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Convolution Structures of 2DFMT & QFMT

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Abstract: Generalization of integral transform from real and complex numbers to quaternion algebra. Quaternion is the generalization of a complex number. A complex number has two components: the real and the imaginary part. However, the quaternion has four components i.e. one real part and three imaginary parts. It transforms a real 2D signal into a quaternion-valued frequency domain signal. The four quaternion components separate four cases of symmetry in real signals instead of only two in the complex integral transform.

In this paper, we have proved convolution structure of generalized two dimensional Fourier-Mellin transform. Also we have defined the Quaternion Fourier-Mellin Transform and proved its convolution structure. Here, we have generalized the two Dimensional Fourier-Mellin transform into Quaternion Fourier-Mellin transform using quaternion algebra.

Key Words: Two Dimensional Fourier-Mellin Transform(2DFMT), Quaternion Fourier-Mellin transform (QFMT), Fourier Transform, Mellin Transform.

Introduction

Fourier Transformation play a vital role in diverse areas of science and technology such as electric analysis, communication engineering, control engineering, linear system, analysis, statistics, optics, quantum physics, solution of partial differential operation etc. and also applicable to electric circuits, signal design and solution to related problems [6]. Mellin transform is used in place of Fourier's transform when scale invariance is more relevant than shift invariance then Mellin's transform suggests new formal treatments. The Mellin transform is applied in signal analysis and imaging techniques [2]. The Mellin transform is applicable in the area of random variables, Probability and statistics. It is also used in computer science for analysis of algorithms, analytic number theory, theory of functions, number theory and partial differential equations [3].

In the present work we have proved convolution structure of two dimensional Fourier-Mellin transform in conventional way and the two dimensional Distributional Fourier-Mellin Transform with parameters s, u, p, v of $f(t, l, x, y)$ is given by-

$$FM\{f(t, l, x, y)\} = F(s, u, p, v)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(t, l, x, y)\ e^{-i(st+ul)}\ x^{p-1}y^{v-1}\ dt\ dl\ dx\ dy$$

Recently it has become popular to generalize the Integral transform from real and complex numbers to quaternion algebra. It transforms a real two-dimensional signal into a quaternion valued frequency domain signal. The four quaternion components separate four cases of symmetry in real signals instead of only two in the complex integral transform [1].

Preliminaries

The concept of the quaternion was introduced by Sir. William Hamilton in 1843. It is the generalization of a complex number. A complex number has two components: the real and the imaginary part. However, the quaternion has four components i.e. one real part and three imaginary parts and can be represented in Cartesian form as by [5,8,12]:

$$\mathbb{H} = \{q/w + ix + jy + kz,\ \ w, x, y, z \in R\}$$

Where w, x, y and z are real numbers and $i, j$and k are complex operators which obey the following rules;
$ij = -ji = k, jk = -kj = i, ki = -ik = j$
From these rules, it is clear that multiplication is not commutative. The quaternion conjugate is,
$q = w - ix - jy - kz$
and the modulus of a quaternion is given by,
$|q| = \sqrt{q\bar{q}} = \sqrt{w^2 + x^2 + y^2 + z^2}$
A quaternion with zero real part is called a pure quaternion and a quaternion with unit modulus is called a unit quaternion.
The imaginary part of a quaternion has three components and may be associated with a 3-space vector. For this reason, it is sometimes useful to consider the quaternion as composed of a vector part and a scalar part .Thus q can be expressed as;
$q = s(q) + V(q)$
Where, $s(q)$ is the real or scalar part i.e $s(q) = w$ and $V(q)$ is the vector part which is a composition of three imaginary components $V(q) = xi + yj + zk$.
Bas, Le Bihan and Chassery [7] used the QFT to design a digital color image watermarking scheme. Bayro et al. [4] applied the QFT in image pre-processing and neural computing techniques for speech recognition. The quaternion Fourier transform (QFT) plays a vital role in the representation of signals [1]. Recent work of quaternion and after studying it, we have motivated and generalized two Dimensional Fourier-Mellin transform to Quaternion Fourier-Mellin transform using the quaternion algebra.
In the present paper, section 1 define Quaternion Fourie-Mellin Transform. Some properties of quaternion Fourier-Mellin Transform is proved in section 2. Lastly we have concluded the paper.

