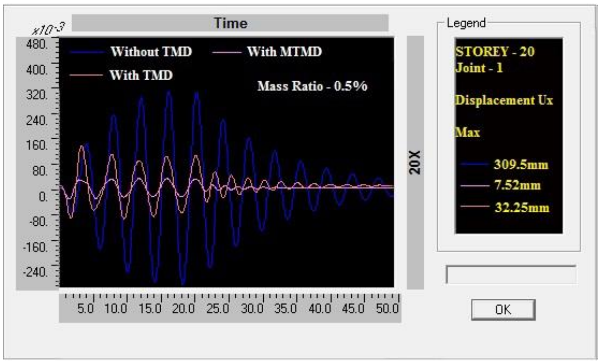
From the table-4 it can be concluded, that the response of 20 storey frame structure decreases with the increase in the TMD mass ratio. The maximum response reduction of the sinusoidal ground acceleration is much more significant compare to any other earthquake loadings, It can be seen from the above table that the displacement of the structure due to sinusoidal ground acceleration having a time period of 4.239 sec, using single TMD of 10% mass ratio approximately 99% of reduction can be achieved. This is due to the reason that the TMD can be tuned to the same frequency as the sinusoidal acceleration frequency; hence much reduction in the displacement can be achieved.

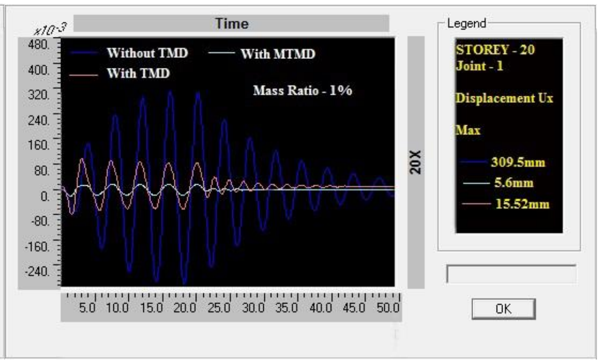
It can also be seen from the results that the response of the frame structure is reduced under different earthquake loading conditions when equipped with single TMD with the increase in mass ratio but as we can see that for the earthquake BHUJ and KOBE the displacement is increased when the mass ratio is 10% compare to 5% mass ratio i'e from 70% reduction of displacement in 5% mass ratio has reduced to 63% in 10% mass ratio. Hence from this we can conclude that the response of the structure decreases using single TMD with the increase in mass ratio but upto certain limit. The maximum percentage of response reduction takes place under Kobe earthquake for all the mass ratios.

3.1 Structure Equipped with Multiple Tuned Mass Dampers with uniform Mass Ratio

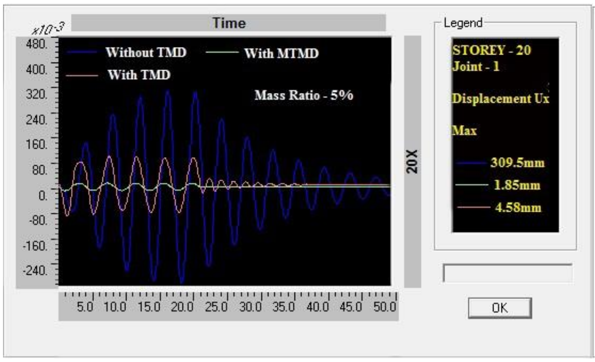
3.1.1 Comparative study on the Maximum displacement of the frame structure at the 20th floor with and without Multiple TMD with variation of mass ratio.

