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The effect of the order Hilbert matrix On Detected components based on luminance for color image

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Abstract

In this paper, firstly, study Hilbert matrix and some applications in image processing. Hilbert matrix are used both mathematics and computational sciences ,its plays an important role in dealing with detected components based on luminance for color image, filtering the noise image and encryption the color image. And it applies in Mat lab program for any color image and any format without convert image to gray.

The results proved that use Order Hilbert matrix highly efficient in the lighting effect of the components and words increased its grade Order Hilbert matrix. Whenever the light is covered on the image the concealment of the cocoons was also very clear.

1. Introduction

Feature detection for color image is a paramount tool in computer visibility [1].We show the set of existent tool founded features than we can apply on luminance images and display how we can extend to the color domain. Therefore, we start the main concepts of Hilbert matrix.

Hilbert matrix in linear algebra had come in by Hilbert (1894), who is defined as a square matrix and the elements are being the unit fractions. [2]

$$H_{ij} = \frac{1}{i+j-1} \qquad (1)$$

.We take the Hilbert matrix H_n ($n \times n$ matrix).If we use the order n=5 for Hilbert matrix in Mat lab we can write

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format rat

hn=hilb(5)
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$$H_5 = \begin{bmatrix} 1 & 1/2 & 1/3 & 1/4 & 1/5 \\ 1/2 & 1/3 & 1/4 & 1/5 & 1/6 \\ 1/3 & 1/4 & 1/5 & 1/6 & 1/7 \\ 1/4 & 1/5 & 1/6 & 1/7 & 1/8 \\ 1/5 & 1/6 & 1/7 & 1/8 & 1/9 \end{bmatrix} \quad (2)$$

We can be obtained the Hilbert matrix from the integral

$$H_{ij} = \int_0^1 x^{i+j-2} dx \quad (3)$$

It appears from the least squares of any functions that give our polynomials.

We illustrate the properties of the Hilbert matrix as follows. [3][4][5]

1. The Hilbert matrix is positive definite and symmetric.
2. It is totally positive (it means that the determinant of every sub matrix is being positive).
3. The determinant of the Hilbert matrix can be expressed in form

$$\det(H) = \frac{c_n^4}{c_{2n}} \quad (4) \qquad \text{Where} \qquad c_n = \prod_{i=1}^{n-1} i^{n-i}$$

4. We obtained the inverse of the Hilbert matrix from the formula

$$(H^{-1})_{ij} = (-1)^{i+j}(i+j-1)\binom{n+i-1}{n-j}\binom{n+j-1}{n-i}\binom{i+j-2}{i-1}^2 \quad (5)$$

Where n is the order of the matrix.

5. Hilbert matrix are well known of extremely of ill-conditioned matrices.

2. Illumination [6]

Absent illumination there is no color that is a simple fact. Under down illumination grade the human visibility system is ultimately mono- varicolored, as a black-and-white camera. We can see color under sufficient illumination levels but the recognize color of thingummy depends of course on the property of the illumination source. It interacts with the material, when the illuminating light arrives an object.

3. The proposed method

In the following algorithm which obtain Hlibert matrix from the Mathematical formula.

$$H_{ij} = \int_0^1 x^{i+j-2} dx \quad (6)$$

Step 1: if n=3

$$h_{11}=\int_0^1 1dx=1\ ;\ h_{12}=\int_0^1 xdx=\frac{1}{2}\ ;\ h_{13}=\int_0^1 x^2dx=\frac{1}{3}\ ;\ h_{21}=\int_0^1 xdx=\frac{1}{2}\ ;h_{22}=\int_0^1 x^2dx=\frac{1}{3}$$

$$h_{23}=\int_0^1 x^3dx=\frac{1}{4};\ h_{31}=\int_0^1 x^2dx=\frac{1}{3};h_{32}=\int_0^1 x^3dx=\frac{1}{4}\ ;\ h_{33}=\int_0^1 x^4dx=\frac{1}{5}$$

Now, the Hilbert matrix of order 3 is

$$H_{3\times3}=\begin{bmatrix}1 & \frac{1}{2} & \frac{1}{3}\\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4}\\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5}\end{bmatrix}$$