1. Convolution Theorem of Two Dimensional Fourier-Mellin Transform

1.1 Definition of Convolution of 2DFMT

For any function $f(t,l,x,y)$, let the functions $\tilde{f}(t,l,x,y) = f(t,l,x,y)e^{-i(st+ul)}x^{p-1}y^{v-1}$. Now the convolution of two functions $f(t,l,x,y)$ and $g(t,l,x,y)$ denoted by $f * g$ is defined as,

$$h(t,l,x,y) = (f * g)(t,l,x,y)$$
$$= e^{i(st+ul)}x^{-p+1}y^{-v+1}(\tilde{f} * \tilde{g})(t,l,x,y) \qquad (1.1.1)$$

where $' * '$ is the usual convolution operation.
Also $(\tilde{f} * \tilde{g})(t,l,x,y) =$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \frac{1}{cm}\tilde{f}(a,b,c,m)\tilde{g}\left(t-a, l-b, \frac{x}{c}, \frac{y}{m}\right) da\, db\, dc\, dm \qquad \text{\{by theorem\}}$$

1.2 Theorem

Let $h(t,l,x,y) = (f * g)(t,l,x,y)$ and $F(s,u,p,v), G(s,u,p,v)$and $H(s,u,p,v)$ denote the two dimensional Fourier-Mellin transform of f, g and h respectively, then $H(s,u,p,v) = F(s,u,p,v)G(s,u,p,v)$ (1.2.1)

Proof:

$$H(s,u,p,v) = FM\{h(t,l,xy)\}(s,u,p,v)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} h(t,l,xy)\; e^{-i(st+ul)}x^{p-1}\, y^{v-1}\, dt\, dl\, dx\, dy$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} (f * g)(t,l,x,y)\; e^{-i(st+ul)}x^{p-1}\, y^{v-1}\, dt\, dl\, dx\, dy$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} e^{i(st+ul)}x^{-p+1}y^{-v+1}(\tilde{f} * \tilde{g})(t,l,x,y)\, e^{-i(st+ul)}x^{p-1}y^{v-1}dtdldxdy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} (\tilde{f} * \tilde{g})(t,l,x,y)\ dt\ dl\ dx\ dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \frac{1}{cm}\ \tilde{f}(a,b,c,m)\tilde{g}\left(t-a, l-b, \frac{x}{c}, \frac{y}{m}\right) da\ db\ dc\ dm\right]$$

$$dtdldxdy \qquad \{\text{by } (1.1.1)\}$$

Replacing, $t - a = \xi\ \therefore a = t - \xi, l - b = \eta\ \therefore b = l - \eta,$
$\frac{x}{c} = \sigma'\ \therefore c = \frac{x}{\sigma'}, \frac{y}{m} = \rho'\ \therefore m = \frac{y}{\rho'}$
and
$da = -d\xi, db = -d\eta, dc = \frac{-x}{(\sigma')^2}d\sigma', dm = \frac{-y}{(\rho')^2}\ d\rho'$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} dt\ dl\ dx\ dy \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \frac{1}{\frac{x}{\sigma'}\frac{y}{\rho'}}\ \tilde{f}\left(t-\xi, l-\eta, \frac{x}{\sigma'}, \frac{y}{\rho'}\right)\right.$$

$$\left.\tilde{g}(\xi,\eta,\sigma',\rho')(-d\xi)(-d\eta)\left(\frac{-x}{(\sigma')^2}d\sigma'\right)\left(\frac{-y}{(\rho')^2}\ d\rho'\right)\right]$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \left[\int_{0}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \frac{1}{\sigma'\rho'}\tilde{f}\left(t-\xi, l-\eta, \frac{x}{\sigma'}, \frac{y}{\rho'}\right)\tilde{g}(\xi,\eta,\sigma',\rho')\right.$$

$$\left. d\xi\ d\eta\ d\sigma' d\rho'\right] dt\ dl\ dx\ dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} dt\ dl\ dx\ dy \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \frac{1}{\sigma'\rho'} f\left(t-\xi, l-\eta, \frac{x}{\sigma'}, \frac{y}{\rho'}\right) e^{-is(t-\xi)}e^{-iu(l-\eta)}\right.$$

$$\left(\frac{x}{\sigma'}\right)^{p-1}\left(\frac{y}{\rho'}\right)^{v-1} g(\xi,\eta,\sigma',\rho')e^{-is\xi}e^{-iu\eta}(\sigma')^{p-1}(\rho')^{v-1}\ d\xi\ d\eta\ d\sigma'\ d\rho' \qquad (1.2.2)$$

put $t - \xi = w', l - \eta = q, \frac{x}{\sigma'} = n, \frac{y}{\rho'} = r$
$\therefore t = \xi + w', l = \eta + q, x = n\sigma', y = \rho' r$
and $dt = dw',\ dl = dq,\ dx = \sigma' dn,\ dy = \rho' dr$
(1.2.2),

$$H(s,u,p,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(w',q,n,r)e^{-isw'}e^{-iuq}n^{p-1}r^{v-1}dw'\ dq\ dn\ dr$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} g(\xi,\eta,\sigma',\rho')e^{-is\xi}e^{-iu\eta}(\sigma')^{p-1}\ (\rho')^{v-1}\ d\xi\ d\eta\ d\sigma'\ d\rho'$$

