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# *STUDY OF LIE THEORY ACCESSIBLE TO MATHEMATICAL CONCEPT OF MATRIX*

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**Abstract**

*It is actually supposed to make a very first view of Lie theory accessible to mathematics undergraduates. While the prerequisites are saved as amount that is lower as possible, the content is advanced and includes a lot of the important themes of the older subject. To be able to accomplish this we confine ourselves to matrix groups, i.e., closed subgroups of general linear groups. One of the primary outcomes that we prove shows that every matrix team is actually actually a Lie subgroup, the proof being modelled on that in the expository paper.*

***Keywords: Lie, theory, matrix, subgroups, linear, etc.***

## 1. INTRODUCTION

Obviously, the classical Lie groups are readily released at undergraduate level, and it's possible to examine a lot of the functions of theirs. The spinor organizations are also created and through them the role of global topology. Closely related as synonyms are actually the words matrix and linear algebra about the matrix, we all knew that the arrangement of entries in systematic manner, though it was systematically released by an English mathematician Joseph Sylvester in the season 1850. It was during the period when there was a British rule on important region of world. Many statistical figures which arrived from various corners were to be systematically placed and meaningful deduction was anticipated by the authorities. Along with this a mathematician Cayley also devoted the majority of the work time of his in the improvement of the topic. It, as they described a matrix means "Something that presents the time or maybe location from which another thing originates or perhaps develops". As written in the first note, it's, at a stage onwards, hard to distinguish between linear algebra and Matrix theory.

## 2. DIFFERENTIAL EQUATIONS IN MATRICES

Let $A \in M_n(R)$. Let $(a, b) \subseteq R$ be the open interval with endpoints a, b and a < b; we will usually assume that $a < 0 < b$. We are going to use the standard notation.

$$a'(t) = \frac{d}{dt}a(t)$$

Imagine the original order differential equation

$$a'(t) = a(t)A \qquad (1)$$

Where $\alpha: (a, b) \longrightarrow M_n(R)$ is assumed to be a differentiable function.

If $n = 1$ then taking A to be a non-zero real number we know that the general solution is $\alpha(t) = ce^{At}$ where$\alpha(0) = c$. Hence there's a unique option subject to this particular boundary condition. Actually this particular remedy is provided by a power series

$$\alpha(t) = \sum_{k\geq 0} \frac{tk}{k!} a(0)$$

This is indicative of the general situation.

Theorem 1. For $A, C \in M_n(R)$ with A non-zero, and $a < 0 < b$, the differential equation of (1) has a unique solution $\alpha: (a, b) \longrightarrow M_n(R)$ for which $\alpha(0) = C$. Furthermore, if $C$ is invertible then so is $\alpha(t)$ for $t \in (a, b)$, hence $\alpha: (a, b) \longrightarrow GL_n(R)$.

Proof. First we are going to solve the situation subject to the boundary condition $\alpha(0) = I$. For $t \in (a, b)$, by Chapter 1 Section 8 the series

$$\sum_{k\geq 0} \frac{tk}{k!} Ak = \sum_{k\geq 0} \frac{1}{k!} (tA)k = exp(tA)$$

converges, so the function

$$\alpha: (a, b) \longrightarrow M_n(R);\ \alpha(t) = exp(tA),$$

is defined and differentiable with

$$\alpha'(t) = \sum_{k\geq 0} \frac{t(k-1)}{(k-1)!} Ak = \exp(tA)\, A = Aexp(tA)$$

Hence α satisfies the above differential equation with boundary condition α(0) = I. Notice also that whenever $s, t, (s + t) \in (a, b)$

$$\alpha(s + t) = \alpha(s)\alpha(t).$$

In particular, this shows that α(t) is always invertible with $\alpha(t)^{-1} = \alpha(-t)$

One solution subject to $\alpha(0) = C$ is easily seen to be $(t) = Cexp(tA)$. If β is a second such solution then $\gamma(t) = \beta(t)\, exp(-tA)$ satisfies

$$\gamma'(t) = \beta'(t)\, exp(-tA) + \beta(t) \frac{d}{dt} exp(-tA)$$

$$= \beta\, 0\, (t)\, exp(-tA) - \beta(t)\, exp(-tA)A$$

$$= \beta(t)Aexp(-tA) - \beta(t)\, exp(-tA)A$$

$$= O.$$

Hence $\gamma(t)$ is a constant function with $\gamma(t) = \gamma(0) = C$. Thus$\beta(t) = Cexp(tA)$, and this is the unique solution subject to$\beta(0) = C$. If C is invertible so is $Cexp(tA)$ for all t.

## 3. ONE PARAMETER SUBGROUPS

Let $G \leq GL_n(k)$ be a matrix group and let $\varepsilon > 0\ or \varepsilon = \infty$.

Definition 2: A one parameter semigroup in G is a continuous function $\gamma : (-\varepsilon, \varepsilon) \longrightarrow G$ which is differentiable at 0 and satisfies

$$\gamma(s + t) = \gamma(s)\gamma(t)$$

whenever $s, t, (s + t) \in (-\varepsilon, \varepsilon)$. We will refer to the last condition as the homomorphism property.

If $\varepsilon = \infty$ then $\gamma : R \longrightarrow G$ is called a one parameter group in G or one parameter subgroup of G.

Notice that for a one parameter semigroupin $G, \gamma(0) = I$.

