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# Sum Square Prime Labeling for Some Path Related Graphs

**Sunoj B S[1],**
Assistant Professor,
Department of Mathematics, Government Polytechnic College, Attingal, Kerala, India

**Mathew Varkey T K[2],**
Assistant Professor,
Department of Mathematics, T K M College of Engineering, Kollam, Kerala, India

## Abstract

Sum square prime labeling of a graph is the labeling of the vertices with 0,1,2---------,p-1 and the edges with square of the sum of the labels of the incident vertices. The greatest common incidence number of a vertex (***gcin***) of degree greater than one is defined as the greatest common divisor(gcd) of the labels of the incident edges. If the gcin of each vertex of degree greater than one is one, then the graph admits sum square prime labeling. Here we identify some path related graphs for sum square prime labeling.

***Keywords:*** Graph labeling, greatest common incidence number, prime labeling, path.

## 1. INTRODUCTION

All graphs in this paper are simple, connected, finite and undirected. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [1],[2] and [3]. Some basic concepts are taken from [1] and [4]. In this paper we investigated sum square prime labeling of some path related graphs.

**Definition: 1.1** Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

## 2. MAIN RESULTS

**Definition 2.1** Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection f : V(G) $\rightarrow$ {0,1,2,3,----------------,p-1} by $f(v_i) = i-1$ , for every i from 1 to p and define a 1-1 mapping $f_{ssp}^*$ : E(G) $\rightarrow$ set of natural numbers N by $f_{ssp}^*(uv)$ = $\{f(u) + f(v)\}^2$ . The induced function $f_{ssp}^*$ is said to be a sum square prime labeling, if for each vertex of degree at least 2, the ***gcin*** of the labels of the incident edges is 1.

**Definition 2.2** A graph which admits sum square prime labeling is called a sum square prime graph.

**Theorem 2.1** Path $P_n$ admits sum square prime labeling.

Proof: Let G = $P_n$ and let $v_1,v_2$,----------------,$v_n$ are the vertices of G

Here |V(G)| = n and |E(G)| = n-1

Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------},n-1\}$ by

$f(v_i) = i-1$ , $i = 1,2,\text{------},n$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{ssp}$ is defined as follows

$f^*_{ssp}(v_i \ v_{i+1}) = (2i-1)^2,$ $i = 1,2,\text{------------},n-1$

Clearly $f^*_{ssp}$ is an injection.

***gcin*** of $(v_{i+1})$ $= \text{gcd of } \{f^*_{ssp}(v_i \ v_{i+1}),\ f^*_{ssp}(v_{i+1} \ v_{i+2})\}$

$= \text{gcd of } \{(2i-1)^2, (2i+1)^2\}$

$= \text{gcd of } \{2i-1, 2i+1\} = 1,$ $i = 1,2,\text{------------},n-2$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $P_n$ , admits sum square prime labeling. ■

**Theorem 2.2** $(P_n)^2$ admits sum square prime labeling.

Proof: Let G = $(P_n)^2$ and let $v_1,v_2,\text{----------------},v_n$ are the vertices of G

Here $|V(G)| = n$ and $|E(G)| = 2n-3$

Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------},n-1\}$ by

$f(v_i) = i-1$ , $i = 1,2,\text{------},n$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{ssp}$ is defined as follows

$f^*_{ssp}(v_i \ v_{i+1}) = (2i-1)^2,$ $i = 1,2,\text{------------},n-1$

$f^*_{ssp}(v_i \ v_{i+2}) = (2i)^2,$ $i = 1,2,\text{------------},n-2$

Clearly $f^*_{ssp}$ is an injection.

***gcin*** of $(v_{i+1})$ $= 1,$ $i = 1,2,\text{------------},n-2$

***gcin*** of $(v_1)$ $= \text{gcd of } \{f^*_{ssp}(v_1 \ v_2),\ f^*_{ssp}(v_1 \ v_3)\}$

$= \text{gcd of } \{1, 4\}$

***gcin*** of $(v_n)$ $= \text{gcd of } \{f^*_{ssp}(v_n \ v_{n-1}),\ f^*_{ssp}(v_n \ v_{n-2})\}$

$= \text{gcd of } \{(2n-3)^2, (2n-4)^2\}$

$= \text{gcd of } \{(2n-3), (2n-4)\} = 1$

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $(P_n)^2$, admits sum square prime labeling. ■

**Theorem 2.3** Two tuple graph of path $P_n$ admits sum square prime labeling, when n is not a multiple of 3.

Proof: Let G = $T^2(P_n)$ and let $v_1,v_2,\text{----------------},v_{2n}$ are the vertices of G

Here $|V(G)| = 2n$ and $|E(G)| = 3n-2$

Define a function $f : V \rightarrow \{0,1,2,3,\text{----------------},2n-1\}$ by

$f(v_i) = i-1$ , $i = 1,2,\text{------},2n$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{ssp}$ is defined as follows

$f^*_{ssp}(v_{2i-1} \ v_{2i+1}) = (4i-2)^2,$ $i = 1,2,\text{------------},n-1$

$f^*_{ssp}(v_{2i} \ v_{2i+2}) = (4i)^2,$ $i = 1,2,\text{------------},n-1$

$f^*_{ssp}(v_{2i-1} \ v_{2i}) = (4i-3)^2,$ $i = 1,2,\text{------------},n$

Clearly $f^*_{ssp}$ is an injection.