Step 2: if n=4, the Hilbert matrix is the same Hilbert matrix of order n=3 and we additional components like this

$$h_{14}=\int_0^1 x^3dx=\frac{1}{4}\ ;\ h_{24}=\int_0^1 x^4dx=\frac{1}{5}\ ;\quad h_{34}=\int_0^1 x^5dx=\frac{1}{6}\ ;\quad h_{44}=\int_0^1 x^6dx=\frac{1}{7}$$

Since $h_{41}=h_{14}$; $h_{42}=h_{24}$ and $h_{43}=h_{34}$ then Hilbert matrix of order (4) becomes

$$H_{4\times4}=\begin{bmatrix}1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4}\\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5}\\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6}\\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7}\end{bmatrix}$$ And so on for $H_{5\times5}$,$H_{6\times6}$ and $H_{7\times7}$.

Step 3: find the inverse Hilbert matrix from the formula

$$(H^{-1})_{ij}=(j+i-1)\left[\prod_{\substack{k=1\\k\neq j}}^{n}\frac{(1-i-k)}{(j-k)}\right]\left[\prod_{\substack{l=1\\l\neq i}}^{n}\frac{(j+l-1)}{(l-i)}\right] \qquad (7)$$

If n=3

$$h_{11}^{-1}=1\left[\frac{(1-1-2)(1-1-3)}{(1-2)(1-3)}\right]\left[\frac{(1+2-1)(1+3-1)}{(2-1)(3-1)}\right]=\frac{36}{4}=9$$

$$h_{12}^{-1}=2\left[\frac{(1-1-1)(1-1-3)}{(2-1)(2-3)}\right]\left[\frac{(2+2-1)(2+3-1)}{(2-1)(3-1)}\right]=2(\frac{3}{-1})(\frac{12}{2})=-36$$

$$h_{13}^{-1}=3\left[\frac{(1-1-1)(1-1-2)}{(3-1)(3-2)}\right]\left[\frac{(3+2-1)(3+3-1)}{(2-1)(3-1)}\right]=3(\frac{2}{2})(\frac{20}{2})=30$$

$$h_{21}^{-1}=2\left[\frac{(1-2-2)(1-2-3)}{(1-2)(1-3)}\right]\left[\frac{(1+1-1)(1+3-1)}{(1-2)(3-2)}\right]=2(\frac{12}{2})(\frac{3}{-1})=-36$$

$$h_{22}^{-1}=3\left[\frac{(1-2-1)(1-2-3)}{(2-1)(2-3)}\right]\left[\frac{(2+1-1)(2+3-1)}{(1-2)(3-2)}\right]=3(\frac{8}{-1})(\frac{8}{-1})=192$$

$$h_{23}^{-1}=4\left[\frac{(1-2-1)(1-2-2)}{(3-1)(3-2)}\right]\left[\frac{(3+1-1)(3+3-1)}{(1-2)(3-2)}\right]=4(\frac{6}{2})(\frac{15}{-1})=-180$$

$$h_{31}^{-1}=3\left[\frac{(1-3-2)(1-3-3)}{(1-2)(1-3)}\right]\left[\frac{(1+1-1)(1+2-1)}{(1-3)(2-3)}\right]=3(\frac{20}{2})(\frac{2}{2})=30$$

$$h_{32}^{-1}=4\left[\frac{(1-3-1)(1-3-3)}{(2-1)(2-3)}\right]\left[\frac{(2+1-1)(2+2-1)}{(1-3)(2-3)}\right]=4(\frac{15}{-1})(\frac{6}{2})=-180$$

$$h_{33}^{-1}=5\left[\frac{(1-3-1)(1-3-2)}{(3-1)(3-2)}\right]\left[\frac{(3+1-1)(3+2-1)}{(1-3)(2-3)}\right]=5(\frac{12}{2})(\frac{12}{2})=180$$

Hence, the inverse Hilbert matrix of order (3) is

$$H_{3\times3}^{-1}=\begin{bmatrix} 9 & -36 & 30 \\ -36 & 192 & -180 \\ 30 & -180 & 180 \end{bmatrix}$$

In the same way we can find the inverse of Hilbert matrix of order $H_{4\times4}, H_{5\times5}, H_{6\times6}$, $\mathrm{H}_{7\times7}$ and so on.

We can see the inverse of Hilbert matrix is also symmetric.