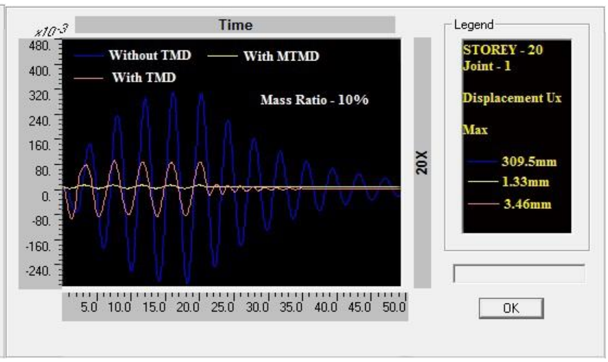
(a) Sinusoidal acceleration





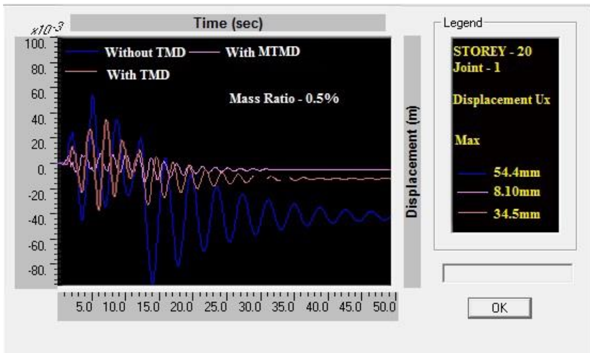
Mass ratio 0.5%Mass ratio 1%

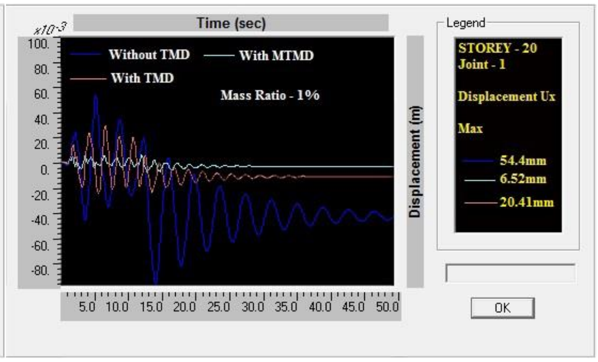




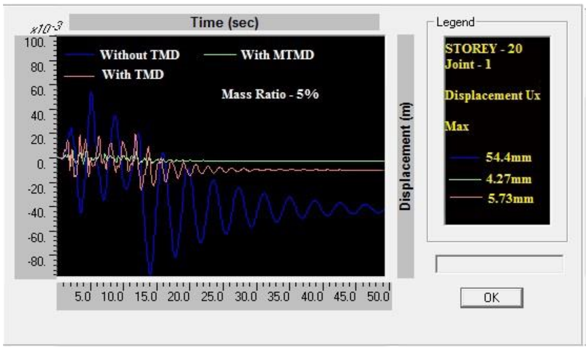
Mass ratio 5% **Mass ratio 10%**

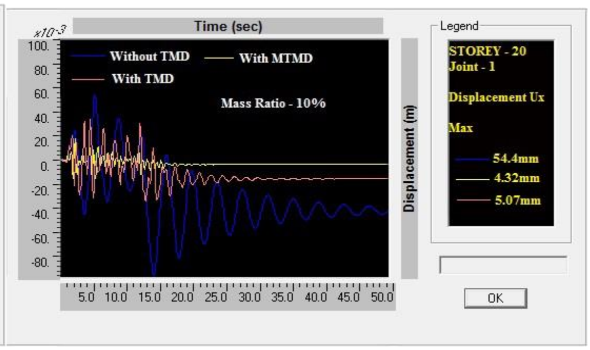
b. El-Centro Earthquake





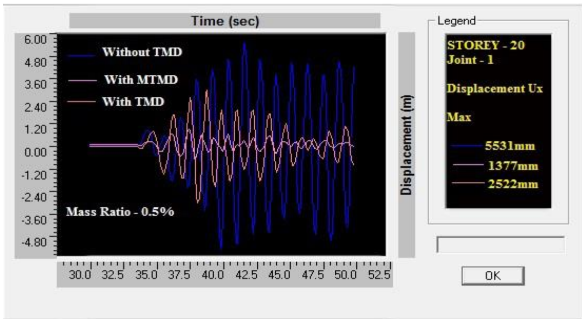
Mass ratio 0.5% **Mass ratio 1%**

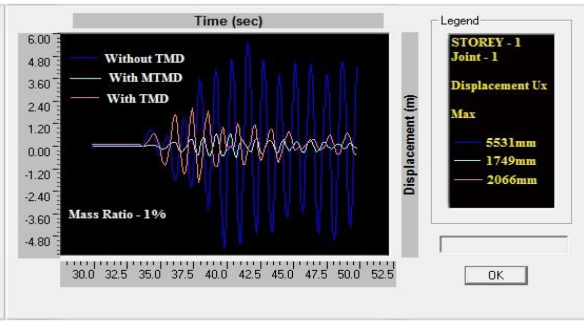




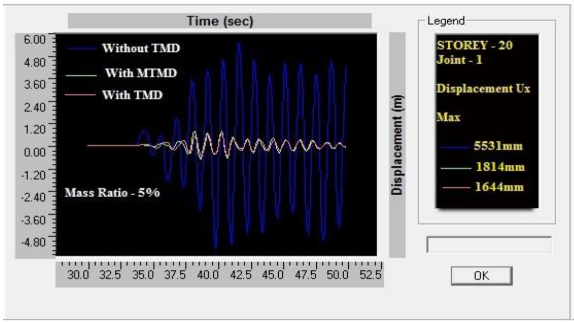