***gcin*** of $(v_{2i-1})$ $= \text{gcd of } \{f^*_{ssp}(v_{2i} \ v_{2i-1}),\ f^*_{ssp}(v_{2i-1} \ v_{2i+1})\}$

= gcd of $\{(4i-3)^2, (4i-2)^2\}$
= gcd of $\{(4i-3), (4i-2)\}$ = 1, i = 1,2,------------,n-1

***gcin*** of ($v_{2i+2}$) = gcd of $\{f_{ssp}^*(v_{2i}\ \ v_{2i+2}),\ f_{ssp}^*(v_{2i+2}\ \ v_{2i+1})\}$
= gcd of $\{(4i)^2, (4i+1)^2\}$
= gcd of $\{(4i), (4i+1)\}$ = 1, i = 1,2,------------,n-1

***gcin*** of ($v_2$) = gcd of $\{f_{ssp}^*(v_1\ \ v_2),\ f_{ssp}^*(v_2\ \ v_4)\}$
= gcd of $\{1, 4^2\}$

***gcin*** of ($v_{2n-1}$) = gcd of $\{f_{ssp}^*(v_{2n-3}\ \ v_{2n-1}),\ f_{ssp}^*(v_{2n-1}\ \ v_{2n})\}$
= gcd of $\{(4n-6)^2, (4n-3)^2\}$
= gcd of $\{(4n-6), (4n-3)\}$
= gcd of {3, 4n-6}
= g c d of {3,n} = 1

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $T^2(P_n)$, admits sum square prime labeling. ∎

**Theorem 2.4** Duplicate graph of path $P_n$ admits sum square prime labeling.
Proof: Let G = $D(P_n)$ and let $v_1, v_2$,----------------,$v_{2n}$ are the vertices of G
Here |V(G)| = 2n and |E(G)| = 2n-2
Define a function f : V → {0,1,2,3,----------------,2n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f_{ssp}^*$ is defined as follows

$f_{ssp}^*(v_i\ \ v_{i+1})$ = $(2i-1)^2$, i = 1,2,------------,n-1
$f_{ssp}^*(v_{n+i}\ \ v_{n+i+1})$ = $(2n+2i-1)^2$, i = 1,2,------------,n-1

Clearly $f_{ssp}^*$ is an injection.

***gcin*** of ($v_{i+1}$) = 1, i = 1,2,------------,n-2
***gcin*** of ($v_{n+i+1}$) = 1, i = 1,2,------------,n-2

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $D(P_n)$, admits sum square prime labeling. ∎

**Theorem 2.5** Middle graph of path $P_n$ admits sum square prime labeling.
**Proof:** Let G = $M(P_n)$ and let $v_1, v_2$,----------------,$v_{2n-1}$ are the vertices of G
Here |V(G)| = 2n-1 and |E(G)| = 3n-4
Define a function f : V → {0,1,2,3,----------------,2n-2 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n-1
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f_{ssp}^*$ is defined as follows

$f_{ssp}^*(v_i\ \ v_{i+1})$ = $(2i-1)^2$, i = 1,2,------------,2n-2
$f_{ssp}^*(v_{2i}\ \ v_{2i+2})$ = $(4i)^2$, i = 1,2,------------,n-2

Clearly $f_{ssp}^*$ is an injection.

***gcin*** of ($v_{i+1}$) = 1, i = 1,2,------------,2n-3

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $M(P_n)$, admits sum square prime labeling. ∎

**Theorem 2.6** Total graph of path $P_n$ admits sum square prime labeling.

**Proof:** Let G = T($P_n$) and let $v_1, v_2$,----------------,$v_{2n-1}$ are the vertices of G
Here |V(G)| = 2n-1 and |E(G)| = 4n-5.
Define a function f : V → {0,1,2,3,----------------,2n-2 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n-1
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f_{ssp}^*$ is defined as follows
$f_{ssp}^*(v_i \ v_{i+1})$ = $(2i-1)^2$, i = 1,2,------------,2n-2
$f_{ssp}^*(v_{2i} \ v_{2i+2})$ = $(4i)^2$, i = 1,2,------------,n-2
$f_{ssp}^*(v_{2i-1} \ v_{2i+1})$ = $(4i-2)^2$, i = 1,2,------------,n-1
Clearly $f_{ssp}^*$ is an injection.
***gcin*** of ($v_{i+1}$) = 1, i = 1,2,------------,2n-3
***gcin*** of ($v_1$) = gcd of {$f_{ssp}^*(v_1 \ v_2)$ , $f_{ssp}^*(v_1 \ v_3)$ }
= gcd of {1, 4}
***gcin*** of ($v_{2n-1}$) = gcd of {$f_{ssp}^*(v_{2n-3} \ v_{2n-1})$ , $f_{ssp}^*(v_{2n-1} \ v_{2n-2})$ }
= gcd of {$(4n-6)^2$, $(4n-5)^2$}
= gcd of {(4n-6), (4n-5)} = 1.
So, ***gcin*** of each vertex of degree greater than one is 1.
Hence T($P_n$), admits sum square prime labeling. ■