Step 4: The diagonal of Hilbert matrix has the form

$$H_{ii}=\frac{1}{2i-1} \qquad (8)$$

Hence the diagonal of Hilbert matrix of order 3 is

$$h_{11}=\frac{1}{2-1}=1 \quad ,\mathrm{h}_{22}=\frac{1}{4-1}=\frac{1}{3} \quad ,\mathrm{h}_{33}=\frac{1}{6\text{-}1}=\frac{1}{5}$$

Also, we can find the diagonal elements of Hilbert matrix for any order

4.Experimental Results

In this section, the results will be discussed .The system was executed on a database of 50 color images with any size and any format Figure (1) shown sample of color images.

Figure (1): sample of color images.

The illumination concept depends on the direction lighting and the direction of the object surfaces. Any ways, spatial diversity in illumination it's the reason for changes of the ferocity in the observed images. However, it has been a latest change when we use Hilbert matrix of order (3×3) .We can note an image of a prospect is a setting up of illumination and reflectance composition, the components have been clear, less illumination and lighter. We can show this in figure 2.

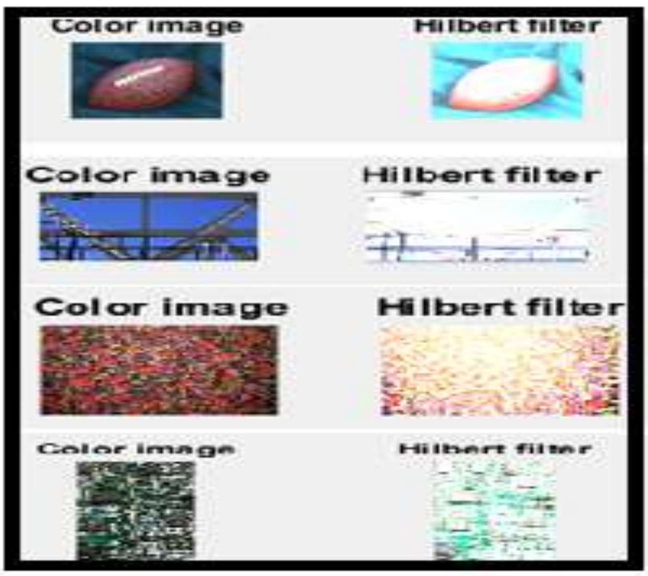


Figure (2): Effectiveness the Hilbert matrix of order (3) on the color image

In this paper, the proposed method is related the diagonal matrix of Hilbert matrix. However, we find the diagonal of Hilbert matrix have significant impact illumination and reflectance on color image and its detected components for color image .we can show in the figure (3).[2]

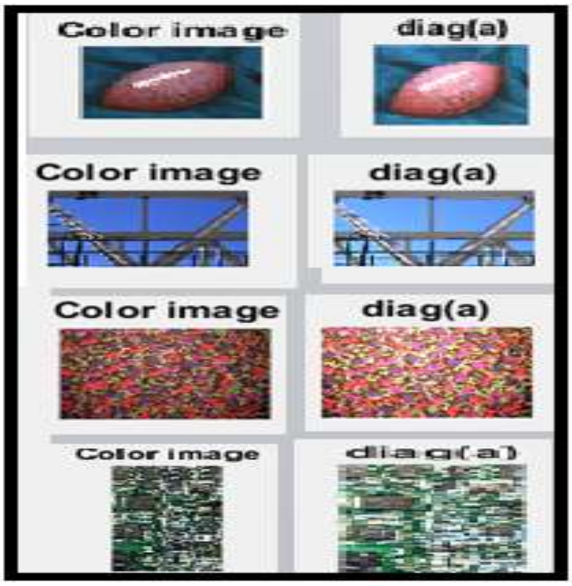


Figure (3): Effective the diagonal of Hilbert matrix on color image

Recently, we study image segmentation and effective the other order of Hilbert matrix on color image. The results of segmentation are highly favorable with the original images and the reflection of light increases the brightness when we use the bigger order of Hilbert matrix because the component is smallest nearest the zero when the order of Hilbert matrix is large. Therefore, the effective of the Hilbert matrix of order (4) is greater than the Hilbert matrix of order (3) ,also the effective of the Hilbert matrix of order(5) is brightness and greater illumination than the Hilbert matrix of order (4).finally, we can see the effective of Hilbert matrix of order (7) is increases brightness than the effective of Hilbert matrix of order (5).We can show that in figure(4)