Mass ratio 0.5% **Mass ratio 1%**

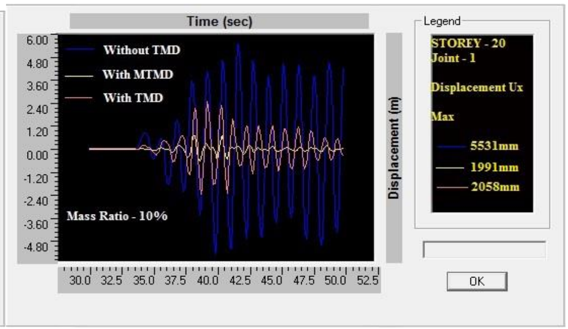
c. BHUJ Earthquake





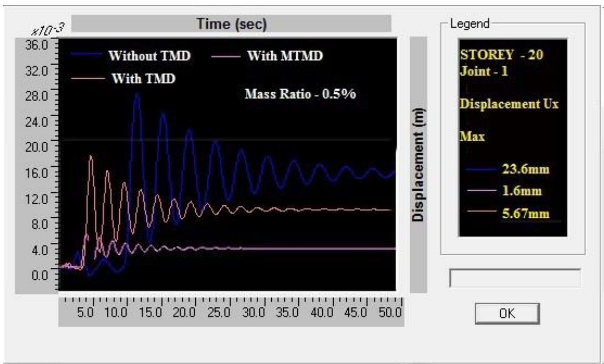
Mass ratio 0.5% **Mass ratio 1%**

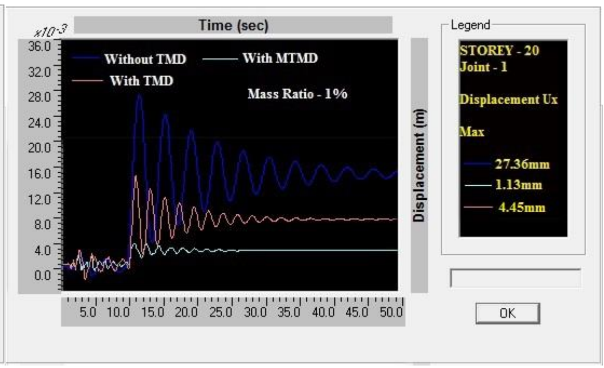




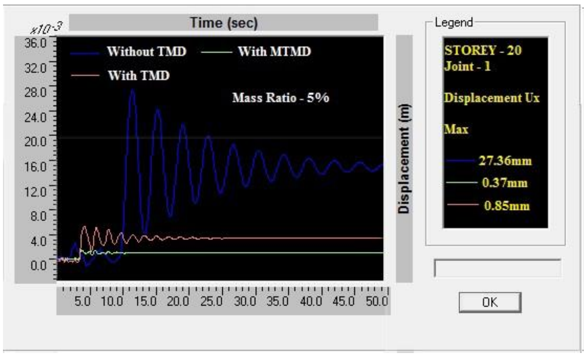
Mass ratio 5% **Mass ratio 10%**

d. KOBE earthquake

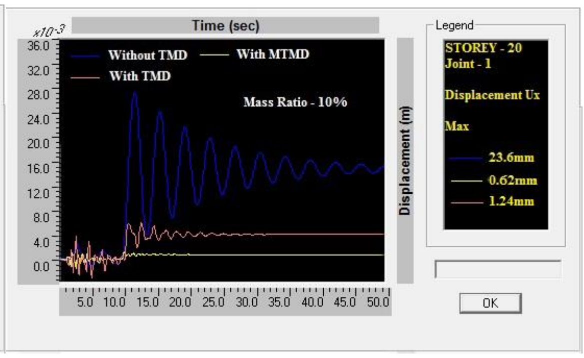




Mass ratio 0.5% **Mass ratio 1%**



Mass ratio 5%



Mass ratio 10%

Tabulation of Comparative study on the Maximum displacement of the frame structure