Proposition 3: Let $\gamma : (-\varepsilon, \varepsilon) \longrightarrow G$ be a one parameter semigroup in G. Then γ is differentiable at every $t \in (-\varepsilon, \varepsilon)$ and

$$\gamma'(t) = \gamma'(0)\gamma(t) = \gamma(t)\gamma'(0).$$

Proof. For small$h \in R$ we have

$$\gamma(h)\gamma(t) = \gamma(h + t) = \gamma(t + h) = \gamma(t)\gamma(h)$$

Hence

$$\gamma'(t) = \frac{lim}{h \to 0}\ 1\, h\, (\gamma(t + h) - \gamma(t)$$

$$\frac{lim}{h \to 0}\ 1\, h\, (\gamma(t + h) - \gamma(t)$$

$$= \gamma'(0)\gamma(t),$$

and similarly

$$\gamma'(t) = \gamma(t)\gamma'(0)$$

Proposition 4. Let $\gamma : (-\varepsilon, \varepsilon) \longrightarrow G$ be a one parameter semigroup in G. Then there is a unique extension to a one parameter group $\bar{\gamma}: R \longrightarrow G$ in G, i.e., such that for all$t \in (-\varepsilon, \varepsilon), \bar{\gamma}e(t) = \gamma(t)$.

Proof.Let $t \in R$. Then for a large enough natural number $m, t/m \in (-\varepsilon, \varepsilon)$. Hence

$\gamma(t/m), \gamma(t/m)^{m} \in G.$

Similarly, for a second such natural number $n$,

$\gamma(t/n), \gamma(t/n)^{n} \in G.$

Then since $mn \geq m, n$ we have $t/mn \in (-\varepsilon, \varepsilon)$ and

$\gamma(t/n)^n = \gamma(mt/mn)^n$

$= \gamma(t/mn)^{mn}$

$= \gamma(nt/mn)^{m}$

$= \gamma(t/m).^{m}$

So $\gamma(t/n)^{n} = \gamma(t/m)^m$ showing we get a well defined element of $G$ for every true number $t$. This describes a function

$\bar{\gamma}: R \longrightarrow G;\ \bar{\gamma}(t) = \bar{\gamma}(t/n)^n$ for large n.

We are able to today establish the type of all one parameter groups in G

Theorem 5. Let γ : R −→ G be a one parameter group in G. Then it has the form

$\gamma(t) = exp(tA)$

for some $A \in M_n(k)$.

Proof.Let $A = \gamma'(0)$. By Proposition this means that γ satisfies the differential equation

$\gamma'(t) = A, \gamma(0) = I.$

## 4. SOME LIE ALGEBRAS OF MATRIX GROUPS

The Lie algebras of $GL_n(R)$and $GL_n(C)$. Let us start with the matrix group $GL_n(R) \subseteq M_n(R)$. For $A \in M_n(R)$ and ε > 0 there is a differentiable curve

$\alpha: (-\varepsilon, \varepsilon) \longrightarrow M_n(R);\ \alpha(t) = I + tA.$

For $t \neq 0$, the roots of the equation $det(t^{-1}I + A) = 0$ are of the form $t = -1/\lambda$ where λ is a non-zero eigenvalue of A. Hence if

$$\varepsilon < min\left\{\frac{1}{|\lambda|} : \lambda anon - zeroeigenvalueof A\right\}$$

then $im\alpha \subseteq GL_n(R)$, so we might as well view α a function $\alpha: (-\varepsilon, \varepsilon) \dashrightarrow GL_n(R)$. Calculating the derivative we find that $\alpha'(t) = A$, hence$\alpha'(0) = A$. This shows that $A \in T_I GL_n(R)$. Since $A \in M_n(R)$ was arbitrary, we have

$$\begin{cases} \mathrm{gl_n(R) = T_I\, GL_n(R) = M_n(R)} \\ \quad \mathrm{dim\, GL_n(R) = n_2} \ . \end{cases} \quad (2)$$

$$\begin{cases} \mathrm{gl_n(C) = T_I\, GL_n(C) = M_n(C)}, \\ \quad \mathrm{dim_C\, GL_n(C) = n_2} \\ \quad \mathrm{dim\, GL_n(C) = 2n_2} \end{cases} \quad (3)$$

For $SL_n(R) \leq GL_n(R)$, suppose that $\alpha: (a,b) \dashrightarrow SL_n(R)$ is a curve lying in $SL_n(R)$ and satisfying $\alpha(0) = I$. For $t \in (a,b)$ we have$det\alpha(t) = 1$, so

$$\frac{d(det\alpha(t))}{dt} = 0.$$

Lemma . We have

$$\frac{d(det\alpha(t))}{dt\,|t=0} = tr\alpha'(0)$$

Proof. Recall that for $A \in M_n(k)$,

$$trA = \sum_{i=1}^{n} Aii$$

It is easy to verify that the operation $\partial = \frac{d}{dt\,|t=0}$ on functions has the derivation property

$$\partial(\gamma_1\gamma_2) = (\partial\gamma_1)\gamma_2(0) + \gamma_1(0)\partial\gamma_2. \quad (4)$$

Put $a_{ij} = \alpha(t)_{ij}$ and notice that when $t = 0, a$

$a_{ij} = \delta_{ij}$ .

## 5. CONCLUSION

It was during the period when there was a British rule on important region of world. Many statistical figures which arrived from various corners were to be systematically placed and meaningful deduction was anticipated by the authorities. Along with this a mathematician Cayley also devoted the majority of the work time of his in the improvement of the topic. It, as they described a matrix means "Something that presents the time or maybe location from which another thing originates or perhaps develops". As written in the first note, it's, at a stage onwards, hard to distinguish between linear algebra and Matrix theory

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