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IMPLICATIONS OF NUMERICAL INTEGRATION AND ITS SIGNIFICANCE

Sonia Rani
Kurukshetra University Kurukshetra

ABSTRACT

Integration is considered as an important branch of mathematics. Numerical integration is known as to compute the definite integral value from a range of integrand numerical values. As compared to numerical differentiation, in numerical integration while calculating the value of the definite integral $\int_b^a f(x)dx$, an interpolation formula having differences takes place of the function f(x).

Round-off error can be reduced by reducing the number of evaluations of the integrand. The property of nesting is found among the quadrature rules. The values of integrand can be reused with the sub-division of each interval at all current points. The current article highlights the implications of numerical integration.

KEYWORDS:

Integration, Integral, Quadrature

INTRODUCTION

There is a common class of integration techniques which is known as interpolatory quadrature where the value of integration of f (interpolant) is evaluated that is supposed to be the closest to the real integral.

Here, a function g is known as interpolating function of f if satisfied:

g(xi) = f (xi)

where, i= 1,2,3……n

and

(xi,f (xi))

where, i= 1,2,3……n.

Numerical integration is done in the situation where the integrand f(x) is known at some points. There are many other occasions like when integrand formula is known but the evaluation of elementary function i.e. anti-derivative is difficult.

The computation of numerical approximation is quite easier than that of the anti derivative. Numerical integration is a process of getting an integral approximation by combining the results of the integrand. Here, the finite sets of points to which the evaluation of integrand is done; are known as integral points and an approximate integral is obtained by the summation of these values.

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In numerical integration, it is also very important to study the nature of the approximation error. Here, the superiority among the methods is judged by the range of error i.e. the method yielding small errors for a small number of evaluations will be considered as superior to other methods yielding big errors.

The problem of evaluating the integral

$$F(x) = \int_a^x f(u)du$$

can be reduced to an initial value problem for an ordinary differential equation by applying the first part of the fundamental theorem of calculus. By differentiating both sides of the above with respect to the argument *x*, it is seen that the function *F* satisfies

$$\frac{dF(x)}{dx} = f(x),\ F(a) = 0.$$

There are following main categories for numerical integration:

1. Newton-Cotes formulas

In this case, we obtain methods for numerical integration which can be derived from the Lagrange interpolating method. Alternatively the formulas can also be derived from Taylor expansion. The idea is similar to the way we obtain numerical differentiation schemes. We can easily derive not just integration formulas but also their errors using this technique. The schemes which we develop here will be based on the assumption of equidistant points.

2. Composite, Newton - Cotes formulas (open and closed)

These methods are composite since they repeatedly apply the simple formulas derived previously to cover longer intervals. This idea allows for piecewise estimates of the integral thus improving the error of our integration. (we will also assume equidistant nodes in our presentation).

3. Romberg Integration

This method allows us to improve the error of our integration methods by doing minimal extra work. The idea is really based on the Richardson extrapolation which we saw earlier in the numerical differentiation section.

4. Adaptive Integration

Here, we are free to choose the points over which we calculate the numerical integral of f(x) so as to minimize our error. Adaptive integration does not therefore require equidistant nodes. Thus if the function is not very smooth at some interval the step size h of the numerical integration method decreases to make sure we do not accumulate too much error in our calculation.

5. Gaussian Integration

We explore methods which can achieve optimal error reduction provided we place the nodes at specific locations. Computing the best weights for our numerical quadratures guarantees optimal approximation of our integral.

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IMPLICATIONS OF NUMERICAL INTEGRATION

It is observed that there are basically 3 steps of a quadrature scheme. First, f (integrand) is evaluated within a given range of limit. After that, a function is obtained which is helpful in interpolating f.

Generally, interpolant and integrand (f) both are independent of each other as there is no need of the integrand ,f, while evaluating the interpolant.