Loading conditions	Displacement (mm)			% Reduction		
	Without TMD (A)	With TMD of mass ratio 0.5% (B)	With MTMD of mass ratio 0.5% (C)	A & B	A & C	B & C
Sinusoidal acceleration	309.5	32.25	7.52	90%	98%	77%
El-Centro Earthquake	54.4	34.5	8.1	37%	85%	77%
BHUJ Earthquake	5531	2522	1377	54%	75%	45%
KOBE earthquake	27.36	5.67	1.6	79%	94%	72%

Table – 3.1 Mass Ratio – 0.5%

Loading conditions	Displacement (mm)			% Reduction		
	Without TMD (A)	With TMD of mass ratio 1% (B)	With MTMD of mass ratio 1% (C)	A & B	A & C	B & C
Sinusoidal acceleration	309.5	15.52	5.6	95%	98%	64%
El-Centro Earthquake	54.4	20.41	6.52	62%	88%	68%
BHUJ Earthquake	5531	2066	1749	63%	68%	15%
KOBE earthquake	27.36	4.45	1.13	84%	96%	75%

Table – 3.2 Mass Ratio – 1%

Loading conditions	Displacement (mm)			% Reduction		
	Without TMD (A)	With TMD of mass ratio 5% (B)	With MTMD of mass ratio 5% (C)	A & B	A & C	B & C
Sinusoidal acceleration	309.5	4.58	1.85	98.5%	99.4%	59.61%
El-Centro Earthquake	54.4	5.73	4.27	89%	92%	25.48%
BHUJ Earthquake	5531	1644	1814	70%	67%	-10.34%
KOBE earthquake	27.36	0.85	0.37	97%	99%	56.47%

Table – 3.3 Mass Ratio – 5%

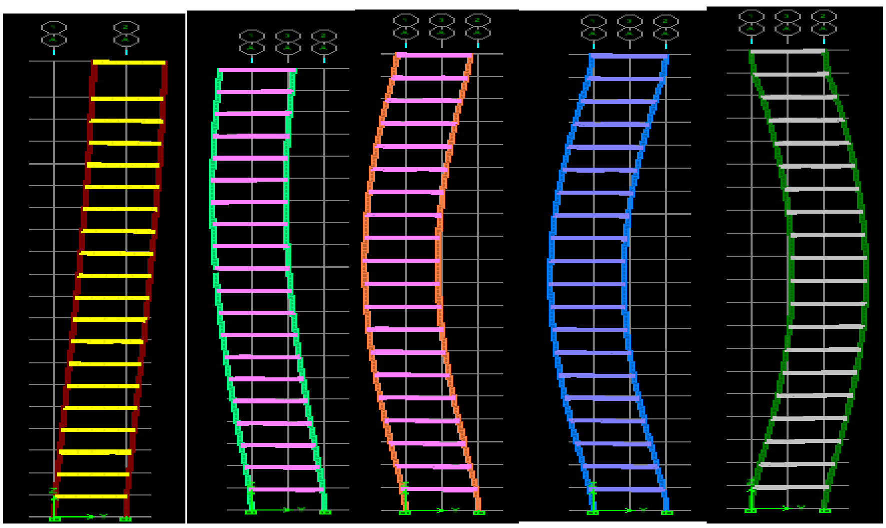
Loading conditions	Displacement (mm)			% Reduction		
	Without TMD (A)	With TMD of mass ratio 10% (B)	With MTMD of mass ratio 10% (C)	A & B	A & C	B & C
Sinusoidal acceleration	309.5	3.46	1.33	99%	100%	62%
El-Centro Earthquake	54.4	5.07	4.32	91%	92%	15%
BHUJ Earthquake	5531	2058	1991	63%	64%	3%
KOBE earthquake	27.36	1.24	0.62	95%	98%	50%

Table 3.4 Mass Ratio – 10%

From the tables 3.1, 3.2, 3.3 & 3.4it can be concluded that when multiple tuned mass dampers with uniform mass ratio are used to control the vibration in the structure instead of only single tuned mass damper, it can be observed that the effectiveness of multiple are much more effective in controlling the vibration than a single TMD of same mass ratio. The reduction in response goes on increasing with the increase in the mass ratio of the TMD with the total weight of the structure but upto certain limit after which increase in mass ratio is not much effective. As we can when used multiple tuned mass damper of mass ratio 0.5% the displacement has reduced to larger extent when compare to other mass ratio. But for certain loading conditions like Bhuj earthquake when the mass ratio increased to 5 to 10% it is not much effective when compared to 0.5 to 1% mass ratio.

