CONTRIBUTION OF GEOMETRY IN THE DEVELOPMENT OF SCIENCE

Dr. Krishna Nandan Prasad

Dept Of Mathematics

P.B.S. College, Banka

Abstract:

Mathematics and science are interrelated since time immemorial. As in science, the early history of mathematics is sketchy. We know that the lunar and solar cycles were counted by the Babylonians and Egyptians in an organized fashion. Early Indian mathematicians are credited with many astronomical observations as well as the beginning of algebra. The use of decimals and numbers with 9 figures and a zero are also attributed to the Indians. Their work spread to the Arabic nations, where the term Arabic numbers first emerged. The Chinese are credited with the inventions of spills and abacus, which are both counting machines. Multiplications tables were used at least from the 6th century BC. Chinese mathematics was used for the solutions of practical problems in engineering and business. The Greeks' conception of numbers as the elements of all things and of the heavens made Geometrical relationships a respected field of study. Since the existence of the earth geometry played a significant role in the development of all branches of science.

Key Words: Geometry, Relationship, Development, Science, Manifolds, Astronomy.

Introduction:

The Geometry has a long history of interaction with other fields of science and technology. This interaction provides tools and insights to help those other fields advance; at the same time, the efforts of those fields to push research frontiers routinely raise new challenges for the Geometry themselves. One way of evaluating the state of the Geometry is to examine the richness of this interplay[1]. A compelling illustration of how much other fields rely on the Geometry arises from examining those fields' own assessments of promising directions and identifying the directions that are dependent on parallel progress in the Geometrical sciences. Though every field of science is benefitted by geometry however, some of the most important fields are detailed here.

Geology and **Astronomy**:

The notion of space is central to the Geometrical sciences, to the physical sciences, and to engineering and recent technology. There are entire branches of theoretical mathematics devoted to studying spaces, with different branches focusing on different aspects of spaces or on spaces endowed with different characteristics or structures. By contrast, in geometry one studies spaces in which, first of all, one can differentiate, leading to notions such as tangent vectors, and, second, for which the notion of lengths and angles of tangent vectors are defined[2]. These concepts were first introduced by Riemann in the 1860s in his thesis "The hypotheses that underlie geometry," and the resulting structure is called a Riemannian metric. Intuitively, one can imagine that to a topologist spaces are made out of rubber or a substance like taffy, while to a geometer they are made out of steel. Although we have no direct visual representation of spaces of higher dimension, they exist as Geometrical objects on the same footing as lower-dimensional spaces that we can directly see, and this extension from physical space has proved very useful[3]. Topological and geometric spaces are central objects in the Geometrical sciences. They are also ubiquitous in the physical sciences, in computing, and in engineering, where they are the context in which problems are precisely formulated and results are expressed.

Geology and Computer Science:

Science, engineering and society today are the building of Geometrical models to represent complex processes[4]. For example, aircraft and automotive manufacturers routinely use Geometrical representations of their vehicles as surrogates for building physical prototypes during vehicle design, relying instead on computer simulations that are based on those Geometrical models. The economic benefit is clear. A prototype automobile that is destroyed in a crash test, for example, can cost $300,000, and many such prototypes are typically needed in a testing program, whereas a computer model of the automobile that can be virtually crashed under many varying conditions can often be developed at a fraction of the cost[5]. The Geometrical modeling and computational science that underlies the development of such simulators of processes has seen amazing advances over the last two decades, and improvements continue to be made.

Geometry and Behavioural **Science:**

The emergence of online social networks is changing behavior in many contexts, allowing decentralized interaction among larger groups and fewer geographic constraints. The structure and complexity of these networks have grown rapidly in recent years. At the same time, online networks are collecting social data at unprecedented scale and resolution, making social networks visible that in the past could only be explored via in-depth surveys[6]. Today, millions of people leave digital traces of their personal social networks in the form of text messages on mobile phones, updates on Facebook or Twitter, and so on. The Geometrical analysis of networks is one of the great success stories of applying the Geometry to an engineered system, back to the days when AT&T networks were designed and operated based on graph theory, probability, statistics, discrete mathematics, and optimization[7]. However, since the rise of the Internet and social networks, the underlying assumptions in the analysis of networks have changed dramatically. The abundance of such social network data, and the increasing complexity of social networks, is changing the face of research on social networks. These changes present both opportunities and challenges for Geometrical and statistical modeling[8].