**Theorem 2.7** Z graph of path $P_n$ admits sum square prime labeling, when n is odd and not a multiple of 3.
**Proof:** Let G = Z($P_n$) and let $v_1, v_2$,----------------,$v_{2n}$ are the vertices of G
Here |V(G)| = 2n and |E(G)| = 3n-3.
Define a function f : V → {0,1,2,3,----------------,2n-1 } by
$f(v_i)$ = i-1 , i = 1,2,------,2n
Clearly f is a bijection.
For the vertex labeling f, the induced edge labeling $f_{ssp}^*$ is defined as follows
$f_{ssp}^*(v_i \ v_{i+1})$ = $(2i-1)^2$, i = 1,2,------------,n-1
$f_{ssp}^*(v_{n+i} \ v_{n+i+1})$ = $(2n+2i-1)^2$, i = 1,2,------------,n-1
$f_{ssp}^*(v_{i+1} \ v_{n+i})$ = $(n+2i-1)^2$, i = 1,2,------------,n-1
Clearly $f_{ssp}^*$ is an injection.
***gcin*** of ($v_{i+1}$) = 1, i = 1,2,------------,n-2
***gcin*** of ($v_{n+i+1}$) = 1, i = 1,2,------------,n-2
***gcin*** of ($v_n$) = gcd of {$f_{ssp}^*(v_{n-1} \ v_n)$ , $f_{ssp}^*(v_{2n-1} \ v_n)$ }
= gcd of {$(2n-3)^2$, $(3n-3)^2$}
= gcd of {(2n-3), (3n-3)}
= gcd of {(2n-3), n}
= g c d of {n, n-3}
= g c d of {3, n-3} =1.
***gcin*** of ($v_{n+1}$) = gcd of {$f_{ssp}^*(v_2 \ v_{n+1})$ , $f_{ssp}^*(v_{n+1} \ v_{n+2})$ }
= gcd of {$(n+1)^2$, $(2n+1)^2$}
= gcd of {(n+1), (2n+1)}
= g c d of {(n+1, n} =1.
So, ***gcin*** of each vertex of degree greater than one is 1.

Hence Z($P_n$), admits sum square prime labeling. ■

**Theorem 2.8** Splitting graph of path $P_n$ admits sum square prime labeling, when n is odd and not a multiple of 3.

**Proof:** Let G = SP($P_n$) and let $v_1,v_2$,----------------,$v_{2n}$ are the vertices of G

Here |V(G)| = 2n and |E(G)| = 3n-3.

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$f(v_i) = i-1$ , i = 1,2,------,2n

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f_{ssp}^*$ is defined as follows

$f_{ssp}^*(v_i\ \ v_{i+1})$ = $(2i-1)^2$, i = 1,2,------------,n-1

$f_{ssp}^*(v_{n+i}\ \ v_{n+i+1})$ = $(2n+2i-1)^2$, i = 1,2,------------,n-1

$f_{ssp}^*(v_{2i}\ \ v_{n+2i-1})$ = $(n+4i-3)^2$, i = 1,2,------------,$\frac{n-1}{2}$

$f_{ssp}^*(v_{2i}\ \ v_{n+2i+1})$ = $(n+4i-1)^2$, i = 1,2,------------,$\frac{n-1}{2}$

Clearly $f_{ssp}^*$ is an injection.

***gcin*** of ($v_{i+1}$) = 1, i = 1,2,------------,n-2

***gcin*** of ($v_{n+i+1}$) = 1, i = 1,2,------------,n-2

***gcin*** of ($v_{n+1}$) = gcd of {$f_{ssp}^*(v_{n+1}\ \ v_{n+2})$ , $f_{ssp}^*(v_2\ \ v_{n+1})$ }

= gcd of {$(2n+1)^2$, $(n+1)^2$}

= gcd of {(2n+1), (n+1)}

= g c d of {n+1, n} =1.

***gcin*** of ($v_{2n}$) = gcd of {$f_{ssp}^*(v_{2n-1}\ \ v_{2n})$ , $f_{ssp}^*(v_{n-1}\ \ v_{2n})$ }

= gcd of {$(4n-3)^2$, $(3n-3)^2$}

= g c d of {(4n-3), (3n-3)}

= gcd of {3n-3, n}

= g c d of {n-3, n} = 1.

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence SP($P_n$), admits sum square prime labeling. ■

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