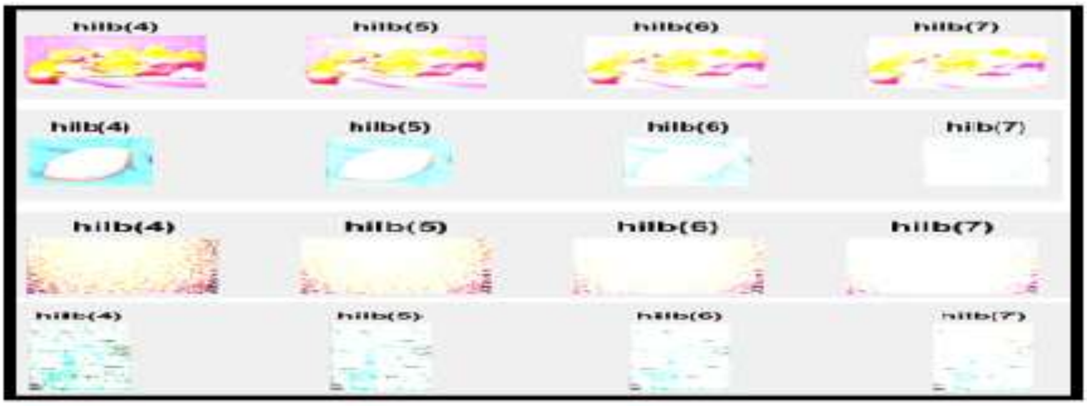


Figure (4): Effective Hilbert matrix of order (4),(5),(6) and(7) on color image

Anyway, spatial diversity in illumination that give the reason for changes of the intensities .We can notice in the color images when we use the inverse of Hilbert matrix. The options it's like encryption, we can see that in figure (5).

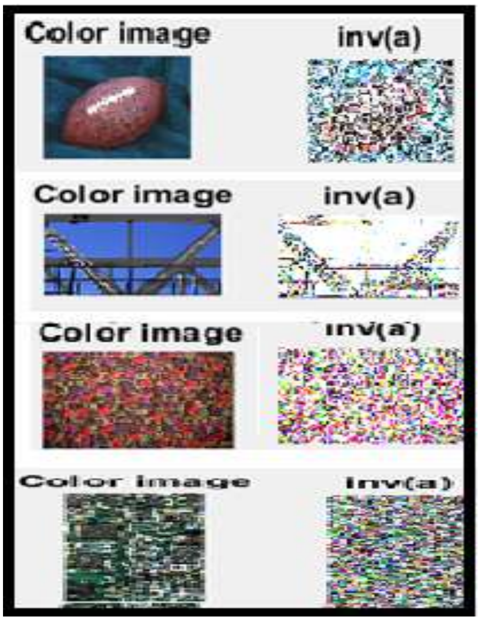
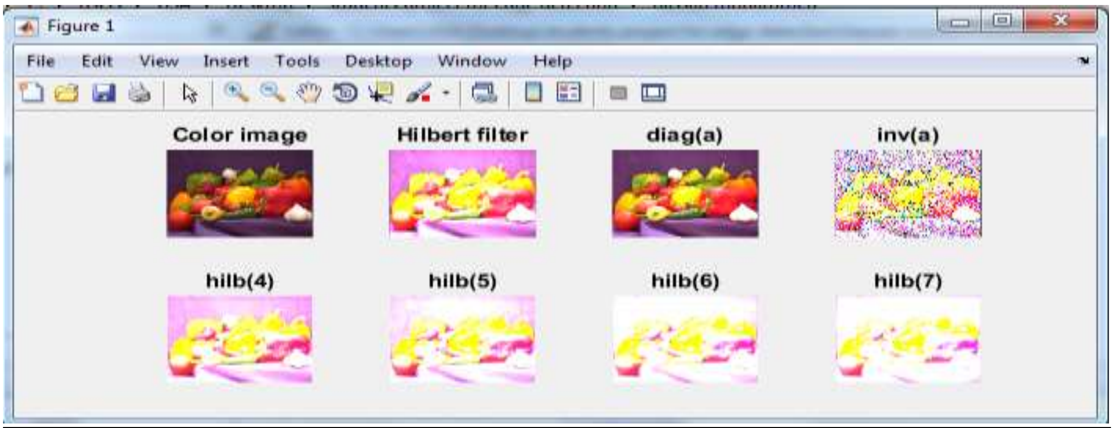
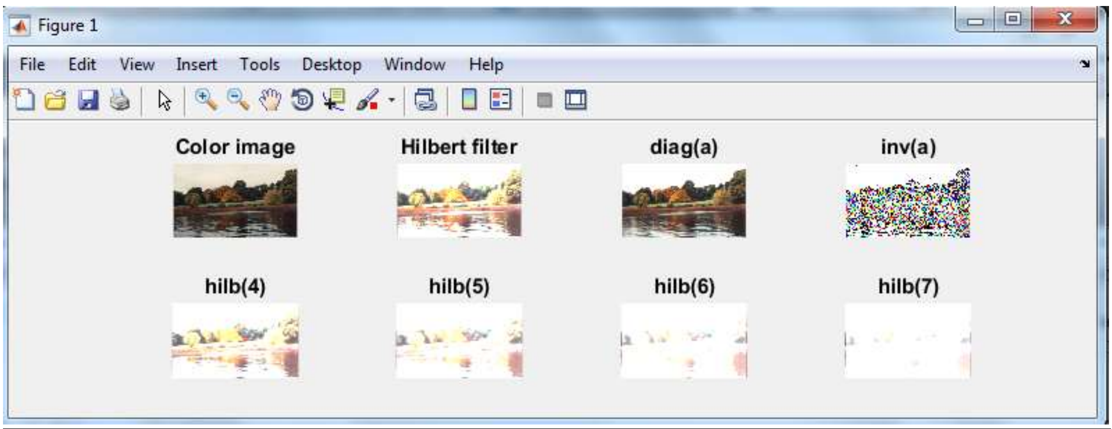