Geology and Advance Biology:

Knowing the shape of a protein is an essential step in understanding its biological function. Nobel winner, biologist Christian Anfinsen showed that an unfolded protein could refold spontaneously to its original biologically active conformation[9]. This observation led to the famous conjecture that the one-dimensional sequence of amino acids of a protein uniquely determines the protein's three-dimensional structure[10]. That in turn led to the almost 40-year effort of quantitative scientists in searching for computational strategies and algorithms for solving the so-called "protein-folding problem," which is to predict a protein's three-dimensional structure from its primary sequence information. Some sub problems include how a native structure results from the interatomic forces of the sequence of amino acids and how a protein can fold so fast[11]. Although the protein-folding conjecture has been shown to be incorrect for a certain class of proteins, for example, sometimes enzymes called "chaperones" are needed to assist in the folding of a protein, scientists have observed that more than 70 percent of the proteins in nature still fold spontaneously, each into its unique three-dimensional shape. Conceptually, the protein-folding problem is straightforward. Given the positions of all atoms in a protein, one would calculate the potential energy of the structure and then find a configuration that minimizes that energy[12]. However, such a goal is technically difficult to achieve owing to the extreme complexity of the means by which the energy depends on the structure. A more attractive strategy, termed "molecular dynamics," has a clear physical basis.

Geometry and Physics:

There is a long history of interactions of mathematics, and in particular geometry, with theoretical physics. At one point in the mid-nineteenth century the fields were one and the same[13]. For example, Dirichlet's argument for the existence of harmonic functions on the disk with given boundary values on the circle was an appeal to physical intuition about electrostatics. One model for the interaction is seen in both the final formulations of quantum mechanics and of general relativity. By the late 1920s, after many false starts and partial formulations, quantum mechanics was finally formulated in terms of Hilbert spaces and operators acting on these spaces[14]. These Geometrical objects had been introduced by Hilbert in the 1880s for purely Geometrical reasons having nothing to do with quantum mechanics, which had not even been conceived of then. It is interesting to note, though, that Hilbert called the decomposition of his operators the "spectral decomposition" because it reminded him of the spectrum of various atoms, something that was mysterious at the time and finally explained by quantum mechanics. Einstein struggled for many years to formulate general relativity without finding the appropriate Geometrical context. Finally, he learned of Riemann's work on what is now called Riemannian geometry. Interactions such as these led the physicist Eugene Wigner to wonder what accounts for the unreasonable effectiveness of mathematics in physics[15].

A more recent version of the same basic pattern is the Yang-Mills theory. Here again the physicists were struggling to develop a Geometrical framework to handle the physical concepts. Much of the recent history of quantum field theory has turned this model of interaction on its head. When quantum field theory was introduced in the 1940s and 1950s there was no appropriate Geometrical context[16]. Nevertheless, physicists were able to develop the art of dealing with these objects, at least in special cases. The situation became even worse with the advent of string theory, where the appropriate Geometrical formulation seems even more remote. But the fact that the Geometrical context for these theories did not exist and has not yet been developed is only part of the way that the current interactions between mathematics and physics differ from previous ones. As physicists develop and explore these theories, for which no rigorous Geometrical formulation is known, they have increasingly used ever more sophisticated geometric and topological structures in their theories[17]. As physicists explore these theories they come across the questions and statements about the underlying geometric and topological objects in terms of which the theories are defined. Some of these statements are well-known Geometrical results, but many turn out to be completely new types of Geometrical statements[18].

Geometry and Statistics:

It is a new age of statistical inference, where technologies now produce high-dimensional data sets, often with huge numbers of measurements on each of a comparatively small number of experimental units. Examples include gene expression microarrays monitoring the expression levels of tens of thousands of genes at the same time and functional magnetic resonance imaging machines monitoring the blood flow in various parts of the brain. The breathtaking increases in data-acquisition capabilities are such that millions of parallel data sets are routinely produced, each with its own estimation or testing problem. This era of scientific mass production calls for novel developments in statistical inference, and it has inspired a tremendous burst in statistical methodology[19]. More importantly, the data flood completely transforms the set of questions that needs to be answered, and the field of statistics has, accordingly, changed profoundly in the last few decades. The shift is so strong that the subjects of contemporary research now have very little to do with general topics of discussion from the early 1990s.