$= FM\{f(w',q,n,r)\}(s,u,p,v)\ FM\{g(\xi,\eta,\sigma',\rho')\}(s,u,p,v)$
$H(s,u,p,v) = F(s,u,p,v)\ G(s,u,p,v)$
Hence the theorem

2. New Convolution Structure for Quaternion Fourier-Mellin Transform

2.1 Definition

For any quaternion signal $f(t,l,x,y)$ is given by
$f(t,l,x,y) = f_r(t,l,x,y) + if_i(t,l,x,y) + jf_j(t,l,x,y) + kf_k(t,l,x,y)$
where $f_r(t,l,x,y), f_i(t,l,x,y), f_j(t,l,x,y)\ \&\ f_k(t,l,x,y)$ are real, the Quaternion Fourier-Mellin transform of $f(t,l,x,y)$ is denoted by given by-
$F^{i,j,k,l}(s,u,p,v) = FM^{i,j,k,l}\{f(t,l,x,y)\}(s,u,p,v)$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} K^{i,k}(t,x,s,p)\ f(t,l,x,y)\ K^{j,l}(l,y,u,v)\ dt\ dl\ dx\ dy \qquad (2.1.1)$$

where, $K^{i,k}(t,x,s,p) = e^{-ist}x^{(\rho-1)+iw}$, where $p = \rho + iw$
and $K^{j,l}(l,y,u,v) = e^{-iul}\, y^{(\sigma-1)+iz}$, where $v = \sigma + iz$.

2.2 Definition

Here we have defined some new definitions on the basis of 7.3.1,

$$\tilde{f}\left(t-\xi, l-\eta, \frac{x}{\sigma'}, \frac{y}{\rho'}\right) = f\left(t-\xi, l-\eta, \frac{x}{\sigma'}, \frac{y}{\rho'}\right) e^{-is(t-\xi)} e^{-iu(l-\eta)} \left(\frac{x}{\sigma'}\right)^{p-1} \left(\frac{y}{\rho'}\right)^{v-1} \quad (2.2.1)$$

$$\tilde{g}(\xi,\eta,\sigma',\rho') = g(\xi,\eta,\sigma',\rho') e^{-is\xi} e^{-iu\eta} (\sigma')^{p-1} (\rho')^{v-1} \quad (2.2.2)$$

2.3 Theorem

For any real, scalar or complex signal $f(t,l,x,y)$ and convolution kernel $g(t,l,x,y)$

$$h(t,l,x,y) = (f * g)(t,l,x,y)$$
$$\triangleq e^{i(st+ul)} x^{(\rho-1)-iw} y^{(\sigma-1)-iz} \left\{ e^{-i(st+ul)} x^{(\rho-1)+iw} y^{(\sigma-1)+iz} f(t,l,x,y) * g(t,l,x,y) e^{-i(st+ul)} x^{(\rho-1)+iw} y^{(\sigma-1)+iz} \right\}$$

where, $'*'$ is the Fourier-Mellin Transform convolution operator, then

$$FM^{i,j,k,l}\{h(t,l,x,y)\}(s,u,p,v) = FM^{i,j,k,l}\{f(t,l,x,y)\}(s,u,p,v) FM^{i,j,k,l}\{g(t,l,x,y)\}(s,u,p,v)$$

Proof: We have,

$$FM^{i,j,k,l}\{h(t,l,x,y)\}(s,u,p,v)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} h(t,l,x,y)\ K^{i,j,k,l}(t,l,x,y,s,u,p,v)\, dt\, dl\, dx\, dy$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} h(t,l,x,y)\ e^{-i(st+ul)} x^{(\rho-1)+iw}\ y^{(\sigma-1)+iz}\, dt\, dl\, dx\, dy$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} e^{-i(st+ul)} x^{(\rho-1)+iw} y^{(\sigma-1)+iz} \left[e^{i(st+ul)} x^{(\rho-1)-iw} y^{(\sigma-1)-iz} \left\{ e^{-i(st+ul)} x^{(\rho-1)+iw} y^{(\sigma-1)+iz} f(t,l,x,y) * g(t,l,x,y) e^{-i(st+ul)} x^{(\rho-1)+iw}\ y^{(\sigma-1)+iz} \right\} \right. dt\, dl\, dx\, dy$$

by using convolution theorem for two dimensional Fourier-Mellin Transform, we have

$$FM^{i,j,k,l}\{h(t,l,x,y)\}(s,u,p,v)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} dt\, dl\, dx\, dy \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \frac{1}{cm} \tilde{f}(a,b,c,m) \tilde{g}\left(t-a, l-b, \frac{x}{c}, \frac{y}{m}\right) da\, db\, dc\, dm \quad \text{(by 1.1)} \quad (2.3.1)$$

putting $t - a = \xi \ \therefore a = t - \xi,\ l - b = \eta \ \therefore b = l - \eta$