After that, this formula with a limit range from a to b is integrated. This process of function of single variable is known as quadrature. A general quadrature formula is given below:

Let $f(x_k) = y_k$ be the nodal value at the tabular point x_k for k = 0,1,………x_n where x_0=a and $x_n=x_o + nh$ = b. Integrand is replaced so as to get general quadrature formula.

$$\int_a^b f(x)dx = \int_a^b \left[y_0 + \frac{\Delta y_0}{h}(x-x_0) + \frac{\Delta^2 y_0}{2!h^2}(x-x_0)(x-x_1) + \frac{\Delta^3 y_0}{3!h^3}(x-x_0)(x-x_1)(x-x_2) \right.$$
$$\left. + \frac{\Delta^4 y_0}{4!h^4}(x-x_0)(x-x_1)(x-x_2)(x-x_3) + \cdots \right] dx$$

Using the transformation $x = x_0 + h$ yields:

$$\int_a^b f(x)dx = h\int_0^n \left[y_0 + u\Delta y_0 + \frac{\Delta^2 y_0}{2!}u(u-1) + \frac{\Delta^3 y_0}{3!}u(u-1)(u-2) \right.$$
$$\left. + \frac{\Delta^4 y_0}{4!}u(u-1)(u-2)(u-3) + \cdots \right] du$$

Then,

$$\int_a^b f(x)dx = h\left[ny_0 + \frac{n^2}{2}\Delta y_0 + \frac{\Delta^2 y_0}{2!}\left(\frac{n^3}{3} - \frac{n^2}{2}\right) + \frac{\Delta^3 y_0}{3!}\left(\frac{n^4}{4} - n^3 + n^2\right) \right.$$
$$\left. + \frac{\Delta^4 y_0}{4!}\left(\frac{n^5}{5} - \frac{3n^4}{2} + \frac{11n^3}{3} - 3n^2\right) + \cdots \right]$$

For n =1, we have

$$\int_a^b f(x)dx = h\left[y_0 + \frac{\Delta y_0}{2}\right] = \frac{h}{2}\left[y_0 + y_1\right].$$

For n=2, we get

$$\int_a^b f(x)dx = h\left[2y_0 + 2\Delta y_0 + \left(\frac{8}{3} - \frac{4}{2}\right)\frac{\Delta^2 y_0}{2}\right]$$

$$= 2h\left[y_0 + (y_1 - y_0) + \frac{1}{3} \times \frac{y_2 - 2y_1 + y_0}{2}\right] = \frac{h}{3}\left[y_0 + 4y_1 + y_2\right].$$

Here, an interpolating polynomial is used in place of the integrand over the whole interval [a,b] and integrated with the given range of limit. In general, formulae for numerical integration are obtained by dividing the interval [a,b] in n sub-intervals $[x_k, x_{k+1}]$, where, $x_k = x_o + kh$ for k=0,1,……n with $x_0 = a$, $x_n = x_0 + nh = b$.

The propagation and cooperation of spatially localized pulses (the alleged light bullets) with particle highlights in a few space measurements are of both physical and mathematical interests. Such light bullets have been seen in the numerical reenactments of the full Maxwell system with immediate Kerr ($\chi^{(3)}$ or cubic) nonlinearity in two space measurements (2D).

Despite the fact that direct numerical reproductions of the full Maxwell system are inspiring, asymptotic estimate is vital for analysis in a few space measurements. The estimate of 1D Maxwell system has been broadly examined. Long pulses are all around approximated through envelope guess by the cubic centering nonlinear Schrodinger (NLS) for $\chi^{(3)}$ medium.

In situations where a positive arrangement ought to stay positive in time; the spurious numerical oscillations may bring about the answer for change sign. For this situation, one can fall into a badly postured district of the equation, and the numerical arrangement will stop to speak to the arrangement of the current equation.

There have been a few endeavors in the writing to address the complex numerical issues. For instance, solutions of the compaction equation, K(2, 2), were acquired with limited distinction methods.

CONCLUSION

Methods developed for ordinary differential equations, such as Runge–Kutta methods, can be applied to the restated problem and thus be used to evaluate the integral. For instance, the standard fourth-order Runge–Kutta method applied to the differential equation yields Simpson's rule from above.

The differential equation $F'(x) = f(x)$ has a special form: the right-hand side contains only the dependent variable (here *x*) and not the independent variable (here *F*). This simplifies the theory and algorithms considerably. The problem of evaluating integrals is thus best studied in its own right.

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