Geometry and Medicine:

The Geometry contribute to medicine in a great many ways, including algorithms for medical imaging, computational methods related to drug discovery, models of tumor growth and angiogenesis, health informatics, comparative effectiveness research, epidemic modeling, and analyses to guide decision making under uncertainty. With the increasing availability of genomic sequencing and the transition toward more widespread use of electronic health record systems, we expect to see more progress toward medical interventions that are tailored to individual patients. Statisticians will be deeply involved in developing those capabilities. To illustrate just one way in which these endeavors interact, consider some Geometrical science challenges connected to the diagnosis and planning of surgery for cardiac patients. One of the grand challenges of computational medicine is how to construct an individualized model of the heart's biology, mechanics, and electrical activity based on a series of measurements taken over time[20]. Such models can then be used for diagnosis or surgical planning to lead to better patient outcomes. Two basic Geometrical tasks are fundamental to this challenge. Both are much-studied problems in applied mathematics, but they need to be carefully adapted to the task at hand. The first of these tasks is to extract cardiac motion from a time-varying sequence of three-dimensional computerized tomography (CT) or magnetic resonance imaging (MRI) patient images. To tease out the appropriate mapping, one must choose a "penalty function" for the amount of stretching that is required to bring the successive images into approximate alignment[21]. The second Geometrical task is to employ the extracted cardiac motion as observational data that drive the solution of an inverse problem. In this case, the inverse problem is to infer the parameters for the bio-electromechanical properties of the cardiac model based on the motion observed externally through the imaging.

The cardiac biophysical model draws on another area of mathematics, partial differential equations and must bring together multiple physics components: elasticity of the ventricular wall, electrophysiology, and active contraction of the myocardial fibers[22]. The full-blown setting of this problem is analogous to a "blind deconvolution" problem, in the sense that neither the model nor the source is fully known. As such, this presents enormous difficulty for the inversion solvers; as in the image registration case, it requires careful formulation and regularization, as well as large-scale computational solvers that are tolerant of ill-conditioning. Recent research is following a hybrid approach that interweaves the solution of the image registration and model determination problems.

Geometry and Chemistry:

The world of chemistry involves the creation of molecules from the atoms that occur naturally in the world. There are 92 such elements and they have complex properties - some are gases, some are liquid, and some are solid at "room" temperature. From the very beginnings of chemistry, mathematics was used to create quantitative and qualitative models for helping comprehend the world of chemistry by understanding the elements that make up molecules[23]. An atom is made up of particles which are known as protons, neutrons, and electrons. Measurement issues concerning these particles are a big part of what chemistry is about. Protons, neutrons, and electrons have mass and they have electrical charge and mass and charge can be measured. Patterns in the mass and charge of atomic particles helped chemists get insight into the nature of atoms and the molecules these atoms can form. Methane is an example of a hydrocarbon, a molecule made up of hydrogen and carbon atoms. From early on chemists have used geometrical diagrams to help them think through issues involving molecules. For example, here is the way a methane molecule might be drawn[24]. Other diagrams might give more of a sense that the methane molecule is three-dimensional, and give a more accurate feel for the molecule because the labels give information such as bond length or angles between bonds.

Graph theory in chemistry: The value of graph theory to chemistry started to become apparent in the 19th century. Work by two British mathematicians, Arthur Cayley (1821-1895) and James

Joseph Sylvester (1814 -1897), laid the ground for a long tradition of successful use of graph-theoretical ideas in chemistry. For better or worse, when the term graph is used in mathematics it can mean "graph" of a function like $y = x_2$ or a dots and lines diagram. It was Sylvester that we have to thank for introducing this second sense of the word "graph." Perhaps surprisingly, the first use of the word "graph" for a curve is relatively recent, too.

Wiener Index: Harry Wiener was a more recent pioneer in trying to develop an "index" for the graphs of chemical molecules which would help one grasp the physical properties of a molecule by computing information from the graph of the molecule. Wiener's idea was to look at the distances between the vertices of the graph that represented the molecule. If the molecule is a tree, what graph would best capture the properties of the molecule? One could use the graph of the connections between the carbon molecules only (kenogram), or one could use the full graph with carbons and hydrogen atoms (plerogram) [25].

Geometry and Industry**:**

The role of the Geometry in industry has a long history, going back to the days when the Egyptians used the 3-4-5 right triangle to restore boundaries of farms after the annual flooding of the Nile. The recent period is one of remarkable growth and diversification. Even in old-line industries, the role of the Geometry has expanded. For example, whereas the aviation industry has long used mathematics in the design of airplane wings and statistics in ensuring quality control in production, now the Geometry are also crucial to GPS and navigation systems, to simulating the structural soundness of a design, and to optimizing the flow of production. Instead of being used just to streamline cars and model traffic flows, the Geometry are also involved in the latest developments, such as design of automated vehicle detection and avoidance systems that may one day lead to automated driving[26]. Whereas statistics has long been a key element of medical trials, now the Geometry are involved in drug design and in modeling new ways for drugs to be delivered to tumors, and they will be essential in making inferences in circumstances that do not allow double-blind, randomized clinical trials.