3.1.2 Comparison of Mode shapes with and without Multiple Tuned mass damper of uniform mass ratio

As we have seen from the above comparison table of displacement obtained from the time history analysis that with the addition of MTMD of uniform mass ratio in the structure the displacement has been reduced with the reduction in the time period.

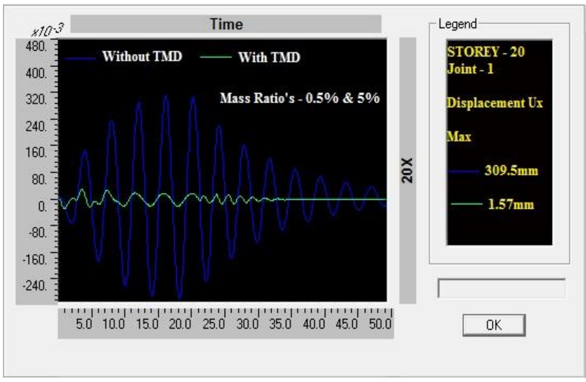


Without TMD T = 4.293 S	With TMD 0.5% MR T = 2.12 S	With TMD 1% MR T = 2.093	With TMD 5% MR T = 1.903 S	With TMD 10% MR T = 1.915 S

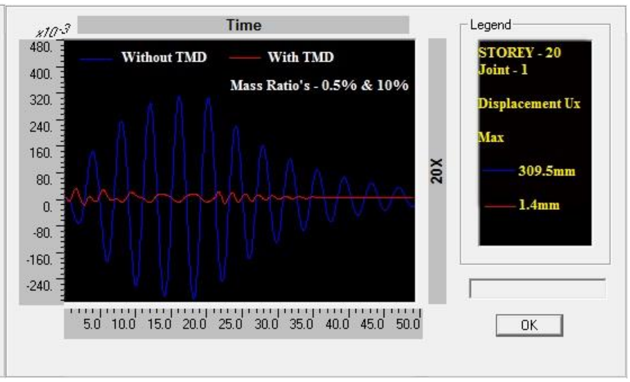
4.1 STRUCTURE EQUIPPED WITH MULTIPLE TUNED MASS DAMPERS WITH NON-UNIFORM MASS RATIO

4.1.1 Comparative study on the Maximum displacement of the frame structure at the 20th floor with and without Multiple TMD with variation of mass ratio.

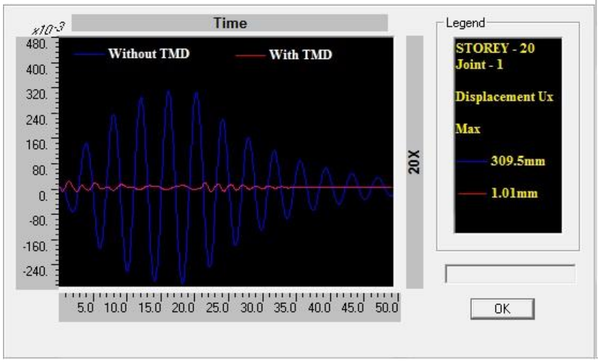
a. Sinusoidal ground acceleration.