$\frac{x}{c} = \sigma' \ \therefore c = \frac{x}{\sigma'}, \quad \frac{y}{m} = \rho' \ \therefore m = \frac{y}{\rho'}$

and $da = -d\xi, db = -d\eta, dc = \frac{-x}{(\sigma')^2} d\sigma', dm = \frac{-y}{(\rho')^2}\, d\rho'$

(1.4.1), (1.4.2) gives,

$$FM^{i,j,k,l}\{h(t,l,x,y)\}(s,u,p,v)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} dt\, dl\, dx\, dy \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \frac{1}{\left(\frac{x}{\sigma'}\right)\left(\frac{y}{\rho'}\right)} \tilde{f}\left(t-\xi, l-\eta, \frac{x}{\sigma'}, \frac{y}{\rho'}\right) \tilde{g}(\xi,\eta,\sigma',\rho')(-d\xi)\,(-d\eta) \left(\frac{-x}{(\sigma')^2} d\sigma'\right) \left(\frac{-y}{(\rho')^2}\, d\rho'\right)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} dt\, dl\, dx\, dy \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \frac{1}{\sigma'\rho'} \tilde{f}\left(t-\xi, l-\eta, \frac{x}{\sigma'}, \frac{y}{\rho'}\right) \tilde{g}(\xi,\eta,\sigma',\rho')$$

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$$d\xi\, d\eta\, d\sigma' d\rho'$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} dt\, dl\, dx\, dy \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \frac{1}{\sigma'\rho'} f\left(t-\xi, l-\eta, \frac{x}{\sigma'}, \frac{y}{\rho'}\right) e^{-is(t-\xi)}$$
$$e^{-iu(l-\eta)} \left(\frac{x}{\sigma'}\right)^{(\rho-1)+iw} \left(\frac{y}{\rho'}\right)^{(\sigma-1)+iz} g(\xi,\eta,\sigma',\rho') e^{-is\xi} e^{-iu\eta}$$
$$(\sigma')^{(\rho-1)+iw} (\rho')^{(\sigma-1)+iz} d\xi\, d\eta\, d\sigma'\, d\rho' \tag{2.3.2}$$

put, $t-\xi = w'$, $\quad l-\eta = q$, $\quad \frac{x}{\sigma'} = n$, $\quad \frac{y}{\rho'} = r$
we have, $t = \xi + w'$, $\quad l = \eta + q, x = n\sigma', y = \rho' r$
and $dt = dw', dl = dq, dx = \sigma' dn, dy = \rho' dr$
(7.4.3) gives,
$FM^{i,j,k,l}\{h(t,l,x,y)\}(s,u,p,v)$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(w',q,n,r) e^{-isw'} e^{-iuq} n^{(\rho-1)+iw}\, r^{(\sigma-1)+iz}\, dw' dq\, dn\, dr$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} g(\xi,\eta,\sigma',\rho') e^{-is\xi} e^{-iu\eta} (\sigma')^{(\rho-1)+iw} (\rho')^{(\sigma-1)+iz} d\xi\, d\eta\, d\sigma'\, d\rho' \tag{2.3.3}$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} K^{i,k}(w',n,s,w)\, f(w',q,n,r)\, K^{j,l}(q,r,u,z) dw' dq\, dn\, dr$$
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} K^{i,k}(\xi,\sigma',s,w) g(\xi,\eta,\sigma',\rho')\, K^{j,l}(\eta,\rho',u,z)\, d\xi\, d\eta\, d\sigma'\, d\rho'$$

Therefore,
$FM^{i,j,k,l}\{h(t,l,x,y)\}(s,u,p,v)$
$$= FM^{i,j,k,l}\{f(w',q,n,r)\} FM^{i,j,k,l}\{g(\xi,\eta,\sigma',\rho')\}$$
By using change of variable property
$FM^{i,j,k,l}\{h(t,l,x,y)\}(s,u,p,v)$
$$= FM^{i,j,k,l}\{f(t,l,x,y)\}(s,u,p,v) FM^{i,j,k,l}\{g(t,l,x,y)\}(s,u,p,v) \tag{2.3.4}$$
Hence the theorem.

Conclusion

In the present work we have proved convolution structures for generalized two dimensional Fourier-Mellin transform(2DFMT) and for Quaternion Fourier-Mellin transform.

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