The “search” industry relies on ideas from the Geometry to make the Internet’s vast resources of information searchable. The social networking industry makes use of graph theory and machine learning. The animation and computer game industry makes use of techniques as diverse as differential geometry and partial differential equations[27]. The biotech industry heavily uses the Geometry in modeling the action of drugs, searching genomes for genes relevant to human disease or relevant to bioengineered organisms, and discovering new drugs and understanding how they might act. The imaging industry uses ideas from differential geometry and signals processing to procure minimally invasive medical and industrial images and, within medicine, adds methods from inverse problems to design targeted radiation therapies and is moving to incorporate the new field of computational anatomy to enable remote surgery. The online advertising industry uses ideas from game theory and discrete mathematics to price and bid on online ads and methods from statistics and machine learning to decide how to target those ads.

The Geometry are now present in almost every industry, and the range of Geometry being used would have been unimaginable a generation ago.

Geometry and National Security**:**

National security is another area that relies heavily on the Geometrical sciences. The National Security Agency (NSA), employs roughly 1,000 Geometrical scientists, although the number might be half that or twice that depending on how one defines such scientists. They include people with backgrounds in core and applied mathematics, probability, and statistics, but people with computer science backgrounds are not included in that count[28]. NSA hires some 40-50 mathematicians per year, and it tries to keep that rate steady so that the Geometry community knows it can depend on that level of hiring. While cryptology is explicitly dependent on mathematics, many other links exist between the Geometry and national security.

One example is analysis of networks, which is very important for national defense. Another is scientific computing. Because national defense relies in part on design and manufacturing of cutting-edge equipment, it also relies on the Geometry through their contributions to advanced engineering and manufacturing. The level of sophistication of these tools has ratcheted steadily upward. The Geometry are also essential to logistics, simulations used for training and testing, war-gaming, image and signal analysis, control of satellites and aircraft, and test and evaluation of new equipment. New devices, on and off the battlefield, have come on stream and furnish dizzying quantities of data, more than can currently be analyzed. Devising ways to automate the analysis of these data is a highly Geometrical and statistical challenge[29]. A very serious threat that did not exist in earlier days is that crucial networks are constantly subject to sophisticated attacks by thieves, mischief-makers, and hackers of unknown origin. Adaptive techniques based on the Geometry are essential for reliable detection and prevention of such attacks, which grow in sophistication to elude every new strategy for preventing them.

The Department of Defense has adopted seven current priority areas for science and technology investment to benefit national security.

1. Data to decisions: Science and applications to reduce the cycle time and manpower requirements for analysis and use of large data sets.

2. Engineered resilient systems: Engineering concepts, science, and design tools to protect against malicious compromise of weapon systems and to develop agile manufacturing for trusted and assured defense systems.

3. Cyber science and technology: Science and technology for efficient, effective cyber capabilities across the spectrum of joint operations.

4. Electronic warfare/electronic protection: New concepts and technology to protect systems and extend capabilities across the electro-magnetic spectrum.

5. Countering weapons of mass destruction (WMD): Advances in DOD's ability to locate, secure, monitor, tag, track, interdict, eliminate, and attribute WMD weapons and materials.

6. Autonomy: Science and technology to achieve autonomous systems that reliably and safely accomplish complex tasks in all environments.

7. Human systems: Science and technology to enhance human-machine interfaces, increasing productivity and effectiveness across a broad range of missions.

While the Geometry is clearly of importance to these priority areas, they also have key roles to play in support of all of the others[30]. Advances in the Geometry that allow simulation-based design, testing, and control of complex systems are essential for creating resilient systems[31]. Improved methods of signal analysis and processing, such as faster algorithms and more sensitive schemes for pattern recognition, are needed to advance electronic warfare and protection[32]. Rapidly developing tools for analyzing social networks, which are based on novel methods of statistical analysis of networks, are being applied in order to sure the safty[33].