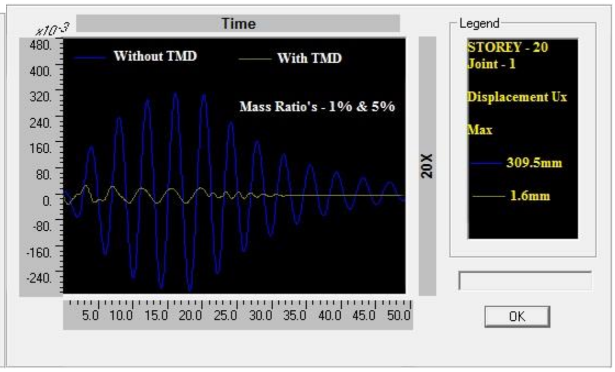
Mass ratio of 0.5% & 5%



Mass ratio of 0.5% & 10%

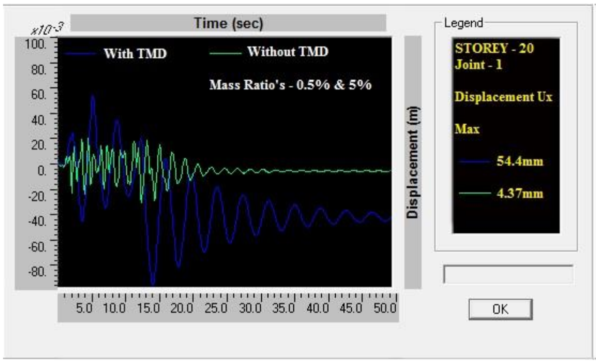


Mass ratio of 1% & 10%

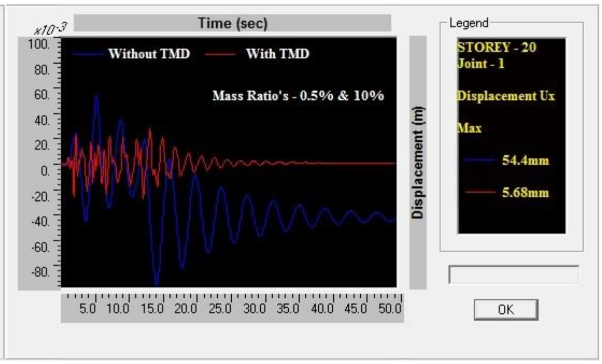


Mass ratio of 1% & 5%

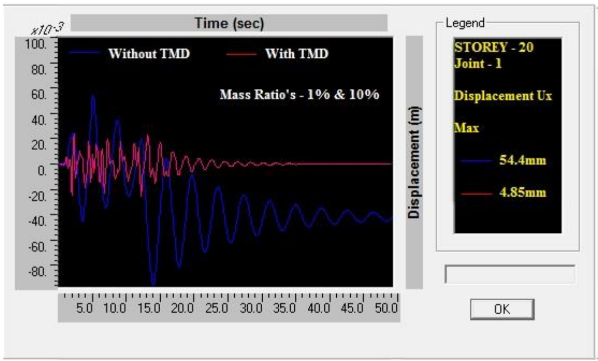
b. El-Centro Earthquake



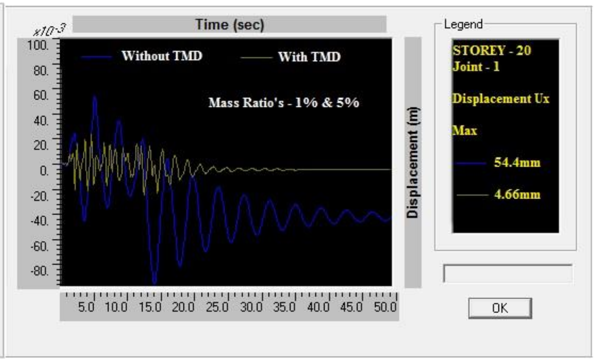
Mass ratio of 0.5% & 5%



Mass ratio of 0.5% & 10%

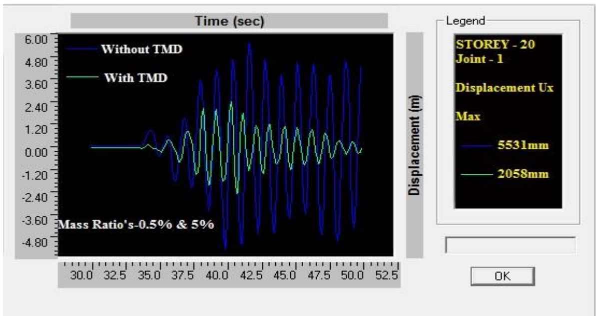


Mass ratio of 1% & 10%

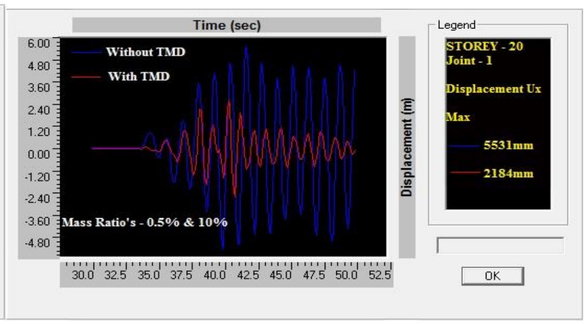


Mass ratio of 1% & 5%

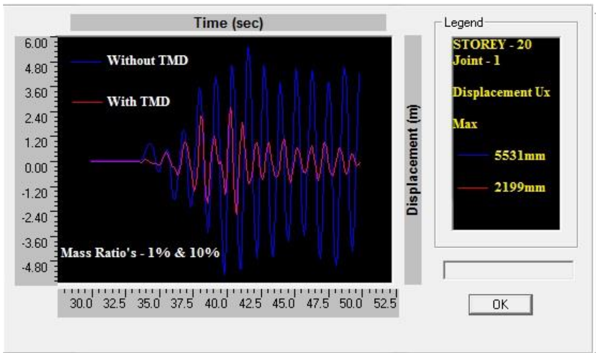
C. BHUJ Earthquake



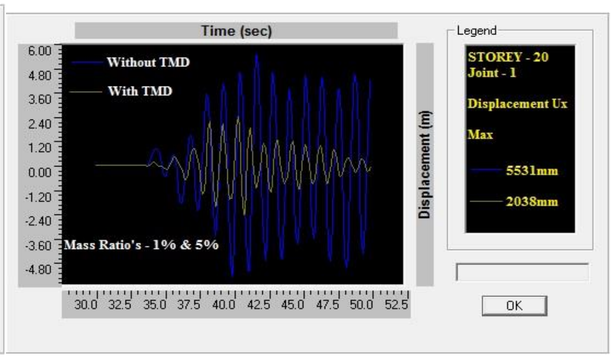
Mass ratio of 0.5% & 5%



Mass ratio of 0.5% & 10%

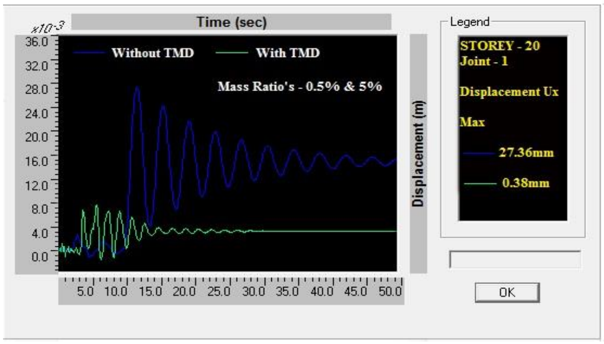


Mass ratio of 1% & 10%

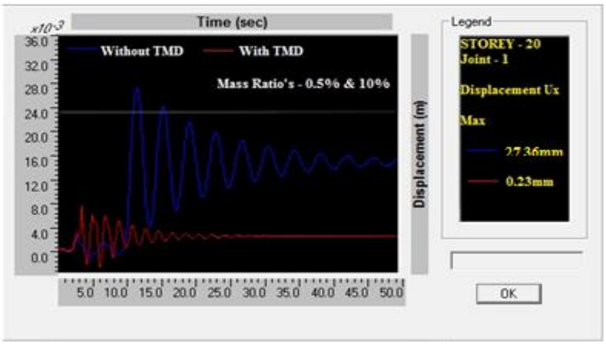


Mass ratio of 1% & 5%

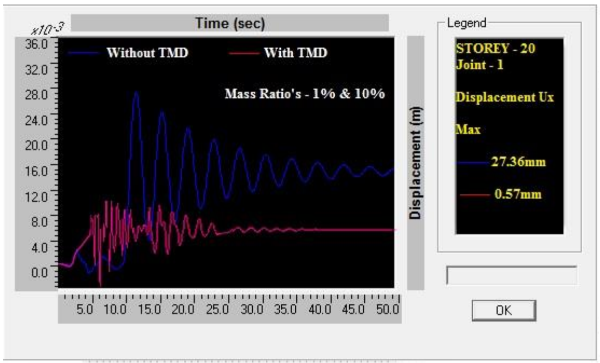
D. KOBE Earthquake



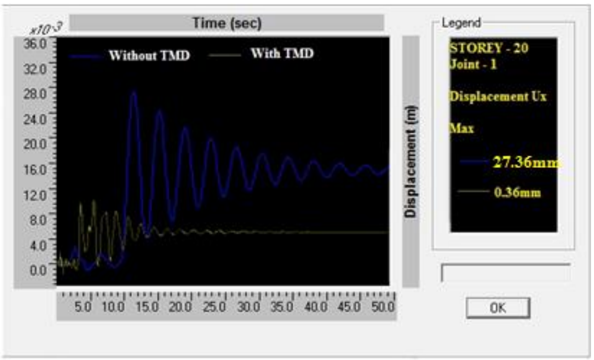
Mass ratio of 0.5% & 5%



Mass ratio of 0.5% & 10%



Mass ratio of 1% & 10%



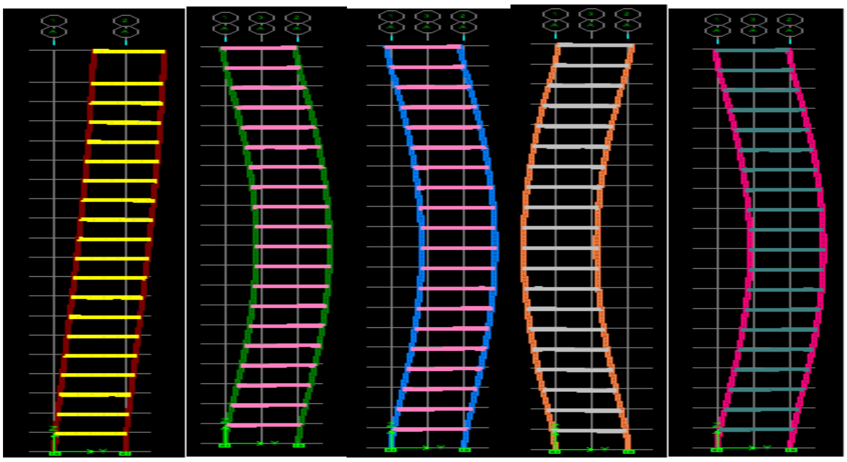
Mass ratio of 1% & 5%

Loading conditions	Displacement (mm)								
	Without TMD (A)	With MTMD of uniform mass ratio				With MTMD of Non- uniform mass ratio			
		0.5%	1%	5%	10%	0.5% & 5%	0.5% & 10%	1% & 10%	1% & 5%
Sinusoidal acceleration	309.5	32.25	5.6	1.85	1.33	1.57	1.4	1.01	1.6
El-Centro Earthquake	54.4	34.5	6.52	4.27	4.32	4.37	5.68	4.85	4.66