Figure (5): Effective the inverse of Hilbert matrix on color image

Finally, we can see the effective the Hilbert matrix of any order on color image, detected components when use the diagonal Hilbert matrix and the image is Confused and unclear as encrypted when we use the inverse Hilbert matrix. The figure (6) explain that.





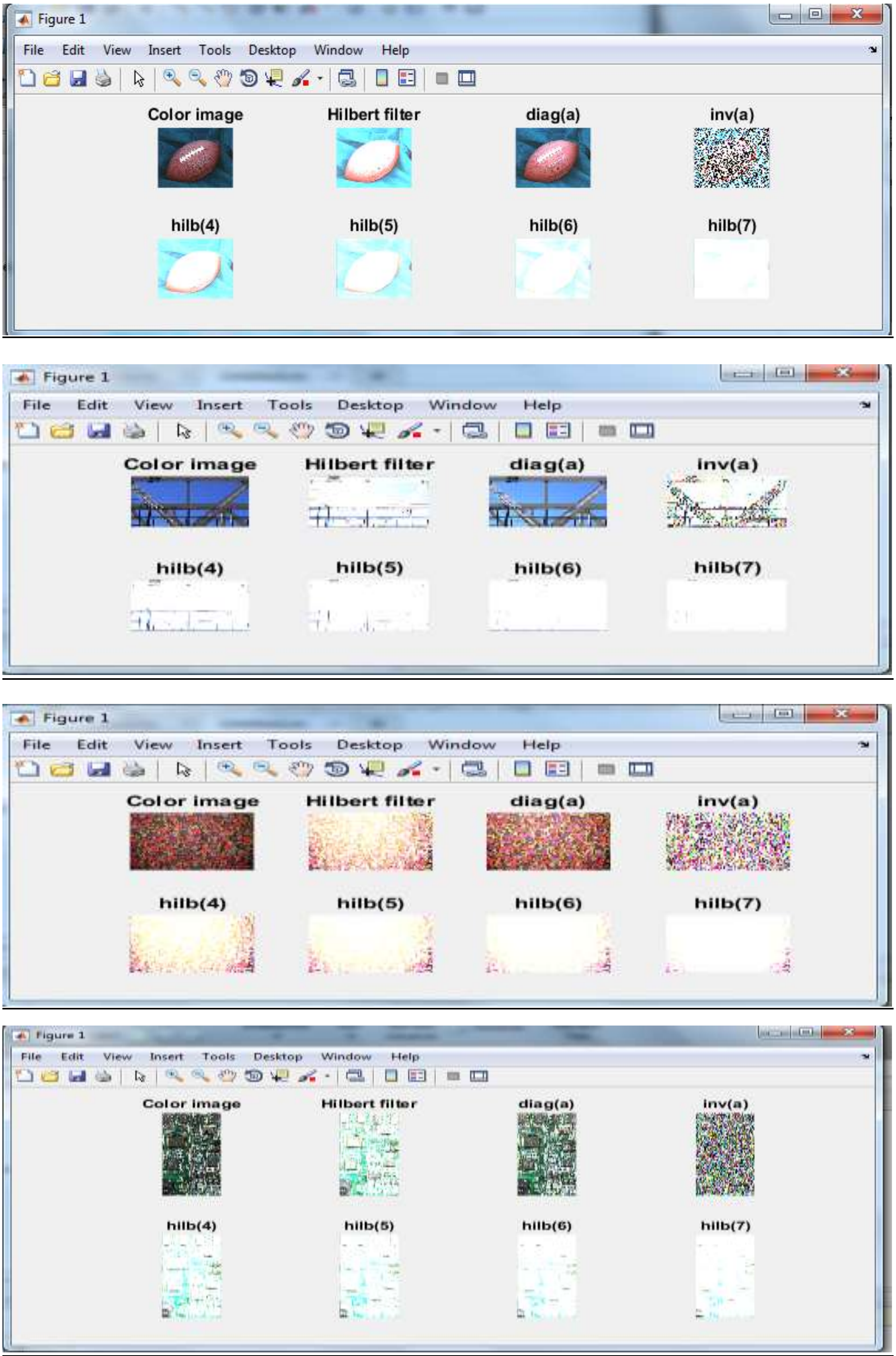
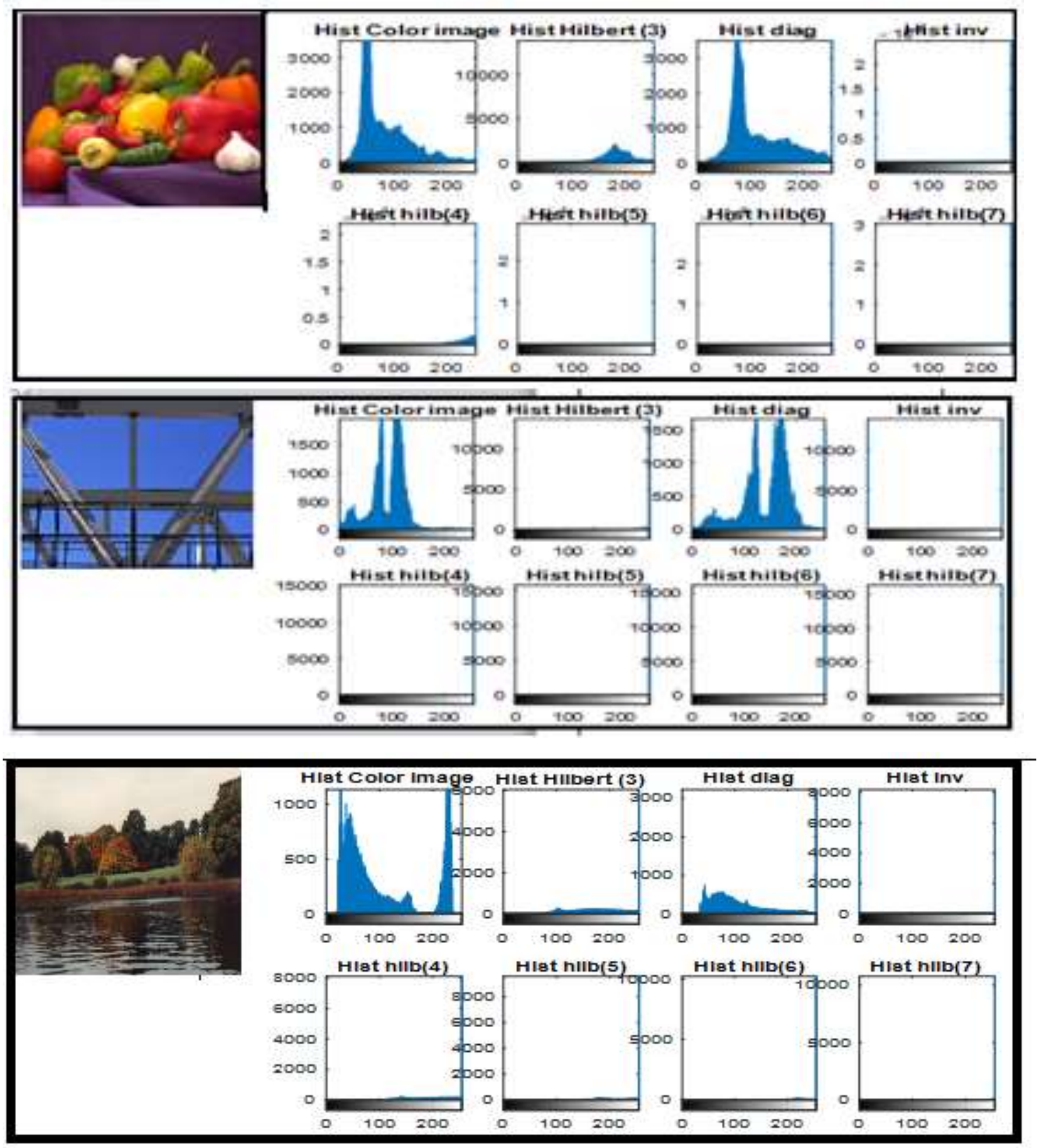