Manifolds and Development of Science:

In mathematics, a manifold is a topological space that locally resembles Euclidean space near each point. More precisely, an *n*-dimensional manifold, or *n-manifold* for short, is a topological space with the property that each point has a neighborhood that is homeomorphic to the Euclidean space of dimension *n*. One-dimensional manifolds include lines and circles, but not figure eights. Two-dimensional manifolds are also called surfaces. Examples include the plane, the sphere, and the torus, which can all be embedded (formed without self-intersections) in three dimensional real space, but also the Klein bottle and real projective plane, which will always self-intersect when immersed in three-dimensional real space[34]. The concept of a manifold is central to many parts of geometry and modern mathematical physics because it allows complicated structures to be described and understood in terms of the simpler local topological properties of Euclidean space. Manifolds naturally arise as solution sets of systems of equations and as graphs of functions. It can be equipped with additional structure. One important class of manifolds is the class of differentiable manifolds; this differentiable structure allows calculus to be done on manifolds[35]. A Riemannian metric on a manifold allows distances and angles to be measured. Simplistic manifolds serve as the phase spaces in the Hamiltonian formalism of classical mechanics, while four-dimensional Lorentzian manifolds model space time in general relativity. Different types of manifolds are:

Circle: After a line, the circle is the simplest example of a topological manifold. Topology ignores bending, so a small piece of a circle is treated exactly the same as a small piece of a line. Consider, for instance, the top part of the unit circle, $x^2 + y^2 = 1$, where the *y*-coordinate is positive. Any point of this arc can be uniquely described by its x-coordinate. So, projection onto the first coordinate is a continuous, and invertible, mapping from the upper arc to the open interval (−1, 1):

Sphere: The sphere is an example of a surface. The unit sphere of implicit equation

$$x^2 + y^2 + z^2 - 1 = 0$$

may be covered by an atlas of six charts: the plane $z = 0$ divides the sphere into two half spheres ($z > 0$ and $z < 0$), which may both be mapped on the disc $x^2 + y^2 < 1$ by the projection on the xy plane of coordinates. This provides two charts; the four other charts are provided by a similar construction with the two other coordinate planes. As for the circle, one may define one chart that covers the whole sphere excluding one point. Thus two charts are sufficient, but the sphere cannot be covered by a single chart. This example is historically significant, as it has motivated the terminology; it became apparent that the whole surface of the Earth cannot have a plane representation consisting of a single map (also called "chart", see nautical chart), and therefore one needs atlases for covering the whole Earth surface.

Enriched circle: Viewed using calculus, the circle transition function T is simply a function between open intervals, which gives a meaning to the statement that T is differentiable. The transition map T, and all the others, are differentiable on (0, 1); therefore, with this atlas the circle is a *differentiable manifold*. It is also *smooth* and *analytic* because the transition functions have these properties as well. Other circle properties allow it to meet the requirements of more specialized types of manifold. For example, the circle has a notion of distance between two points, the arc-length between the points; hence it is a *Riemannian manifold*.

Other curves: Manifolds need not be connected (all in "one piece"); an example is a pair of separate circles. Manifolds need not be closed; thus a line segment without its end points is a manifold. And they are never countable, unless the dimension of the manifold is 0. Putting these freedoms together, other examples of manifolds are a parabola, a hyperbola (two open, infinite pieces), and the locus of points on a cubic curve $y^2 = x^3 - x$ (a closed loop piece and an open, infinite piece). However, excluded are examples like two touching circles that share a point to form a figure-8; at the shared point a satisfactory chart cannot be created. Even with the bending allowed by topology, the vicinity of the shared point looks like a "+", not a line. A "+" is not homeomorphic to a closed interval (line segment), since deleting the center point from the "+" gives a space with four components (i.e. pieces), whereas deleting a point from a closed interval gives a space with at most two pieces; topological operations always preserve the number of pieces.

Manifolds have applications in computer-graphics and augmented-reality given the need to associate pictures (texture) to coordinates (e.g. CT scans). In an augmented reality setting, a picture (tangent plane) can be seen as something associated with a coordinate and by using sensors for detecting movements and rotation one can have knowledge of how the picture is oriented and placed in space. Therefore, without the use of these manifolds the development of science and technology is incomplete.

Conclusion:

The opening years of the twenty-first century have been remarkable ones for the Geometrical sciences. Major breakthroughs have been made on fundamental research problems. The ongoing trend for the Geometry to play an essential role in the field of physical and biological sciences, engineering, medicine, economics, finance, and social science has expanded dramatically. The Geometry has become integral to many emerging industries, and the increasing technological sophistication of our armed forces has made the Geometry central to national defense. A striking feature of this expansion in the uses of the Geometry has been a parallel expansion in the kinds of Geometrical science ideas that are being used. There is a need to build on and solidify these gains. The contribution of geometry in the development of science is a milestone. Geometry is the backbone of the present well developed area of science.

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