BHUJ Earthquake	5531	2522	1749	1814	1991	2085	2184	2199	2038
KOBE earthquake	27.36	5.67	1.13	0.37	0.62	0.38	0.23	0.57	0.36

Fig – 4.1 Tabulation of Comparative study on the Maximum displacement of the frame structure

4.1.2Comparison of Mode shapes with and without Multiple Tuned mass damper

As we have seen from the above comparison table of displacement obtained from the time history analysis that with the addition of MTMD of Non-uniform mass ratio in the structure the displacement has been reduced with the reduction in the time period.



Without TMD T = 4.293 S	With TMD 0.5% & 5% T = 1.930 S	With TMD 0.5% & 10% T = 1.935 S	With TMD 1% & 10% T = 1.926 S	With TMD 1% & 5% T = 1.920 S

The Study made in the above experiments in determining the effectiveness of TMD to control the induced structural vibrations due to sinusoidal ground acceleration and random earthquake loadings. The frame is analyzed using ETABS structural analysis software. Non-Linear time history analysis of the frame has been carried out and the maximum response of the frame subjected to lateral loadings has been obtained and compared the frame response with effect of STMD with variation in mass ratios, MTMD with uniform mass ratios andMTMD with non-uniform variation of mass ratio.

From experimental study carried it can be concluded that.

1) Response of the frame Structure in terms of displacement reduces with the increase in the mass ratio of the single TMD for the applied Sinusoidal ground acceleration and El-Centro earthquake but when the mass ratio increases to 10% there is a slight increase in displacement compared to mass ratio of 5%.
2) From the study it has been found that Single TMD with mass ratio 0f 5% has turned out to be the most effective in controlling the vibration of the structure.
3) From the study it has been found that STMD is much more effective in controlling the structural vibrations when subjected to sinusoidal ground acceleration.
4) When used MTMD with non-uniform distribution of mass ratio it has been observed that it is more effective than single TMD for same mass ratio.
5) It has been observed that when a frame structure is equipped with MTMD with uniform distribution of mass ratio is more effective in controlling the vibrations of the structure compared to STMD as multiple dampers can attenuate different modes.

6) From the comparative study of frame equipped with MTMD with uniform mass ratio and MTMD with non-uniform mass ration it can be seen that MTMD with Non-uniform mass ratio is more efficient than MTMD with uniform mass ratio but it cannot be economical.
7) The response of the frame building equipped with STMD or MTMD's has no effect on the variation of damping ratio of the damper.

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