Figure (6): The General effective of Hilbert matrix on color image

The measure of entropy is stimulate of our method approximation .We can be calculate it from the formula:[7]

$$V(n,\sigma) = n^T (n * g) \qquad (9)$$

When V is the key shrewdness, n is a histogram of x,* is involution, and g is a exclusiveness Gaussian filter. Conclusive, the equation (9) is also warrants us to compute the efficiently of gradient of $V(n,\sigma)$ with respect to n, where (as long as n has been produce using a "smooth" histogramming operation as like as linear interpolation) that allows us to back propagate the propensity onto .we can see that in figure (7)



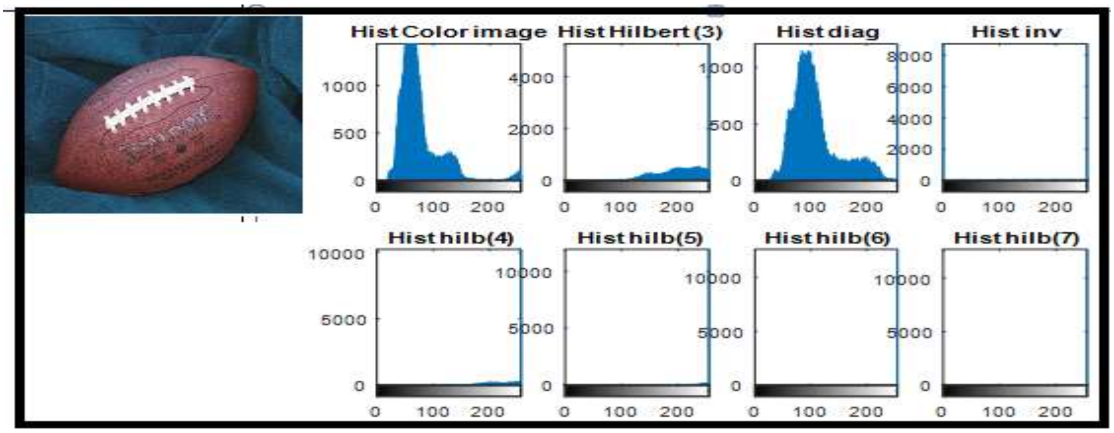


Figure (7): The histogram explain effective of Hilbert matrix

6. Conclusion

In this paper, we have inform a new method to detect twofold illuminants in images based on the Hilbert matrix. By vigor the lights to be examined pair-wise without imperious on accuracy of the illuminant estimation, and the results by processing on a region rather than pixel basis. Results are remarkably good, we have been explain in histogram, we find highly efficient in lighting effect of the components, detected edges components when we use the diagonal Hilbert matrix ,as well as, the components are disappear when the order of Hilbert matrix is higher because the elements of higher order Hilbert matrix are closed to zero.

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